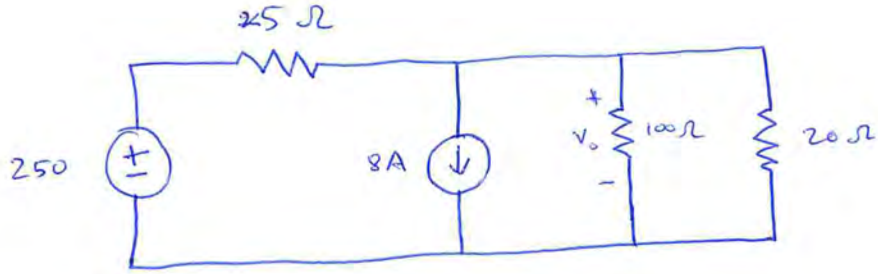
EE-240 - Circuits 1

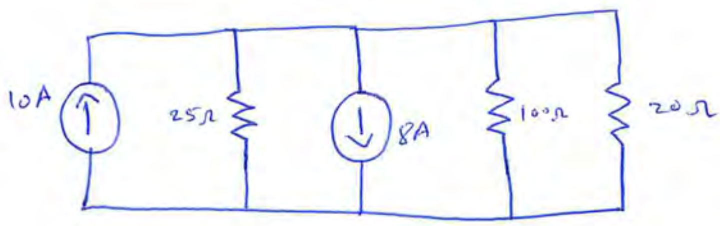
Assignment #2 solution

Ans 1:



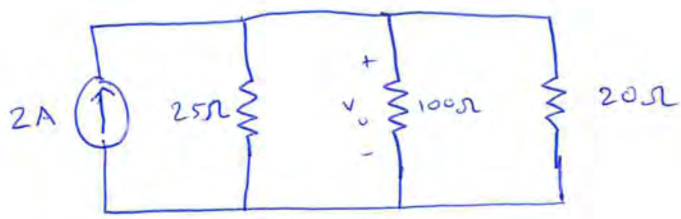
convert voltage to current source

$i = \frac{V}{R} = \frac{250}{25} = 10$



→ Take resultant of parallel current sources.

$10 - 8 = 2A$



$100 // 20 // 25 = 10\Omega$

$V_o = IR$

$\boxed{V_o = 20\Omega}$

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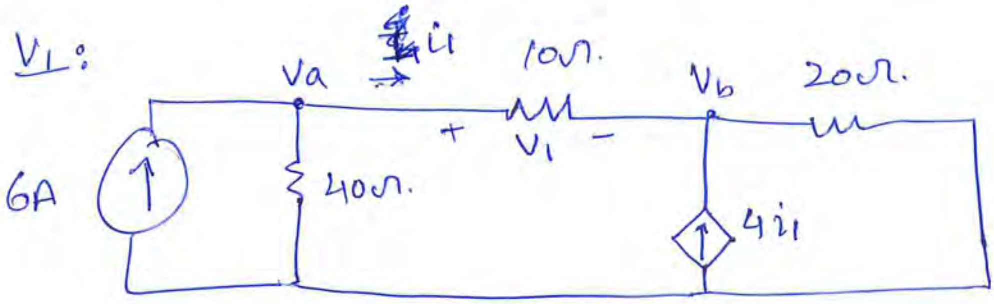
Q2

Let $V_0 = V_1 + V_2$.

$V_1 \Rightarrow$ due to 6A source.

V_2 due to 80 A source.

V_1:



Node a :

