

**LAHORE UNIVERSITY OF MANAGEMENT SCIENCES**  
**Department of Electrical Engineering**

**EE212 Mathematical Foundations of Machine Learning and Data Science**  
**Quiz 08**

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**Total Marks:** 15

**Time Duration:** 45 minutes

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**Question 1** (5 marks)

A continuous random variable  $X$  has the following PDF:

$$f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

Calculate:

- (a) [2 marks]  $E[X]$
- (b) [3 marks]  $E[X^2]$  and hence  $\text{Var}(X)$

**Question 2** (4 marks)

A continuous random variable  $X$  has the following PDF:

$$f(x) = \begin{cases} kx^3 & -2 \leq x \leq 0 \\ 0 & \text{else} \end{cases}$$

- (a) [2 marks] Find the value of  $k$
- (b) [2 marks] Find  $P(-1.5 \leq X \leq 0.5)$

**Question 3** (6 marks)

A movie theatre claims that the mean time to buy a ticket on their website is 60 seconds with a standard deviation of 30 seconds. A random sample of 36 customers attempted to buy a ticket on the website. The mean time to buy was 75 seconds, suggesting that the company's claim might be false.

- (a) [1 mark] Write down the null hypothesis  $H_0$  and alternate hypothesis  $H_a$  associated with this scenario.
- (b) [1 mark] Compute  $z$ -statistic for this scenario.
- (c) [2 marks] Determine the P-value for this scenario.
- (d) [2 marks] Determine what conclusion can be drawn from the P-value for the following significance levels:
  - i  $\alpha = 0.0050$
  - ii  $\alpha = 0.0010$

