EE 240 — Circuits 1

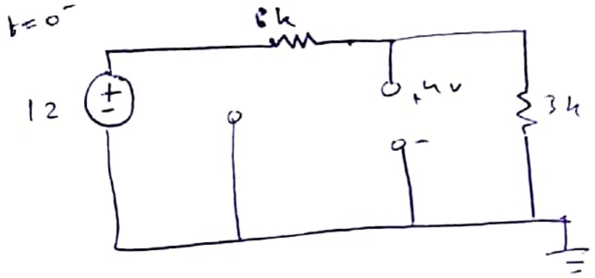
Assignment 4 Solution

Ans 1:

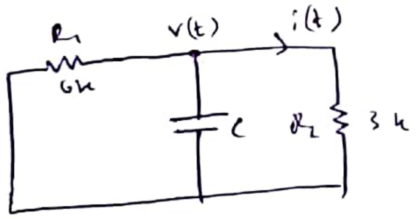
Switch is at '1' for long time, so capacitor charged.
↳ open circuit.

$$V_c(0^-) = 12\left(\frac{3k}{9k}\right) = \boxed{4V}$$

at $t=0^-$



then, for $t>0$:



$$\frac{v(t)}{R_1} + C\frac{dv(t)}{dt} + \frac{v(t)}{R_2} = 0$$

$$\frac{v(t)}{2000} + (1\times10^{-4})\frac{dv(t)}{dt} = 0$$

$$\boxed{\frac{dv(t)}{dt} + 5\,v(t) = 0} \longrightarrow (1)$$

using homogeneous equations:

$$v(t) = K_2 e^{-t/\tau}$$

$$-\frac{1}{\tau} K_2 e^{-t/\tau} + 5 K_2 e^{-t/\tau} = 0$$

$$\frac{1}{\tau} = 5$$

$$\boxed{\tau = 0.2}$$

so,

$$v(t) = K_2 e^{-t/0.2}$$

$$V_c(0^-) = V_c(0^+) = \boxed{4V}$$

$$K_2 e^0 = 4$$

$$\boxed{K_2 = 4}$$

$$\boxed{v(t) = 4e^{-t/0.2} \quad V}$$

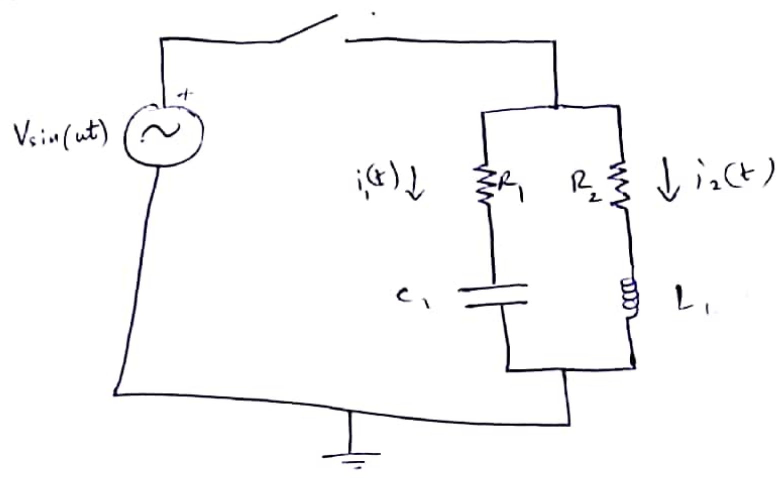
now for $i(t)$;

$$i(t) = \frac{v(t)}{R_2}$$

$$i(t) = \frac{4e^{-t/0.2}}{3000} \quad A$$

$$\boxed{i(t) = \frac{4}{3} e^{-t/0.2} \quad mA}$$

Ans 2:



(a) KVL on inner loop:

$$R_1 i_1 + \frac{1}{C_1}\int i_1 \, dt = V\sin(\omega t)$$

derivate:

$$\boxed{R_1 \frac{di_1}{dt} + \frac{1}{C_1} i_1 = V\omega\cos(\omega t)}$$

at $t=0^+$:

[capacitor is s.c]

$$i_1 = \frac{V\sin(\omega t)}{R_1}$$

$$i_1 = \frac{V\sin(\omega 0^-)}{R_1}$$

$$\boxed{i_1(0^+) = 0A}$$

so,

$$R_1 \frac{di_1(0^+)}{dt} + \frac{1}{C} i_1(0^+) = V\omega\cos(0^+)$$

$$R_1 \frac{di_1(0^+)}{dt} + 0 = V\omega$$

$$\boxed{\frac{di_1(0^+)}{dt} = \frac{V\omega}{R_1}}$$

(b)

$$\left.\begin{aligned} i_L(0^+) &= i_L(0^-) = i_2(0^+) = 0\,A \\ V_c(0^+) &= V_c(0^-) = 0 \end{aligned}\right\} \text{initial conditions}$$

$$i_2 = i_L$$

$$\boxed{i_2(0^+) = 0\,A}$$

Take KVL outer loop:

$$R_2 i_2 + L\frac{di_2}{dt} = V\sin(\omega t)$$

$$R_2 i_2(0^+) + \frac{L\,di_2(0^+)}{dt} = V\sin(0^+)$$

$$0 + \frac{L\,di_2(0^+)}{dt} = 0$$

$$\boxed{\frac{di_2(0^+)}{dt} = 0}$$

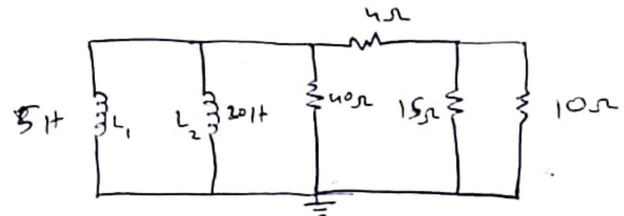
Ans 3:

for $t<0$, L_1 & L_2 are s.c so no current in resistors.

$\boxed{i_3(0^-) = 0}$ $\boxed{i_2(0^-) = i_2(0^+) = -4A}$ $\boxed{i_1(0^+) = i_1(0^-) = -8A}$

→ These are initial conditions.

for $t>0$; circuit becomes:



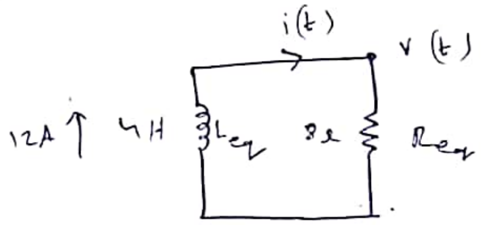
This can be simplified

$$L_{eq} = 5 // 20 = \boxed{4H}$$

$$R_{eq} = \left[\left[(15 // 10) + 4\right] // 40\right] = \boxed{8\,\Omega}$$

combined current is $\boxed{12\text{ A upwards}}$

Redraw:



$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$i(t) = I_0 e^{-t/\tau}$$

$$\boxed{i(t) = 12e^{-2t}\text{ A}}$$

$v(t) = i(t) R_{eq}$

$$\boxed{v(t) = 96e^{-2t}\text{ V}}$$

Now we know the node voltage, this node is common to all, so now we can find currents.

$$i_1(t) = i_1(0^+) + \frac{1}{L_1}\int_0^t v(t')\,dt'$$

$$i_1(t) = -8 + \frac{1}{5}\int_0^t 96e^{-2t'}\,dt'$$

$$\boxed{i_1(t) = 1.6 - 9.6\,e^{-2t}\ A}$$

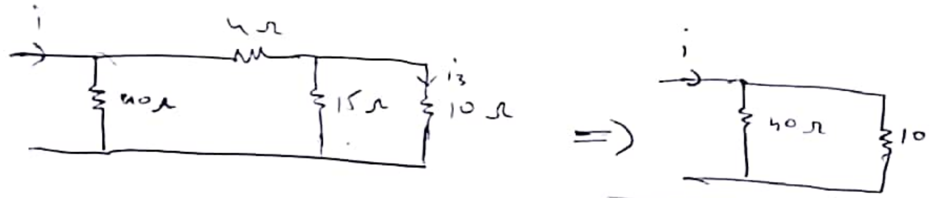
similarly:

$$i_2(t) = i_2(0^+) + \frac{1}{L_2}\int_0^t v(t')\,dt'$$

$$i_2(t) = -4 + \frac{1}{20}\int_0^t 96\,e^{-2t'}\,dt'$$

$$\boxed{i_2(t) = -1.6 - 2.4\,e^{-2t}\ A}$$

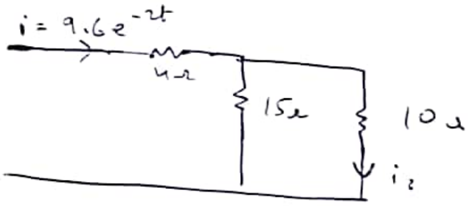
for resistor network:



$\boxed{(15 \parallel 10) + 4 = 10}$

current through equivalent 10 Ω = $\frac{v(t)}{10}$ = $\boxed{9.6e^{-2t}\,A}$

so, analyzing that part



current divides

$$i_3(t) = \frac{15}{25}\,i = \frac{3}{5}(9.6e^{-2t})$$

$$\boxed{i_3(t) = 5.76\,e^{-2t}\ A}$$

now for $t \to \infty$:

L_1 & L_2:

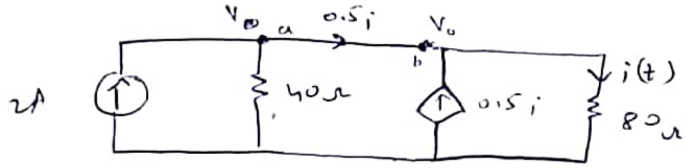
$i_1(t) = 1.6 - 9.6\,e^{-2t}$ → transient dies, $\boxed{i_1(t) = 1.6A}$

$i_2(t) = -1.6 - 9.6e^{-2t}$ → " " , $\boxed{i_2(t) = -1.6A}$

$i_3(t) = 5.76e^{-2t}$; $t \to \infty$, $\boxed{i_3(t) = 0_A}$. Inductors form a closed loop between themselves, & current flows between them, hence no current through resistors.

Ans 4: For $t<0$, capacitor charged so O.C

redraw



node voltage $\boxed{= V_o}$ $V_a = V_b$

for V_o, node equation, current entering = current leaving

so 0.5 i entering from a → b, so $\boxed{0.5i + 0.5i = i}$

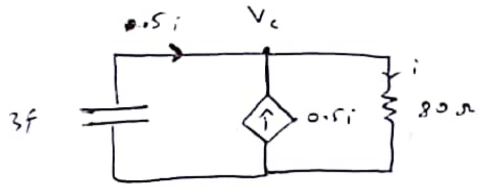
node a:

$$0.5i = 2 - \frac{V_o}{40}$$

$\therefore \boxed{i = \frac{V_o}{80}}$ → so $\frac{V_o}{160} = 2 - \frac{V_o}{40}$ $\boxed{V_o = 64\,V}$

hence, $i = \frac{V_o}{80}$ → $\boxed{i = 0.8A}$ for $\boxed{t<0}$

for $t>0$; redraw



using OPE

$V_c = V_o\, e^{-t/\tau}$

$\boxed{\tau = R_{th} C}$ $\boxed{V_o = 64\,v}$

Find R_{th}:

attach test source of 1V in place of capacitor

$\boxed{I_{test} = 0.5i}$

$i = \frac{V_c}{80}$ $\quad\therefore\quad$ $V_c = 1\,v$

$\boxed{i = \frac{1}{80}}$ $\longrightarrow$ so $\boxed{I_{test} = \frac{1}{160}}$