$$6 = \frac{V_a}{40} + \frac{V_a - V_b}{10}$$

$$240 = 5V_a - 4V_b \quad \text{—(1)}$$

Nodeb

$$-i_1 - 4i_1 + \frac{(V_b - 0)}{20} = 0$$

$$V_b = 100 i_1$$

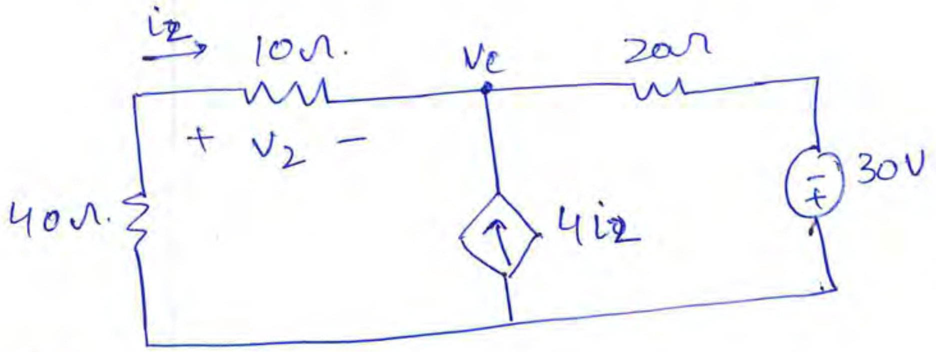
$$i_1 = \frac{V_a - V_b}{10} \qquad V_b = 10(V_a - V_b) \Rightarrow V_b = 0.9091V_a \quad \text{—(2)}$$

Substituting (2) into (1)

$V_a = 175.95$ and $V_b = 159.96$.

$V_1 = V_a - V_b = 15.99V$.

③ Now considering Voltage source only:



Node C

$$4i_2 = \frac{V_c}{50} + \frac{V_c + 30}{20}$$

$$i_2 = \frac{0 - V_c}{50}$$

$$\frac{-4V_c}{50} = \frac{V_c}{50} + \frac{V_c + 30}{20}$$

$$V_c = -10\,V$$

$$i_2 = \frac{0 - V_c}{50} = \frac{0 + 10}{50} = \frac{1}{5}$$

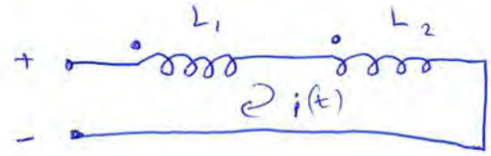
$$V_2 = 10 i_2 = 2\,V$$

$$V_o = V_1 + V_2 = 15.99 + 2 = \boxed{17.99\,V}$$

$$i_o = \frac{V_o}{10} = \boxed{1.799\,A}$$

Ans 3:

a)

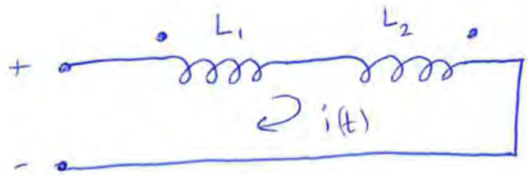


$$V_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$V_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$\boxed{V_T = (L_1 + L_2 + 2M)\frac{di}{dt}} \Rightarrow \text{hence} \quad \boxed{L_{eq} = L_1 + L_2 + 2M}$$

b)



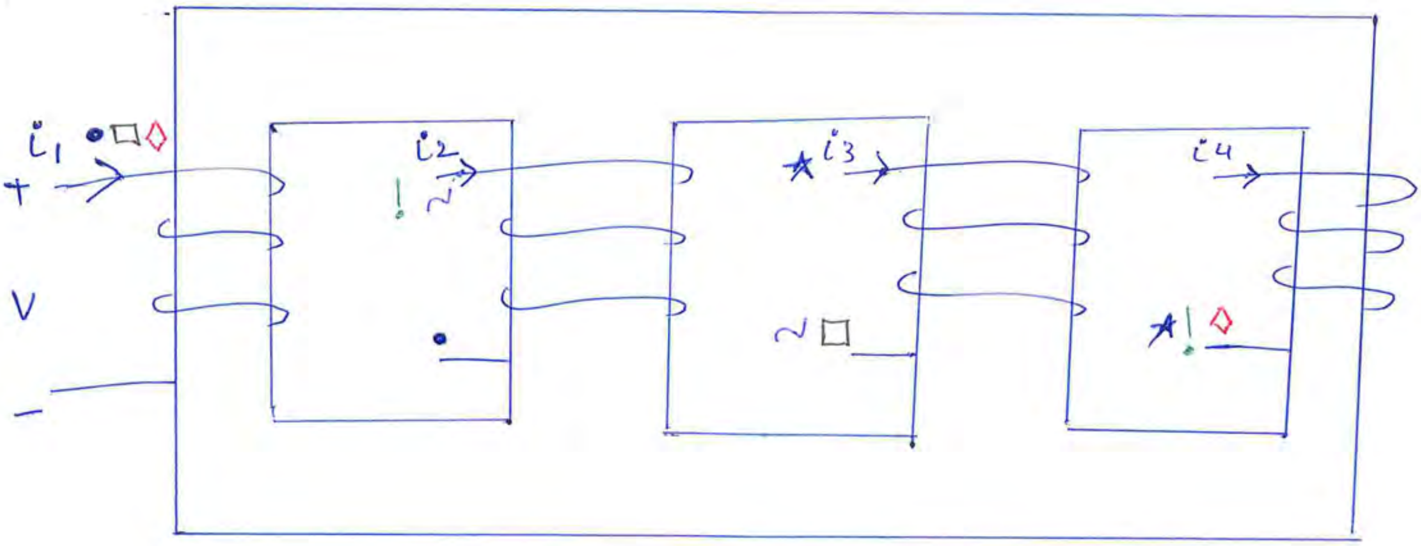
$$V_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}$$

