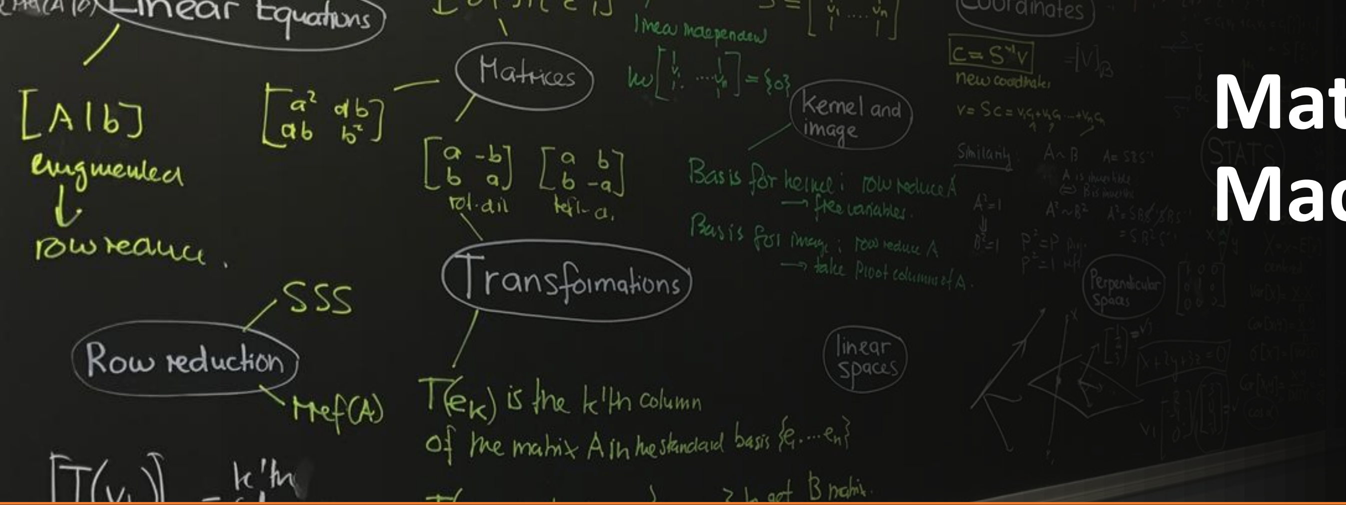

Mathematical Foundations for Machine Learning and Data Science

Analysis and Evaluation of Classifier's Performance

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Outline

- Classification Accuracy (0/1 Loss)
- TP, TN, FP and FN
- Confusion Matrix
- Sensitivity, Specificity, Precision
- F1-Score
- Multi-class Classification

Evaluation of Classification Performance

Classification Accuracy, Misclassification Rate (0/1 Loss):

$$\mathcal{L}_{0/1}(h) = \frac{1}{n}\sum_{i=1}^{n} 1 - \delta_{h(\mathbf{x_i})-y_i}$$

$$\delta_k = \begin{cases} 1, & k = 0 \\ 0 & \text{otherwise} \end{cases}$$

- For each test-point, the loss is either 0 or 1; whether the prediction is correct or incorrect.
- Averaged over n data-points, this loss is a 'Misclassification Rate'.

Interpretation:

- Misclassification Rate: Estimate of the probability that a point is incorrectly classified.
- Accuracy = 1 - Misclassification rate

Issue:

- Not meaningful when the classes are imbalanced or skewed.

Evaluation of Classification Performance

Classification Accuracy (0/1 Loss):

Example:

- Predict if a bowler will not bowl a **no-ball**?
  - Assuming 15 no-balls in an inning, a **model that says 'Yes' all the time** will have **95%** accuracy.
  - Using accuracy as performance metric, we can say that a model is very accurate, but it is not useful or valuable in fact.

Why?

- Total points: 315 (assuming other balls are legal ☺)
- No-ball label: Class 0 (4.76% are from this class)
- Not a no-ball label: Class 1 (95.24% are from this class)

Imbalanced Classes

Evaluation of Classification Performance

TP, TN, FP and FN:

- *Consider a binary classification problem.*

$$D = \{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \ldots, (\mathbf{x_n}, y_n)\} \subseteq \mathcal{X}^d \times \mathcal{Y}$$

$\mathcal{Y} = \{0, 1\}$ (Referring 0 as Negative, 1 as Positive)

y - Actual labels, Ground truth, Gold labels or Standards

We have a classifier (hypothesis function) $h(\mathbf{x}) = \hat{y}$.

