Assignment 1
Solution

Q1

(a) $i = \frac{dq}{dt}$

$\frac{d}{dt}(5t \sin 4\pi t)$

$= 5 \sin 4\pi t + 20\pi t \cos 4\pi t$ mA.

→ At $t = 0.5$

$i = 5 \sin 2\pi + 10\pi \cos 2\pi$

$= 0 + 10\pi$

$\boxed{i = 31.24 \text{ mA.}}$

(b)

$Q_1 = \int_0^1 2\,dt = [2t]_0^1$

$= 2 - 0 = 2$

$Q_2 = \int_1^2 2t^2\,dt = \left[\frac{2t^3}{3}\right]_1^2$

$Q_2 = \frac{16}{3} - \frac{2}{3} = \frac{14}{13}$

$Q_{Total} = Q_1 + Q_2$

$= 2 + \frac{14}{3} = \boxed{6.667 \text{ C}}$

(c)

$Q = \int_1^2 i\,dt = \int_1^2 (3t^2 - t)\,dt$

$= \left[t^3 - \frac{t^2}{2}\right]_1^2 = (8 - 2) - (1 - \frac{1}{2}) = \boxed{5.5 \text{ C}}$

(1d)

Total charge

$\Delta q = i\Delta t = 2 \times 10 = \underline{20C}$

Voltage drop:

$V = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = \boxed{115 V}$

(1e)

(i) $\frac{-30}{2} = \boxed{-15V}$

(ii) $\frac{-30}{-6} = \boxed{5V}$

(1f)

(i) $V = 3i = 15 \cos 60\pi t$

$P = Vi$

$= 75 \cos^2 60\pi t \ W$

At $t = 3ms$.

$P = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi$

$\boxed{= 53.48 W}$

(ii) $V = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t$.

$= -900\pi \sin 60\pi t \ V$.

$P = Vi = -4500\pi \sin 60\pi t \cos 60\pi t \ W$.

At $t = 3ms$.

$P = -4500\pi \sin(0.18\pi) \cos(0.18\pi) \ W$.

$\boxed{= -6.396 kW}$

(Q2)

(a) $i = \frac{dq}{dt}$ A

$i = 40\pi \cos 4\pi t$ mA

$P = Vi$

$P = 80\pi \cos^2 4\pi t$ mW

At $t = 0.3$s.

$P = 80\pi \cos^2 (4\pi \times 0.3)$

$= 164.5$ mW

(b) $W = \int_0^{0.6} P\,dt$

$W = 80\pi \int_0^{0.6} \cos^2 4\pi t\,dt$

$= 40\pi \int_0^{0.6} (1 + \cos 8\pi t)\,dt$ mJ

$W = 40\pi \left[0.6 + \frac{1}{8\pi} \sin 8\pi t \right]_0^{0.6}$

$W = 78.34$ mJ

Q3

(a)

$$Q = \int_2 i\,dt$$

$$Q = \int_0^2 3e^{-2t}\,dt$$

$$Q = \left[\frac{-3}{2}e^{-2t}\right]_0^2$$

$$= -\frac{3}{2}e^{-4} + \frac{3}{2}$$

$$= 1.473\ C$$

(b)

$$P = VI$$

$$V = 5\frac{di}{dt} \Rightarrow \frac{di}{dt} = -6e^{-2t}$$

$$V = -30e^{-2t}$$

$$P = -30e^{-2t} \times 3e^{-2t}$$

$$P = -90e^{-4t}\ W.$$

(c) $E = \int P\,dt$

$$E = \int_0^3 -90e^{-4t}$$

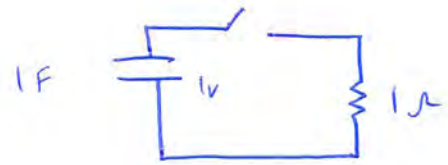
$$\left[\frac{90}{4}e^{-4t}\right]_0^3$$

$$\frac{90}{4}e^{-12} - \frac{90}{4}$$

$$= -22.5\ J$$

Ans 4: $C = 1\,F$ $R = 1\,\Omega$

(a)



$i = e^{-t}$

$e^{-t} = 0.37$

$\boxed{t = 1\,s}$ → we have to find $\frac{dv}{dt}$

$i = C\frac{dV}{dt}$ — (1) mark

$0.37 = 1\,\frac{dv}{dt}$ → (3) marks

$\boxed{\frac{dV}{dt} = 0.37}$ — (2) mark

(b) $Q_{initial} = CV = \boxed{1\,C}$ — (1) mark

charge flowing out $= \int_0^1 i\,dt$

$= \int_0^1 e^{-t}\,dt$

$= \boxed{0.632\,C}$ — (1) mark

(3) marks

$\boxed{Q_{left} = 0.368\,C}$ — (1) mark

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Ans 5:

$d_i = 5 \times 10^{-3}$ m

$C_i = 6 \times 10^{-13}$ f

$V = 8$

$A = ?$

$d_f = 0.2 \times 10^{-3}$ m

$V = 8$ V

$C_f = ?$

$C_i = \frac{\varepsilon_0 A}{d_i} \Rightarrow 6 \times 10^{-3} = \frac{8.85 \times 10^{-12} \cdot A}{5 \times 10^{-3}}$ $\quad \therefore \varepsilon_0 = 8.85 \times 10^{-12}$

$A = 3.39 \times 10^{-4}$ m 1

(2)

$C_f = \frac{\varepsilon_0 A}{d_f} =$ 1.5×10^{-11} F 1

(b) $q_i = C_i V =$ 4.8×10^{-12} C

$q_f = C_f V =$ 1.2×10^{-10} C 1

(2)

change in charge = 1.152×10^{-10} C ⇐ Charge flowing out.

1

(c) $E_i = \frac{1}{2} C_i V^2 =$ 1.92×10^{-11} J

$E_f = \frac{1}{2} C_f V^2 =$ 4.8×10^{-10} J ⇒ Energy increased

(2)

Reason: Energy used to press the key is converted into electrical Energy which is used to send a pulse to computer.

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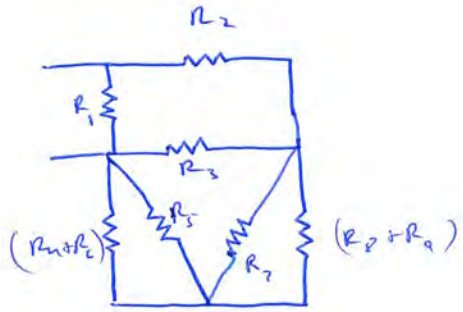
Ans 6:



→ R_8 in series with $R_9 = 112\Omega$

→ R_4 in series with $R_6 = 2k\Omega$

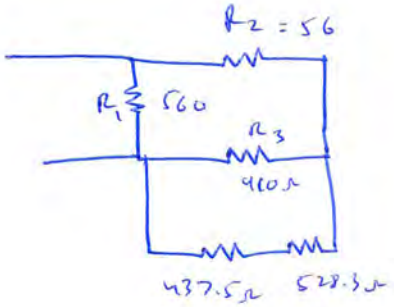
(2)

(2)

→ $(R_8 + R_9) \,//\, R_7 = 528.3\Omega$

→ $(R_4 + R_6) \,//\, R_5 = 437.5\Omega$

} in series



(2)

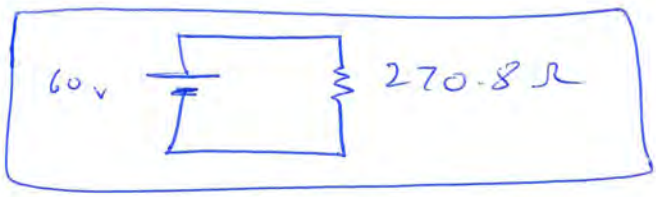
→ These two resultant are in series & parallel with R_3. → 468.5Ω

(2)

→ 468.5Ω in series with R_2 & both are parallel to R_1

So $(R_2 + 468.5) \,//\, R_1$

Final

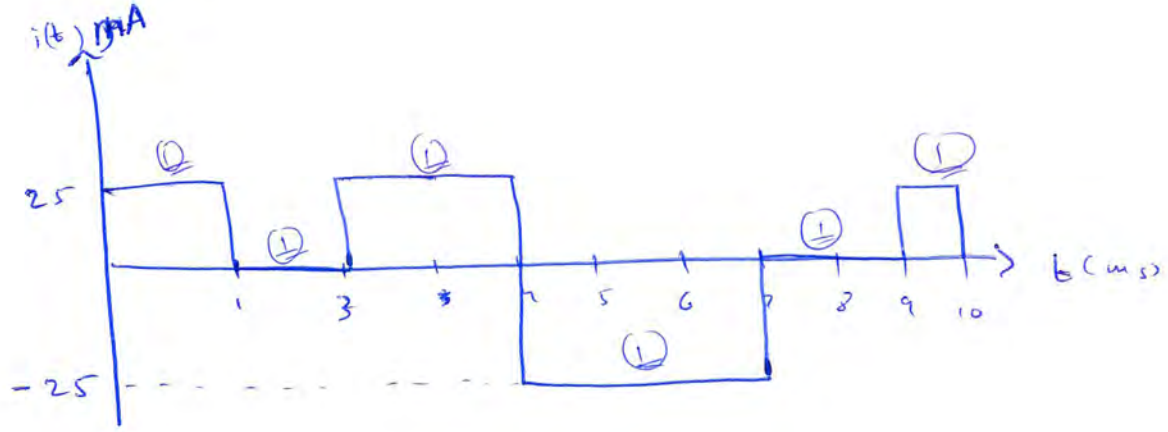


(2)