$$V_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$\boxed{V_T = (L_1 + L_2 - 2M)\frac{di}{dt}} \Rightarrow \text{hence} \quad \boxed{L_{eq} = L_1 + L_2 - 2M}$$

$\longleftrightarrow$

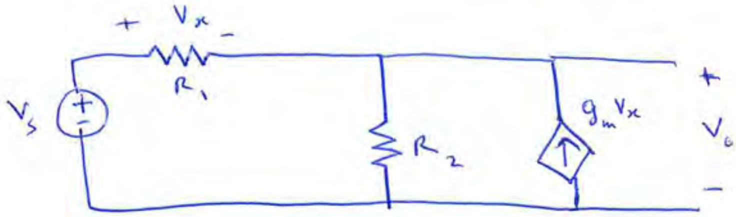
Q4



Key

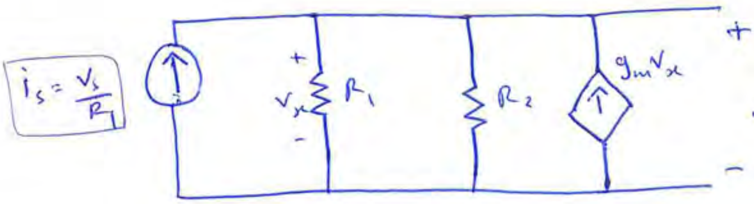
1-2	•
1-3	□
1-4	◇
2-3	~
2-4	!
3-4	★

Ans 5:



a) **Both** sources can be simplified

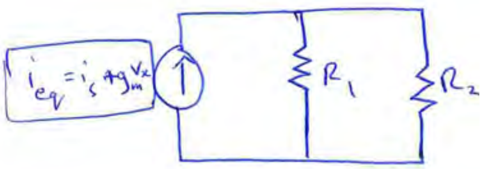
b)



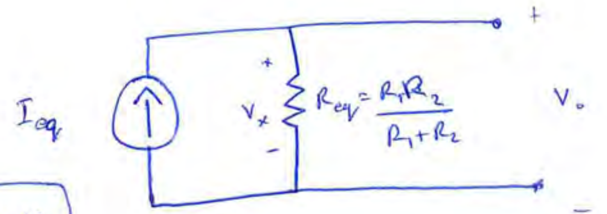
$$R_{eq} = R_1 // R_2$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

→ Current sources in parallel so → simplify



→ replace R_1 & R_2 by R_{eq}



$$\boxed{I_{eq} = \frac{V_s}{R_1} + g_m V_x}$$

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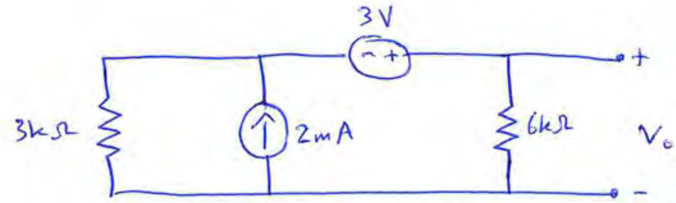
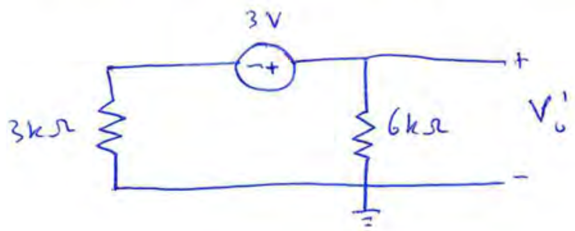
Ans 6:



use <u>Superposition Principle</u>

→ Use voltage source only & treat current source as open circuit



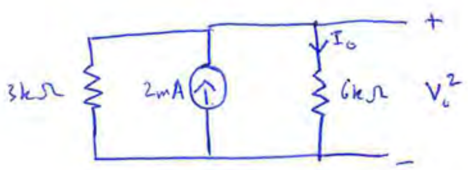
V_o' → voltage due to voltage source

voltage divided b/w 3kΩ and 6kΩ.

$$V_o' = \frac{6k\Omega}{9k\Omega} \times 3$$

$$\boxed{V_o' = 2V}$$

→ Now use current source only & treat voltage source as short circuit



V_o^2 → Voltage due to current source

$$I_o = \frac{3k\Omega}{9k\Omega} \times (2\times10^{-3}) = \boxed{\frac{2}{3} mA}$$

$$V_o^2 = \left(\frac{2}{3}\times10^{-3}\right)\times\left(6\times10^{3}\right) = \boxed{4V}$$

$$V_o = V_o' + V_o^2 = \boxed{6V}$$

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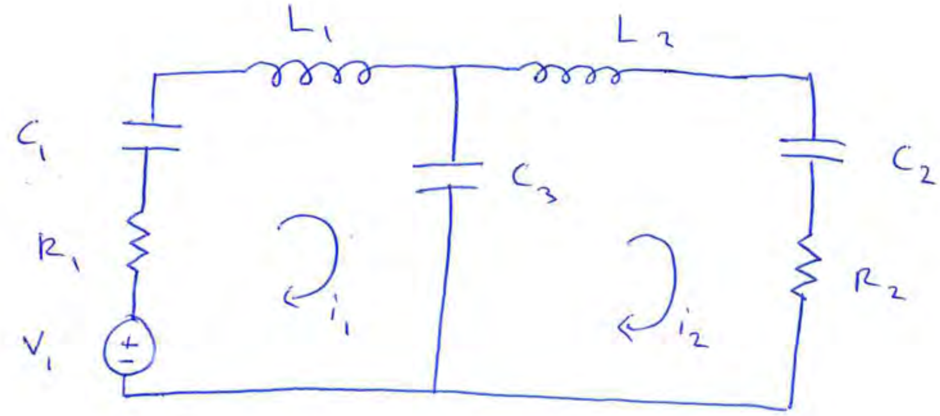
Ans 7:



→ Inductors are not coupled and the only common element is C_3

KVL analysis

Loop 1:

$$L_1 \frac{di_1}{dt} + \frac{1}{C_3}\int (i_1 - i_2)\,dt + i_1 R_1 + \frac{1}{C_1}\int i_1\,dt = V_1$$

Loop 2:

$$L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt + i_2 R_2 + \frac{1}{C_3}\int (i_2 - i_1)\,dt = 0$$

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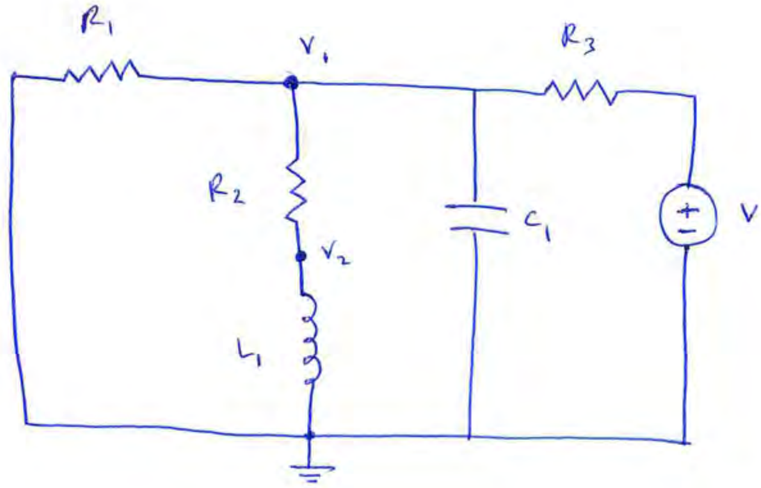
Ans 8:



→ 4 nodes, but 1 is at ground & other between R_3 & V so its voltage is known.