$y, \hat{y}$ - Positive (1) or Negative (0)

$\hat{y}$ - True if $\hat{y} = y$, False if $\hat{y} \neq y$

Evaluation of Classification Performance

TP, TN, FP and FN:

- TP - True Positive
- TN - True Negative
- FP - False Positive
- FN - False Negative

- Number of points with $y = 1$ and are classified as $\hat{y} = 1$
- Number of points with $y = 0$ and are classified as $\hat{y} = 0$
- Number of points with $y = 0$ and are classified as $\hat{y} = 1$
- Number of points with $y = 1$ and are classified as $\hat{y} = 0$

Evaluation of Classification Performance

TP, TN, FP and FN:

Example:

- *Predict if a bowler will not bowl a* ***no-ball****?*
  - *15 no-balls in an inning (Total balls: 315)*
  - *Bowl no-ball (Class 0), Bowl regular ball (Class 1)*
  - *Model(*) predicted 10 no-balls (8 correct predictions, 2 incorrect)*

• TP - True Positive	• TP - 298
• TN - True Negative	• TN - 8
• FP - False Positive	• FP - 7
• FN - False Negative	• FN - 2

** Assume you have a model that has been observing the bowlers for the last 15 years and used these observations for learning.*

Evaluation of Classification Performance

Confusion Matrix (Contingency Table):

- (TP; TN; FP; FN); usefully summarized in a table, referred to as confusion matrix:
  - the rows correspond to predicted class ($\hat{y}$)
  - and the columns to true class (y)

		Actual Labels		
Predicted Labels		**1 (Positive)**	**0 (Negative)**	Total
	1 (Positive)	**TP**	**FP**	Predicted Total Positives
	0 (Negative)	**FN**	**TN**	Predicted Total Negatives
	Total	P= TP+FN Actual Total Positives	N= P+TN Actual Total Negatives	

Evaluation of Classification Performance

Confusion Matrix:

Example:

- Disease Detection :

Given pathology reports and scans, predict heart disease

- Yes: 1, No: 0

		Actual Labels		
		1 (Positive)	0 (Negative)	Total
Predicted Labels	1 (Positive)	TP = 100	FP = 10	110
	0 (Negative)	FN = 5	TN = 50	55
	Total	P = 105	N = 60	

Interpretation:

Out of 165 cases

- Predicted: "Yes" 110 times, and "No" 55 times
- In reality: "Yes" 105 times, and "No" 60 times

Evaluation of Classification Performance

Confusion Matrix:

Example:

- *Predict if a bowler will not bowl a **no-ball**?*

		Actual Labels		
		1 (Positive)	**0 (Negative)**	Total
Predicted Labels	**1 (Positive)**	**TP = 298**	**FP = 7**	305
	0 (Negative)	**FN = 2**	**TN = 8**	10
	Total	P = 300	N = 15	

Interpretation:

Out of 315 balls, we had 15 no-balls.

- *Model predicted 305 regular balls and 10 no-balls (8 correct predictions, 2 incorrect).*

Evaluation of Classification Performance

Confusion Matrix:

Metrics using Confusion Matrix:

	Actual Labels			
		1 (Positive)	0 (Negative)	Total
Predicted Labels	1 (Positive)	TP	FP	Predicted Total Positives
	0 (Negative)	FN	TN	Predicted Total Negatives
	Total	P= TP+FN Actual Total Positives	N= P+TN Actual Total Negatives	

- ***Accuracy***: *Overall, how frequently is the classifier correct?*

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{Total}} = \frac{\text{TP} + \text{TN}}{\text{P} + \text{N}}$$

- ***Misclassification or Error Rate***: *Overall, how frequently is it wrong?*

$$1 - \text{Accuracy} = \frac{\text{FP} + \text{FN}}{\text{Total}} = \frac{\text{FP} + \text{FN}}{\text{P} + \text{N}}$$

- ***Sensitivity or Recall or True Positive Rate (TPR)***: *How often does it predict Positive when it is actually Positive?*

$$TPR = S_e = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{\text{TP}}{\text{P}}$$

Evaluation of Classification Performance

Confusion Matrix:

Metrics using Confusion Matrix:

		Actual Labels		
Predicted Labels		1 (Positive)	0 (Negative)	Total
	1 (Positive)	TP	FP	Predicted Total Positives
	0 (Negative)	FN	TN	Predicted Total Negatives
	Total	P= TP+FN Actual Total Positives	N= P+TN Actual Total Negatives	

