Question 01

Which of the following is not true for standard unit vectors $e_1, e_2, \ldots e_n \in \mathbf{R}^n$

(A) $\langle e_i, e_j \rangle = 1, \quad i \neq j$

(B) $\langle e_i, e_j \rangle = 0, \quad i \neq j$

(C) $\langle e_i, e_i \rangle = 1, \quad i = 1, 2, \ldots, n$

(D) $\langle e_i, e_j \rangle = \langle e_j, e_i \rangle$

Question 02

Given vectors $a_1, a_2, \ldots a_m \in \mathbf{R}^n$ and scalers $\beta_1, \beta_2, \ldots, \beta_m \in \mathbf{R}$, the linear combination of the vectors defined as

$$\beta_1 a_1 + \beta_2 a_2 + \ldots + \beta_m a_m.$$

Which of the following conditions must hold for which the linear combination is referred to as a convex combination of vectors.

1. $\beta_i \geq 0, \quad i = 1, 2, \ldots, m$
2. $\beta_i \leq 1, \quad i = 1, 2, \ldots, m$
3. $\beta_1 + \beta_2 + \ldots + \beta_m = 1$

(A) Condition 1 only

(B) All Conditions

(C) Conditions 1 and 3 only

(D) Conditions 2 and 3 only

Question 03

Compute $\langle a, b \rangle$ for the following two vectors

$$a = \begin{bmatrix} 1 \\ 0 \\ -4 \\ 3 \end{bmatrix} \qquad b = \begin{bmatrix} -3 \\ 100 \\ 0 \\ 1 \end{bmatrix}$$

(A) -6

(B) 0

(C) 6

(D) 3

Question 04

For what value of β, the scaling of the vector by β expands the vector irrespective of the direction?

(A) $\beta = -2$

(B) $\beta = 1$

(C) $\beta = -0.5$

(D) $\beta = 0.5$

Question 05

We want to transform a boolean vector x that only contains 1 or 0 to alternative boolean vector y that only contains on 1 and -1 such that 1 is mapped to 1 and 0 is mapped to -1. Which of the following linear expression yields this linear transformation?

(A) $y = x - 1$

(B) $y = 2x + 0.5$

(C) $y = 2x - 1$

(D) $y = \frac{1}{2}x - 1$