LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations of Machine Learning and Data Science
Quiz 03 Solution

Total Marks: 10 **Time Duration:** 45 minutes

Question 1 (2 marks)

Which of the following vectors span a line (mention all of them):
$u = (2, 1, 2)$, $v = (1, 0, 3)$, $w = (-1, -0.5, -1)$, $x = (6, 3, 6)$

Solution:
u, w and x are all collinear, hence these three span a line.

Question 2 (3 marks)

A hardware shop does it's inventory everyday. The shop owner is very particular about the daily reports. He gives the following instructions to his employees:

"On Monday, I want to know the inventory count of that day. On all other days I want the inventory count as a sum of the count on that day and the day before."

Suppose the inventory count for a pack of nails is stored in a 7-vector v such that:

$$v = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$$

where v_1 corresponds to inventory count on Monday, v_2 corresponds to count on Tuesday and so on. Find a matrix A such that:

$$Av = y$$

where

$$y = (y_1, y_2, y_3, y_4, y_5, y_6, y_7)$$

in which y_1 is the inventory count reported to the owner on Monday, y_2 is the count on Tuesday and so on.

Solution:
We want $y = (v_1, v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4 + v_5, v_5 + v_6, v_6 + v_7)$
Hence,

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Question 3 (3 marks)

Assuming the block matrix

$$U = \begin{bmatrix} I & A^T \\ A & 0 \end{bmatrix}$$

makes sense. Then which of the following are ***always*** true? Provide brief justification.

(a) U is a square matrix

(b) A is a square or wide matrix

(c) The zero submatrix is square

Solution:

(a) Always true; U always has dimensions $(m+n) \times (m+n)$, where m and n can be equal or distinct.

(b) Not always true, since A can be tall too.

(c) Always true; the zero submatrix has dimensions $m \times m$, where m can be any number ≥ 1.

Question 4 (2 marks)

The vector space W is defined as follows:

$$W = \{(x, 0, y, 0) | x, y \in \mathbf{R}\}$$

(a) Provide a basis for W

(b) Find $\dim(W)$

Solution:

(a) $\{(1, 0, 0, 0), (0, 0, 1, 0)\}$

(b) $\dim(W) = 2$