- ***False Positive Rate:*** *Actual Negative, how often does it predict Positive?*

$$FPR = \frac{\mathrm{FP}}{\mathrm{TN}+\mathrm{FP}} = \frac{\mathrm{FP}}{\mathrm{N}}$$

- ***Specificity or True Negative Rate (TNR):*** *When it's actually Negative, how often does it predict Negative?*

$$TNR = S_p = \frac{\mathrm{TN}}{\mathrm{TN}+\mathrm{FP}} = \frac{\mathrm{TN}}{\mathrm{N}} = 1 - FPR$$

- ***Precision:*** *When it predicts Positive, how often is it Positive?*

$$\mathrm{Precision} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}$$

Evaluation of Classification Performance

Confusion Matrix Metrics:

		Actual Labels		
		1 (Positive)	**0 (Negative)**	Total
Predicted Labels	**1 (Positive)**	**TP**	**FP**	*Predicted Total Positives*
	0 (Negative)	**FN**	**TN**	*Predicted Total Negatives*
	Total	P= TP+FN *Actual Total Positives*	N= P+TN *Actual Total Negatives*	

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\frac{\text{TN}}{\text{TN} + \text{FN}}$$

Negative Predicted Value

$$TPR = S_e = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{\text{TP}}{\text{P}}$$

$$TNR = S_p = \frac{\text{TN}}{\text{TN} + \text{FP}} = \frac{\text{TN}}{\text{N}}$$

Evaluation of Classification Performance

Confusion Matrix:

		Actual Labels		
		1 (Positive)	0 (Negative)	Total
Predicted Labels	1 (Positive)	TP = 100	FP = 10	110
	0 (Negative)	FN = 5	TN = 50	55
	Total	P = 105	N = 60	

Metrics using Confusion Matrix (Example: Disease Prediction):

- ***Accuracy***: *Disease/Healthy prediction accuracy*

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{Total}} = \frac{\text{TP} + \text{TN}}{\text{P} + \text{N}}$$ **= (100+50)/165 = 0.91**

- ***Misclassification or Error Rate***: *Disease/Healthy prediction accuracy*

$$1 - \text{Accuracy} = \frac{\text{FP} + \text{FN}}{\text{Total}} = \frac{\text{FP} + \text{FN}}{\text{P} + \text{N}}$$ **= (10+5)/165 = 0.09**

- ***Sensitivity or Recall or True Positive Rate (TPR)***: *When it's positive, how often does the model detected disease?*

$$TPR = S_e = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{\text{TP}}{\text{P}}$$ **= 100/105 = 0.95**

Evaluation of Classification Performance

Confusion Matrix:

Metrics using Confusion Matrix (Example: Disease Prediction):

		Actual Labels		
		1 (Positive)	0 (Negative)	Total
Predicted Labels	1 (Positive)	TP = 100	FP = 10	110
	0 (Negative)	FN = 5	TN = 50	55
	Total	P = 105	N = 60	

- ***False Positive Rate***: *Actually heathy, how often does it predict yes?*

$$FPR = \frac{\mathrm{FP}}{\mathrm{TN} + \mathrm{FP}} = \frac{\mathrm{FP}}{\mathrm{N}}$$ **= 10/60 = 0.17**

- ***Specificity or True Negative Rate (TNR)***: *When it's actually health, how often does it predict healthy?*

$$TNR = S_p = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FP}} = \frac{\mathrm{TN}}{\mathrm{N}}$$ **= 50/60 = 0.83**

- ***Precision***: *When it predicts disease, how often is it correct?*

$$\mathrm{Precision} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FP}}$$ **= 100/110 = 0.91**

Evaluation of Classification Performance

Confusion Matrix:

Metrics using Confusion Matrix:

When to use which?

- Disease Detection: We do not want FN

$$TPR = S_e = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN}} = \frac{\mathrm{TP}}{\mathrm{P}}$$

- Fraud Detection: We do not want FP

$$TNR = S_p = \frac{\mathrm{TN}}{\mathrm{TN} + \mathrm{FP}} = \frac{\mathrm{TN}}{\mathrm{N}}$$

$$\mathrm{Precision} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FP}}$$

		Actual Labels	
		1 (Positive)	0 (Negative)
Predicted Labels	1 (Positive)	TP	FP
	0 (Negative)	FN	TN

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Evaluation of Classification Performance

Confusion Matrix:

Precision and Sensitivity (Recall) Trade-off:

- *Disease Detection:*

Sensitivity or Recall

$$TPR = S_e = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}} = \frac{\mathrm{TP}}{\mathrm{P}}$$