↳ so unknown voltage nodes = 2 i.e V_1 and V_2

KCL Analysis

node 1:

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_3} + C_1 \frac{dV_1}{dt} = 0$$

node 2:

$$\frac{V_2 - V_1}{R_2} + \frac{1}{L_1} \int V_2 \, dt = 0$$

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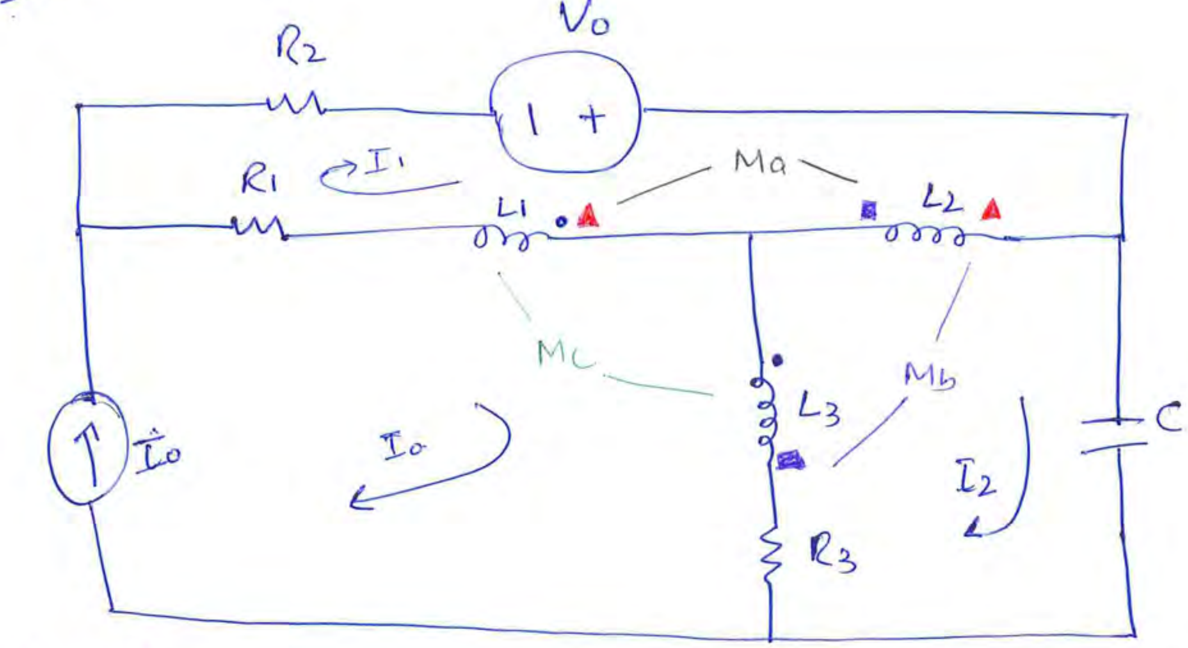
Q9



Loop I_0

$$R_1(I_0 - I_1) + L_1\frac{d}{dt}(I_0 - I_1) + L_3\frac{d}{dt}(I_0 - I_2) + R_3(I_0 - I_2)$$
$$- M_c\frac{d}{dt}(I_0 - I_1) + M_c\frac{d}{dt}(I_0 - I_2) + M_a\frac{d}{dt}(I_2 - I_1) - M_b\frac{d}{dt}(I_2 - I_1) = 0$$

