

**LAHORE UNIVERSITY OF MANAGEMENT SCIENCES**  
**Department of Electrical Engineering**

**EE212 Mathematical Foundations of Machine Learning and Data Science**  
**Quiz 05**

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**Total Marks:** 10

**Time Duration:** 45 minutes

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**Question 1** (3 marks)

$A$  has 4 eigen values, 2 of them are non-zero.

- (a) What is the dimension of null space of  $A$ ?
- (b) Using EVD, show that the dimension of the null space of  $A^2$  is the same as that of null space of  $A$ .

**Question 2** (2 marks)

Define rank of a matrix. How do we determine the rank of a matrix using its singular value decomposition?

**Question 3** (5 marks)

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

The above matrix can be represented as  $A = U\Sigma V^T$ , where  $U$ ,  $\Sigma$  and  $V$  are all  $2 \times 2$  matrices. This matrix has singular values  $\sigma_1 = 2\sqrt{2}$  and  $\sigma_2 = \sqrt{2}$  and corresponding to the first singular value, the right singular vector is:

$$v_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

- (a) Find  $v_2$  which is the second normalized right singular vector.
- (b) Find  $U$ .