Precision

$$\mathrm{Precision} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FP}}$$

- *Recall or Sensitivity (S_e); how good we are at detecting diseased people.*
- ***Precision:** How many have been correctly diagnosed as unhealthy.*
- *If we have diagnosed everyone unhealthy, **S_e=1** (diagnose all unhealthy people correctly) but **Precision may be low** (because TN=0 that increases the value of FP).*

		Actual Labels	
		1 (Positive)	0 (Negative)
Predicted Labels	1 (Positive)	TP	FP
	0 (Negative)	FN	TN

- *We want high **Precision** and high **S_e (=1, Ideally)**.*
- *We should combine precision and sensitivity to evaluate the performance of classifier.*
    - *F1-Score*

Evaluation of Classification Performance

F1-Score:

- We observed trade-off between recall and precision.

$$TPR = S_e = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FN}} = \frac{\mathrm{TP}}{\mathrm{P}} \qquad \mathrm{Precision} = \frac{\mathrm{TP}}{\mathrm{TP} + \mathrm{FP}}$$

- Higher levels of recall may be obtained at the price of lower values of precision.
- We need to define a single measure that combines recall and precision or other metrics to evaluate the performance of a classifier.
- Some combined measures:
  - F1 Score
  - Matthew's Correlation Coefficient
  - 11-point average precision
  - The Breakeven point

Evaluation of Classification Performance

F1 Score:

- *One measure that assesses recall and precision trade-off is weighted harmonic mean (HM) of recall and precision, that is,*

$$F_\beta = \frac{1+\beta^2}{\frac{1}{\text{Precision}} + \frac{\beta^2}{\text{Recall}}}, \quad \beta \geq 0$$

For $\beta = 1$, we have harmonic mean of precision and recall, that is,

$$F_1 = \frac{2}{\frac{1}{\text{Precision}} + \frac{1}{\text{Recall}}} = \frac{2(\text{Precision})(\text{Recall})}{(\text{Precision}) + (\text{Recall})} = \frac{2\text{TP}}{2\text{TP} + \text{FP} + \text{FN}}$$

Evaluation of Classification Performance

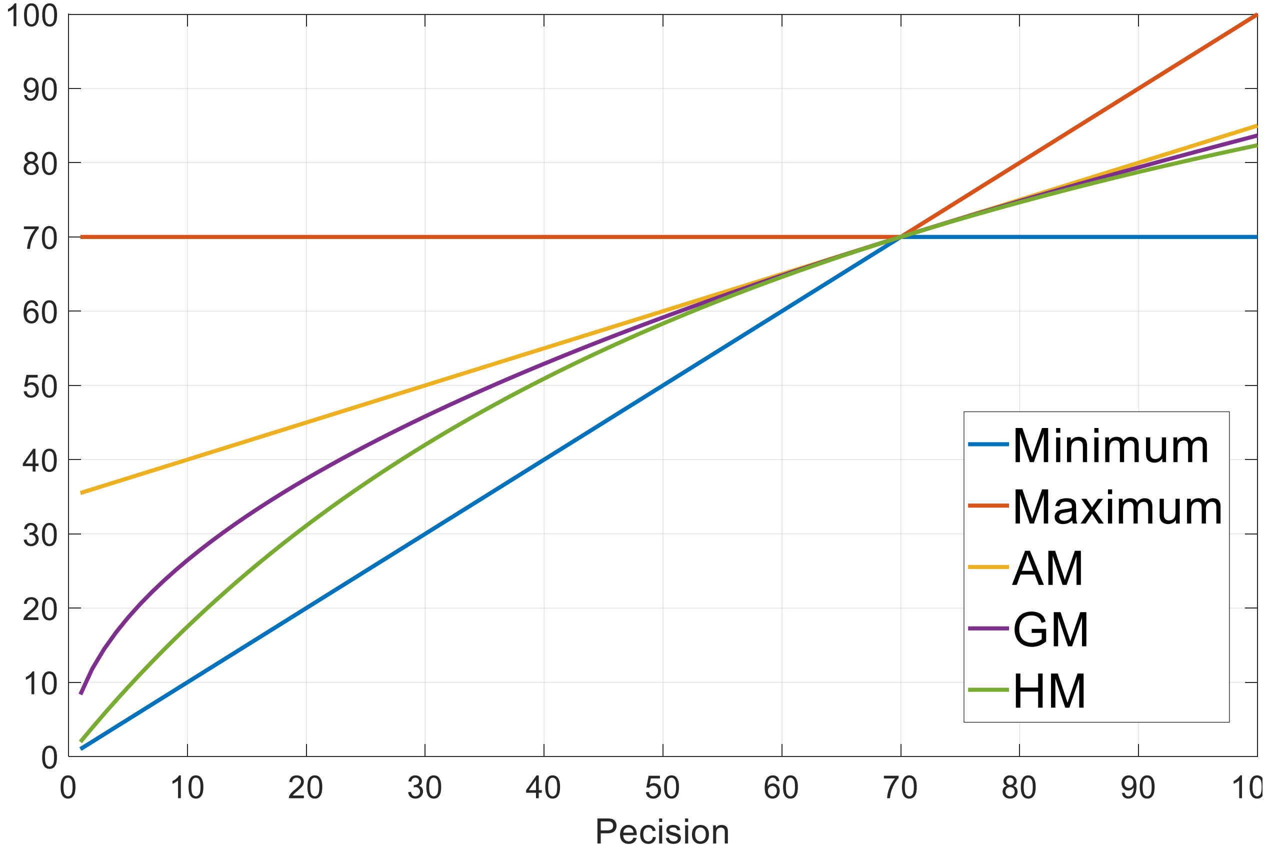
F1 Score:

Why harmonic mean?

- *We could also use arithmetic mean (AM) or geometric mean (GM).*
- *HM is preferred as it penalizes model the most; a conservative average, that is, for two real positive numbers, we have*

$$\mathrm{HM} \leq \mathrm{GM} \leq \mathrm{AM}$$

- *Improvement in HM implies improvement in AM or GM.*



Different means, minimum and maximum against precision. Recall=70% is fixed.

Outline

- Classification Accuracy (0/1 Loss)
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- Multi-class Classification

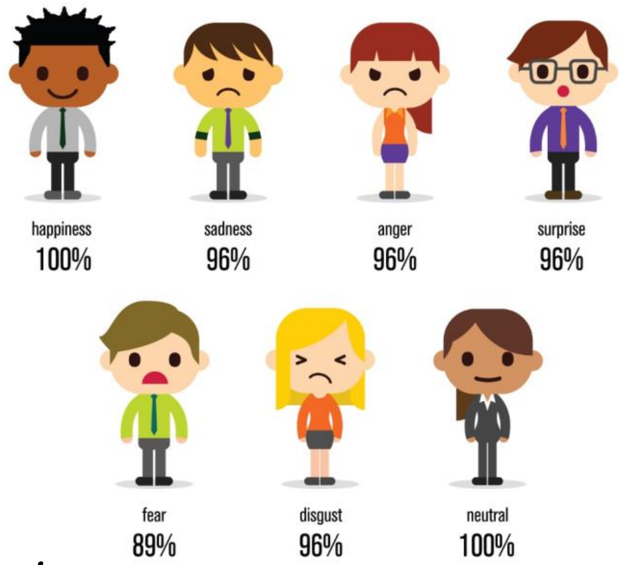
Multi-Class Classification

Formulation:

- We assume we have training data D given by

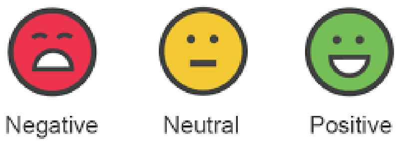
$$D = \{(\mathbf{x_1}, y_1), (\mathbf{x_2}, y_2), \dots, (\mathbf{x_n}, y_n)\} \subseteq \mathcal{X}^d \times \mathcal{Y}$$

- $\mathcal{Y} = \{1, 2, \dots, M\}$ (M-class classification)



Examples:

- Emotion Detection.
- Vehicle Type, Make, model, color of the vehicle from the images streamed by safe city camera.
- Speaker Identification from Speech Signal.
- State (rest, ramp-up, normal, ramp-down) of the process machine in the plant.
- Sentiment Analysis (Categories: Positive, Negative, Neutral), Text Analysis.
- Take an image of the sky and determine the pollution level (healthy, moderate, hazard).
- Record Home WiFi signals and identify the type of appliance being operated.



Multi-Class Classification

Implementation (Possible options using binary classifiers):

Option 1: Build a one-vs-all (OvA) one-vs-rest (OvR) classifier:

Train M different binary classifiers $h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_M(\mathbf{x})$.

Classifier $h_i(\mathbf{x})$ is trained to classify if $\mathbf{x}$ belongs to i-th class or not.

For a new test point $\mathbf{z}$, get scores for each classifier, that is, $s_i = h_i(\mathbf{z})$.

For example, s_i can be assigned the probability that $\mathbf{z}$ belongs to class i.

Predict the label as $\hat{y} = \max_{i=1,2,\ldots,M} \quad s_i$

There can be other options…