LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE212 Mathematical Foundations for Machine Learning and Data Science
Quiz 02 Solutions

Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (3 marks)

Let V be a valid vector space in $\mathbb{R}^2$. Any point in $\mathbb{R}^2$ is defined as (x, y). Let W be a subset of V. Is W a valid subspace, if

(a) [2 marks] W: a set of all points in $\mathbb{R}^2$ such that
$$x^2 + y^2 \leq 1$$

> **Solution:** W is not a subspace. Closure under addition and multiplication is not satisfied.

(b) [1 mark] W: a set of all points such that
$$3x + 4y = 0$$

> **Solution:** W is a subspace. All properties of a subspace are satisfied.

Give explanations for your answers.

Question 2 (2 marks)

The following system of linear equations can be expressed in the form $Ax = y$, which type of inverse exists for A? Explain your answer in 1-2 lines.
$$3x_1 + x_2 = y_1$$
$$3x_2 = y_2$$
$$4x_1 + x_2 = y_3$$
$$-x_1 - x_2 = y_4$$

> **Solution:** Matrix A for the given system of linear equations come out to be:
> $$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \\ 4 & 1 \\ -1 & -1 \end{bmatrix}$$
> Left inverse exists because the the columns of matrix A are linearly independent and the matrix is overdetermined.

Question 3 (2 marks)

For a system of linear equations $Ax = y$, the inverse of A exists and is given by:
$$X = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

If $y = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ Give solution x for the system of linear equations.

Solution: The solution for the system of equations is $x = Xy$.

$$x = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Question 4 (3 marks)

For a wide matrix A (rows less than columns), inverse (either left or right) exists. Which of the following statements hold true for matrix A.

a. Left inverse exists.

b. Right inverse exists.

c. All rows are linearly independent.

d. All columns are linearly independent.

e. The matrix is over-determined

f. The matrix is under-determined.

Solution: b, c and f.