

Total Marks: 15

Time Duration: 45 minutes

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**Question 1** (3 marks)

The following set of equations can be modeled as  $Ax = b$ .

$$\begin{aligned}3x + 5y &= 1 \\x - 2y &= 3 \\2x + 5y - z &= 7 \\3y - 4z &= 2\end{aligned}$$

- (a) Write down the matrix  $A$ .
- (b) Give an *expression* for  $x$  in terms of  $A$  and  $b$ .

**Question 2** (5 marks)

Consider the following matrix:

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

- (a) Compute the gram matrix.
- (b) Determine if the gram matrix is invertible or not.  
*Hint:* You don't need to compute the determinant of the  $3 \times 3$  matrix.
- (c) What role do the eigen-values and eigen-vectors of gram matrix play in the SVD of  $A$ ?

**Question 3** (4 marks)

A 3 by 3 matrix  $B$  is known to have eigenvalues 0, 1, 2. This information is enough to find three of these (give the answers where possible):

- (a) the determinant of  $B^T B$
- (b) the eigenvalues of  $(B^2 + I)^{-1}$

**Question 4** (3 marks)

$A$  is an  $m \times n$  matrix of rank  $r$ . Suppose there are right sides  $b$  for which  $Ax = b$  has *no solution*.

- (a) What are all the inequalities ( $<$  or  $\leq$ ) that must be true between  $m$ ,  $n$  and  $r$ ?
- (b) How do you know that  $A^T y = 0$  has solutions other than  $y = 0$ ?