LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations of Machine Learning and Data Science
Quiz 08 Solution

Total Marks: 15 **Time Duration:** 45 minutes

Question 1 (5 marks)

A continuous random variable X has the following PDF:

$$f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & else \end{cases}$$

Calculate:

(a) [2 marks] $\mathrm{E}[X]$

(b) [3 marks] $\mathrm{E}[X^2]$ and hence $\mathrm{Var}(X)$

Solution:

(a) $\mathrm{E}[X]{=}\int_0^1 x(2x)dx = |\frac{2}{3}x^3|_0^1 = \frac{2}{3}$

(b) $\mathrm{E}[X^2]{=}\int_0^1 x^2(2x)dx = |\frac{1}{2}x^4|_0^1 = \frac{1}{2}$
$\mathrm{Var}(X){=}\frac{1}{2} - (\frac{2}{3})^2 = \frac{1}{18}$

Question 2 (4 marks)

A continuous random variable X has the following PDF:

$$f(x) = \begin{cases} kx^3 & -2 \leq x \leq 0 \\ 0 & else \end{cases}$$

(a) [2 marks] Find the value of k

(b) [2 marks] Find $P(-1.5 \leq X \leq 0.5)$

Solution:

(a) For a valid PDF, it must integrate to 1. So:
$\int_{-2}^{0} kx^3 = 1$ which gives us $k = -\frac{1}{4}$

(b) $P(-1.5 \leq X \leq 0.5) = \int_{-1.5}^{0} f(x)dx$
$= -\frac{1}{4}\int_{-1.5}^{0} x^3 dx = \frac{81}{256} = 0.3164$

Question 3 (6 marks)

A movie theatre claims that the mean time to buy a ticket on their website is 60 seconds with a standard deviation of 30 seconds. A random sample of 36 customers attempted to buy a ticket on the website. The mean time to buy was 75 seconds, suggesting that the company's claim might be false.

(a) [1 mark] Write down the null hypothesis H_0 and alternate hypothesis H_a associated with this scenario.

(b) [1 mark] Compute z-statistic for this scenario.

(c) [2 marks] Determine the P-value for this scenario.

(d) [2 marks] Determine what conclusion can be drawn from the P-value for the following significance levels:

i $\alpha = 0.0050$

ii $\alpha = 0.0010$

Solution:

(a) $H_0 : \mu = 60$
$H_a : \mu > 60$

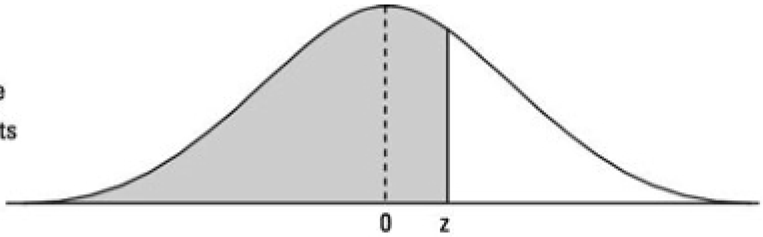
(b) $Z_{stat} = \frac{75-60}{30/\sqrt{36}} = 3$

(c) Using the z-table, we have $P(Z > Z_{stat}) = 1 - P(Z \leq Z_{stat}) = 1 - 0.9987 = 0.0013$. Hence, P-value = 0.0013.

(d) i $0.0013 < \alpha = 0.0050$: reject H_0. The company's claim might be false.

ii $0.0013 > \alpha = 0.0010$: fail to reject H_0. The company's claim might be true.

Number in the table represents $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999