(→)

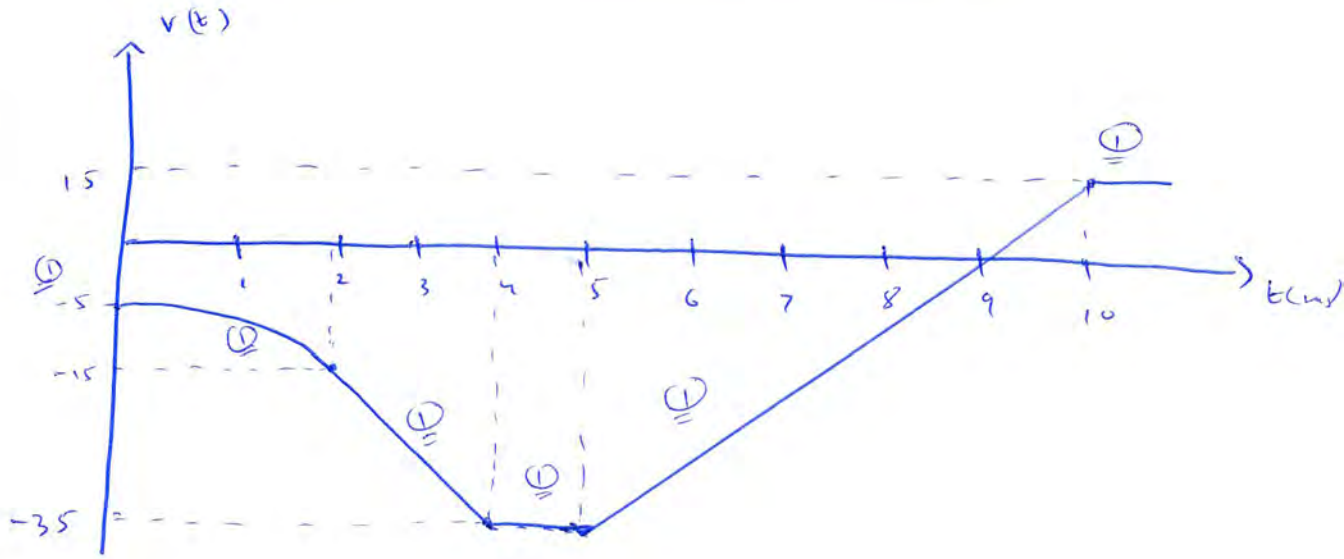
Aus 7:

(a)



$p(t) = i(t)\, v(t) = \frac{25 \times 10}{\times 10^{-6}} = 250\ \mu J$ **Energy = 250 μJ** (2)

(b)



Energy = 312.5 nJ (2)

