LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Methematical Foundations of Machine Learning and Data Science
Quiz 02 Solution

Total Marks: 15 **Time Duration:** 45 minutes

Question 1 (2 marks)

Compute is the ℓ_2 norm of the vector $u = (5, -2, 1, 6)$?
Solution:
$\sqrt{5^2 + (-2)^2 + 1^2 + 6^2} = 8.124$

Question 2 (2 marks)

Express the RMS value of a n-vector u using inner-product notation?
Solution:
$\sqrt{< u(1/n), u >}$ when expanded gives: $\sqrt{\frac{(u^T u)}{n}} = \frac{||u||}{\sqrt{n}}$ which is equal to the RMS value.

Question 3 (4 marks)

How would you interpret the correlation in the data if the correlation coefficient, ρ, between two features has a value:

- $\rho = 0$
- $0 < \rho < 1$
- $-1 < \rho < 0$

Solution:
$\rho = 0$: No correlation
$0 < \rho < 1$: Positive correlation, maximum as it approaches 1, minimum as it approaches 0.
$-1 < \rho < 0$: Negative correlation, maximum as it approaches -1, minimum as it approaches 0.

Question 4 (3 marks)

Show that adding a constant to mean of a vector has no effect on its standard deviation.
Solution:
Recall that the standard deviation of a vector $\mathbf{x}$ is given by:

$$\mathbf{std}(x) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n}}$$

Let us add a constant α to the vector $\mathbf{x}$. Since α is added to all components of $\mathbf{x}$, the mean $\bar{x}$ shifts by α too. Hence, the new standard deviation is:

$$\mathbf{std}(x) = \sqrt{\frac{\sum_{i=1}^{n}((x_i + \alpha) - (\bar{x} + \alpha))}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})}{n}}$$

Hence, proved that adding any constant α to a vector has no effect on its standard deviation.

Question 5 (4 marks)

Convert the vector $u = (4, 2, -1)$ into a standardized vector with 0 mean and unity standard deviation.
Solution:
$\mathbf{avg}(x) = \frac{4+2-1}{3} = \frac{5}{3}$
$\mathbf{std}(x) = \frac{\sqrt{38}}{3}$
Standardized vector $z = \frac{x - \mathbf{avg}(x)}{\mathbf{std}(X)} = \frac{1}{\sqrt{38}}(7, 1, -8)$