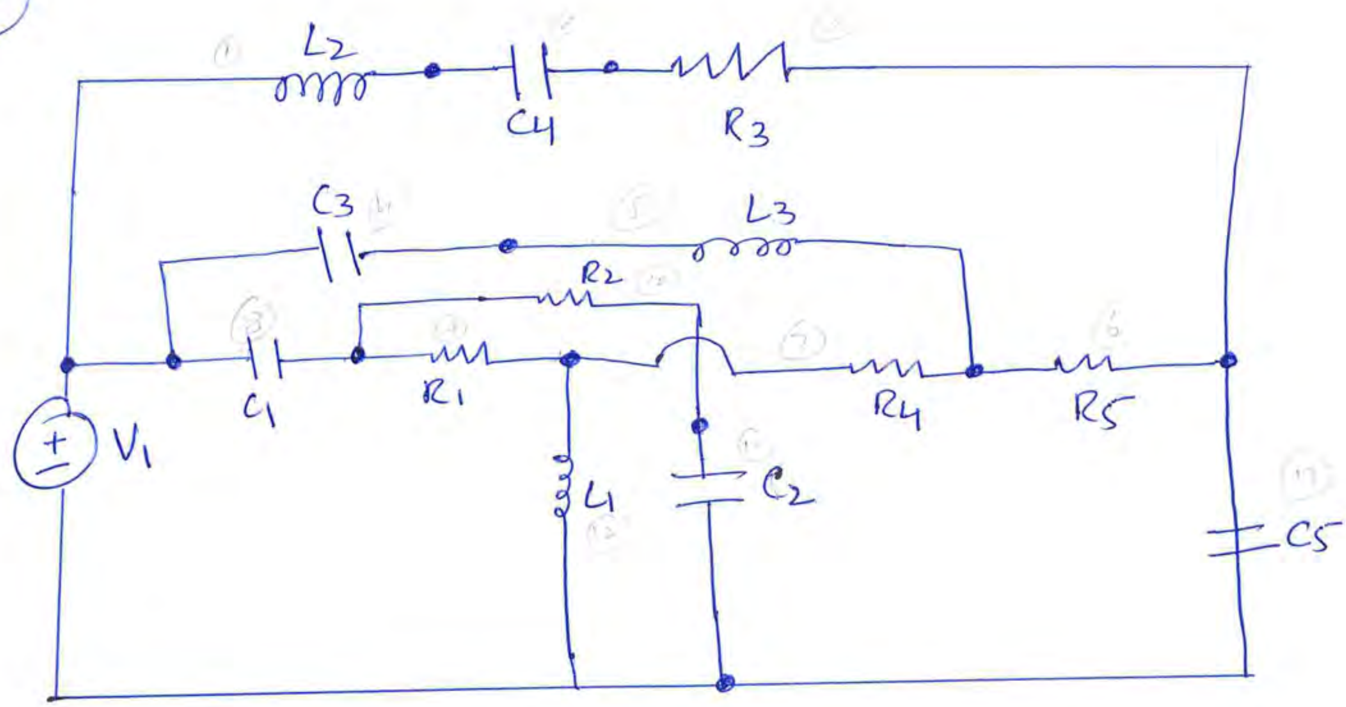
Loop 1

$$V_0 = R_2 I_2 + L_2\frac{d}{dt}(I_1 - I_2) + L_1\frac{d}{dt}(I_1 - I_0) + R_1(I_1 - I_0)$$
$$+ M_a\frac{d}{dt}(I_1 - I_0) - M_a\frac{d}{dt}(I_2 - I_1) - M_b\frac{d}{dt}(I_2 - I_0)$$
$$- M_c\frac{d}{dt}(I_2 - I_0)$$

Loop 2

$$0 = \frac{1}{c}\int I_2\,dt + R_3(I_2 - I_0) + L_3\frac{d}{dt}(I_2 - I_0) + L_2\frac{d}{dt}(I_2 - I_1)$$
$$+ M_b\frac{d}{dt}(I_2 - I_0) + M_b\frac{d}{dt}(I_2 - I_1) + M_c\frac{d}{dt}(I_0 - I_1)$$
$$+ M_a\frac{d}{dt}(I_0 - I_1)$$

(10)



(a) 11 nodes.

(b) 9 $(n-1)-1$ ↳ as voltage source is present

(c) 4 $b-(n-1)$.

$b=14$.