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(Q8) $Q = \int i\,dt$

0 — 6s

$Q = \frac{3t^2}{2}$

$Q = \left[\frac{3t^2}{2}\right]_0^6$

$Q = 54\,C$

6 — 10 s

$Q = [18t]_6^{10}$

$Q = 180 - 108$

$= 72.$

Total Q at 10s = $54 + 72 = 126\,C.$

10 — 15 s

$Q = [-12t]_{10}^{15}$

$= -180 + 120.$

$= -60$

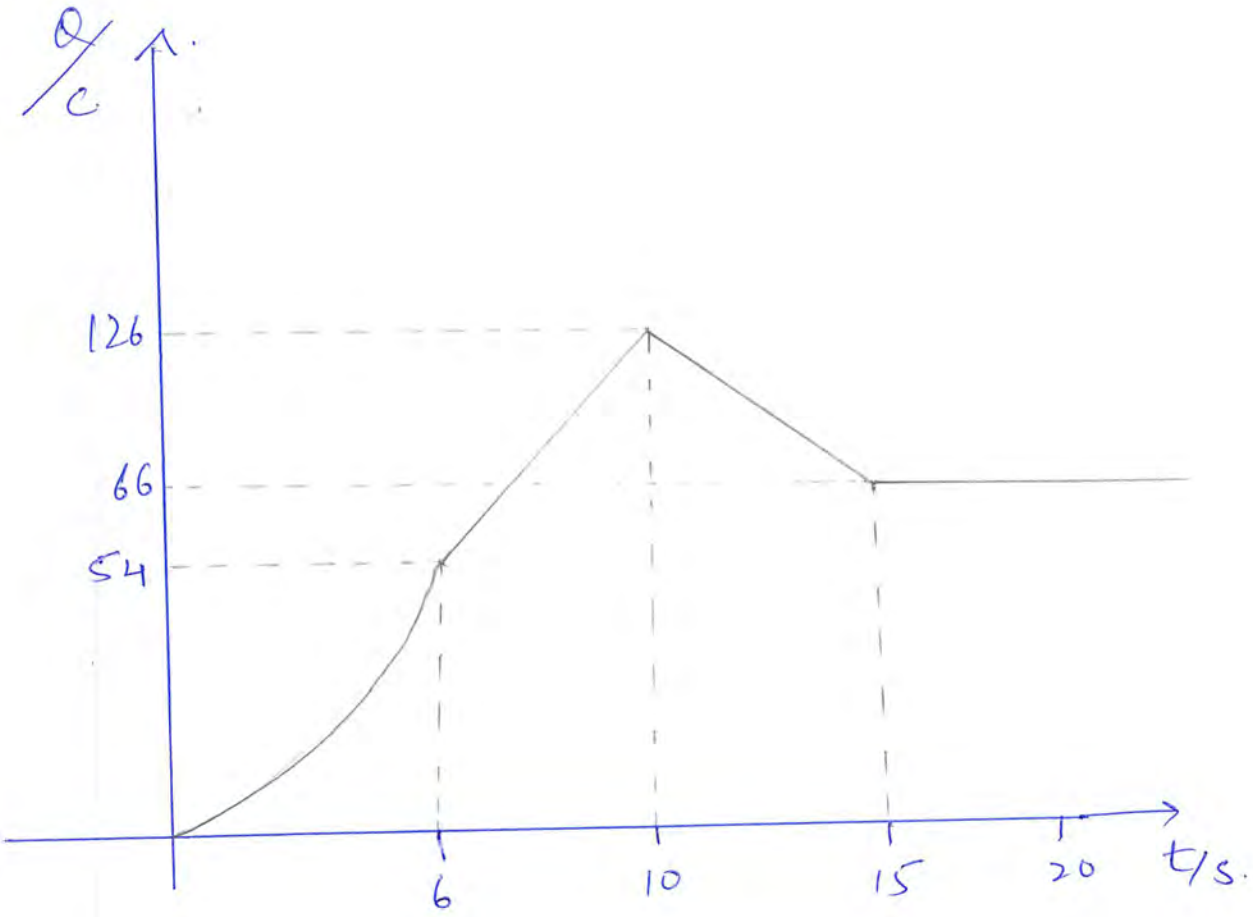
Total Q at 15s = $126 - 60 = 66.$

t > 15s.

~~$Q = \int i\,dt$~~ ~~$= 66$~~

$= 66.$

Graph for Q8



Ans 9:

$$v(t) = \begin{cases} (1-3t)e^{-3t} \text{ mV} & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

(a) $V = L\frac{di}{dt}$ (1)

$$i = \frac{1}{L}\int_0^t v\,dt = 5\int_0^t (1-3t)e^{-3t}\,dt$$

$$i = 5\left[\int_0^t e^{-3t}\,dt - 3\int_0^t t\,e^{-3t}\,dt\right]$$

(1) → integration by parts

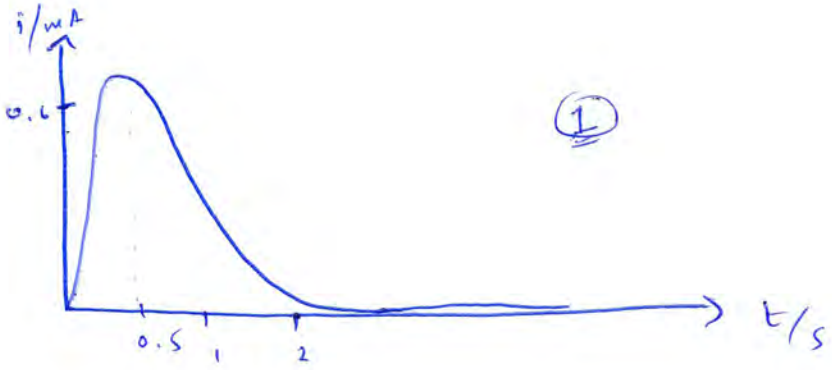
$$i = 5\left[\left|\frac{e^{-3t}}{-3}\right|_0^t - 3\left|\frac{-e^{-3x}(3x+1)}{9}\right|_0^t\right]$$

$$i = 5\left[\frac{-e^{-3t}}{3} + \frac{1}{3} + \frac{e^{-3t}(3t+1)}{3} - \frac{1}{3}\right]$$

$$i = 5\left[\frac{e^{-3t}(3t+1+1)}{3}\right]$$

$$\boxed{i(t) = \begin{cases} 5t\,e^{-3t} \text{ mA} & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}}$$

