LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

AI 501 Mathematics for Artificial Intelligence
Quiz 08 Solutions

Name: _____

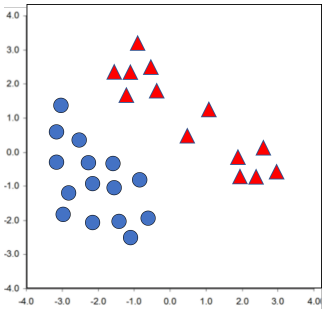
Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (5 marks)

From the given options, select whichever is correct. There may be more than one correct option.

1. Given the data shown below for the classification problem, which kernel can we use in SVM to separate the two classes?



   (a) Linear

   (b) Gaussian

   (c) Polynomial

   (d) None of the above

2. Which of the following best describes a "Hard Margin SVM"?
   (a) It allows misclassifications to occur on the training data.
   (b) It handles noisy data effectively using slack variables.
   (c) It assumes the data is perfectly linearly separable.
   (d) It maps data into higher dimensions using kernel tricks.
3. What is the purpose of slack variables in a Soft Margin SVM?
   (a) To make the optimization problem infeasible.
   (b) To allow some misclassifications for noisy data.
   (c) To reduce the margin between two classes.
   (d) To increase the number of support vectors.
4. What happens when you increase the value of the parameter C in Soft Margin SVM?
   (a) The model allows more misclassifications.
   (b) The margin becomes wider.
   (c) The model becomes more sensitive to outliers.
   (d) The kernel computation becomes faster.
5. What is "Hinge Loss" in the context of Soft Margin SVM?
   (a) A regularization term for reducing model complexity.
   (b) A loss function that penalizes misclassified points.
   (c) A function for computing the inner product.
   (d) A loss function for minimizing squared errors.

Solution:

1. (a), (b), and (c).
2. (c).
3. (b).
4. (c).
5. (b).

Question 2 (5 marks)

Point	x_1	x_2	**Label** y
A	2	3	+1
B	3	3	+1
C	1	1	-1
D	2	1	-1

1. Plot the points A, B, C, and D on a 2D graph. Identify the class labels for each point and visualize the dataset.
2. Based on visual inspection, determine the approximate separating hyperplane.
   Provide the equation of a line (not necessarily SVM) that separates the two classes.
3. Formulate (only) the hard margin SVM optimization problem. Define the variables and Write the four constraints (one for each point in the dataset).

Solution: The data is linearly separable and a possible separating hyperplane is:

$$x_1 + x_2 = 4$$

This line separates the two classes, where:

- Points A and B (Class $+1$) are above the line. - Points C and D (Class -1) are below the line.

Assume that the data is linearly separable. The hard margin SVM optimization problem is given as:

$$\min_{\mathbf{w},\theta} \quad \frac{1}{2}\|\mathbf{w}\|^2$$
$$\text{subject to} \quad y_i(\mathbf{w}^T\mathbf{x}_i - \theta) \geq 1, \quad \forall i = 1, \ldots, N$$

For the given dataset, the constraints are:

$$\begin{aligned}
\text{For A } (2,\,3),\ y = +1: &\quad w_1 2 + w_2 3 - \theta \geq 1 \\
\text{For B } (3,\,3),\ y = +1: &\quad w_1 3 + w_2 3 - \theta \geq 1 \\
\text{For C } (1,\,1),\ y = -1: &\quad w_1 1 + w_2 1 - \theta \leq -1 \\
\text{For D } (2,\,1),\ y = -1: &\quad w_1 2 + w_2 1 - \theta \leq -1
\end{aligned}$$

The objective is to find $\mathbf{w} = (w_1, w_2)$ and θ that satisfy these constraints while minimizing $\frac{1}{2}\|\mathbf{w}\|^2$.