LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations of Machine Learning and Data Science
Quiz 06 Solution

Total Marks: 10 **Time Duration:** 45 minutes

Question 1 (2 marks)

You have developed a regression model for predicting a scalar outcome y from a feature vector x of dimension 20, using a collection of $N = 600$ data points. The mean of the outcome variable y across the given data is 1.85, and its standard deviation is 0.32. For training-test split, you use 5-fold cross-validation that splits the data into 5 parts or folds. If you obtain the following RMS test errors (based on forming a model based on the data excluding fold i, and testing it on fold i).
How would you expect your model to do on new, unseen (but similar) data? Provide a brief explanation.

Fold Excluded	RMS test error
1	0.13
2	0.11
3	0.09
4	0.13
5	0.12

Solution: We observe that the RMS test errors are similar across the folds, so it is reasonable to expect the model to generalize well on similar data, with RMS error about 0.12 or so. We cannot guarantee this prediction; it will hold only to the extent that the new data is similar to the given data.

Question 2 (8 marks)

Consider the data given below:

x	y
0	2
1	3
2	5
3	4
4	6

The data can be modeled to follow a linear model of $y_i = ax_i + b$, where a and b are unknown parameters.

(a) [2 marks] Model the above equation as $y = A\Theta$ and find A. Θ is a 2 x 1 matrix containing a and b, and A is generated using x.

(b) [4 marks] Evaluate a and b.

Information: The inverse of a 2 x 2 *matrix* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *is:* $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

(c) [2 marks] Mark the data points on a graph and sketch the line $y = ax + b$ on the same graph.

Solution:

(a)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} a \\ b \end{bmatrix}$$

(b) $\Theta = A^{\dagger}y$

$$\Theta = \begin{bmatrix} 30 & 10 \\ 10 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \\ 4 \\ 6 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 0.9 \\ 2.2 \end{bmatrix}$$

$a = 0.9$ and $b = 2.2$

(c)