**EE212 Mathematical Foundations of Machine Learning and Data Science**
**Quiz 03**

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**Total Marks:** 10                                                  **Time Duration:** 45 minutes

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**Question 1**  (2 marks)

Which of the following vectors span a line (mention all of them):
$u = (2, 1, 2)$, $v = (1, 0, 3)$, $w = (-1, -0.5, -1)$, $x = (6, 3, 6)$

**Question 2**  (3 marks)

A hardware shop does it's inventory everyday. The shop owner is very particular about the daily reports. He gives the following instructions to his employees:

"On Monday, I want to know the inventory count of that day. On all other days I want the inventory count as a sum of the count on that day and the day before."

Suppose the inventory count for a pack of nails is stored in a 7-vector $v$ such that:

$$v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$$

where $v_1$ corresponds to inventory count on Monday, $v_2$ corresponds to count on Tuesday and so on. Find a matrix $A$ such that:

$$Av = y$$

where

$$y = (y_1, y_2, y_3, y_4, y_5, y_6, y_7)$$

in which $y_1$ is the inventory count reported to the owner on Monday, $y_2$ is the count on Tuesday and so on.

**Question 3**  (3 marks)

Assuming the block matrix

$$U = \begin{bmatrix} I & A^T \\ A & 0 \end{bmatrix}$$

makes sense. Then which of the following are ***always*** true? Provide brief justification.

(a) $U$ is a square matrix

(b) $A$ is a square or wide matrix

(c) The zero submatrix is square

**Question 4**  (2 marks)

The vector space $W$ is defined as follows:

$$W = \{(x, 0, y, 0)|x, y \in \mathbf{R}\}$$

(a) Provide a basis for $W$

(b) Find $\dim(W)$