(2)

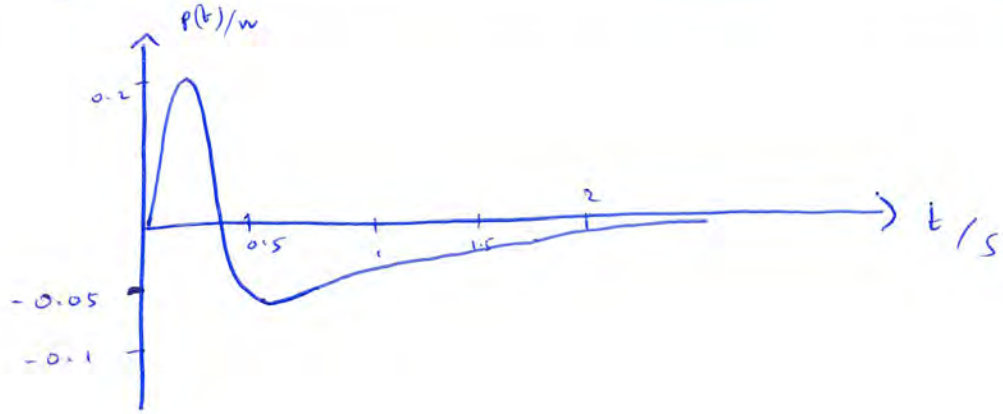


(1)

(b) $p(t) = v(t)\, i(t)$

$$p(t) = \begin{cases} 5t(1-3t)e^{-6t} \ \mu W ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$

(2)

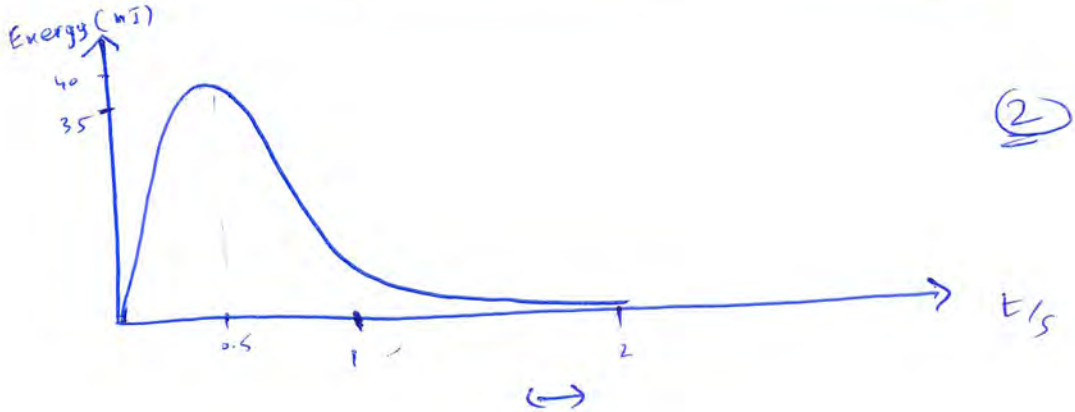


(2)

(c) $w(t) = \frac{1}{2} L\, i(t)^2$

$$w(t) = \begin{cases} 2.5\, t^2 e^{-6t} \ \mu j ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$

(2)



(2)

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Ans 10: $L = 50 \times 10^{-3}$ H

$$i(t) = \begin{cases} 0 & ; t < 0 \\ 2te^{-4t} & ; t > 0 \end{cases}$$

(a) $V = L\frac{di}{dt}$ (1)

$V = L\left[e^{-4t}(2-8t)\right]$ (1)

$\boxed{V(t) = 50\times10^{-3}(2-8t)e^{-4t}}$ (1)

(b) $\frac{di(t)}{dt} = (2-8t)e^{-4t} = 0$ (2)

(1)

$\boxed{t = 1/4\ s}$ (1)

(c) $\frac{dv(t)}{dt} = 0$

(1)

$5\times10^{-2}\left[16e^{-4t}(2t-1)\right] = 0$ (2)

$2t-1 = 0$

$\boxed{t = \frac{1}{2}\ s}$ (1)

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