$R_{th} = \frac{1}{I_{test}} = \boxed{160\,\Omega}$

so $\tau = 160(3)$

$\boxed{\tau = 480}$

so, $\boxed{V_c(t) = 64e^{-t/480}\ v}$

now, current through capacitor = $-0.5i$ [opposite direction]

$$c\frac{dv_c(t)}{dt} = -0.5\,i(t)$$

$$i(t) = -2c\frac{dv_c(t)}{dt}$$

$$i(t) = -6 \times \frac{64}{480} e^{-t/480}$$

$$\boxed{i(t) = 0.8\,e^{-t/480}\ A\ ;\ t > 0}$$

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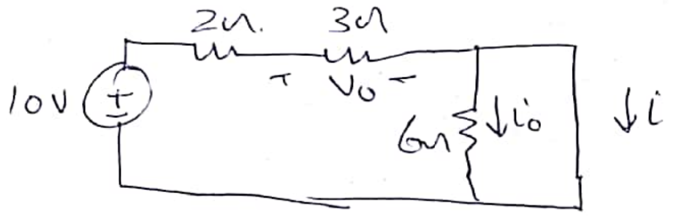
Q5

For $t<0$, The switch is open. Since Inductor acts like a Short circuit to dc, the 6-Ω resistor is also Short circuited



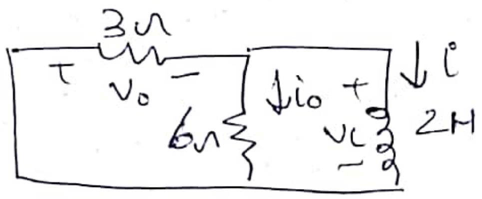
$\boxed{\text{hence } i_o = 0}$

$$i(t) = \frac{10}{2+3} = 2A \quad t<0$$

$$V_o(t) = 3i(t) = 6V \quad t<0$$

$$i(0) = 2$$

For $t>0$ now voltage source is S.C



$$R_{TH} = 3 \| 6 = 2\Omega.$$

Time Constant:

$$\tau = \frac{L}{R_{TH}} = 1s.$$

Hence

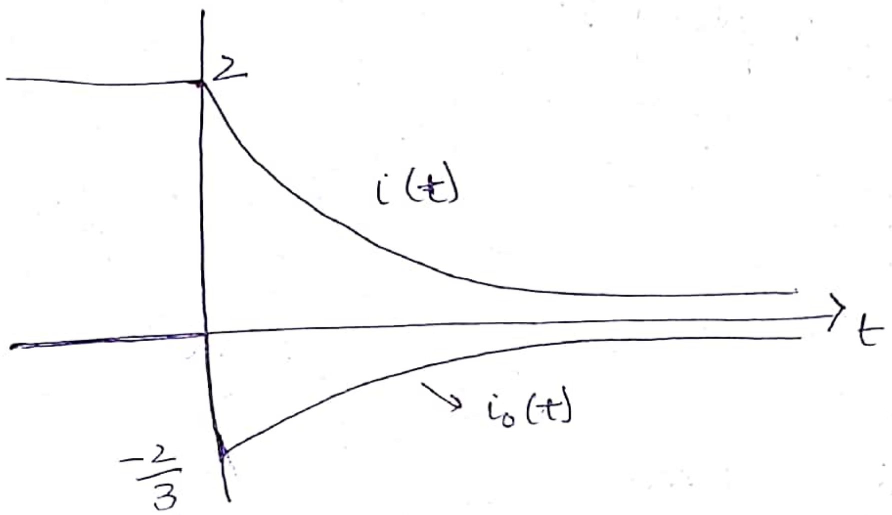
$$i(t) = i(0)e^{-t/\tau} = 2e^{-t} A \quad t>0$$

Since Inductor is parallel with 6-Ω and 3Ω. resistors, :

$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}\,V, \quad t>0$$

and

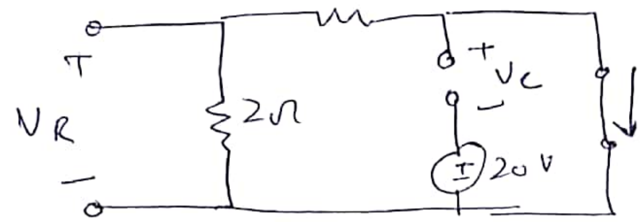
$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t}\,A \qquad t>0$$



Q6

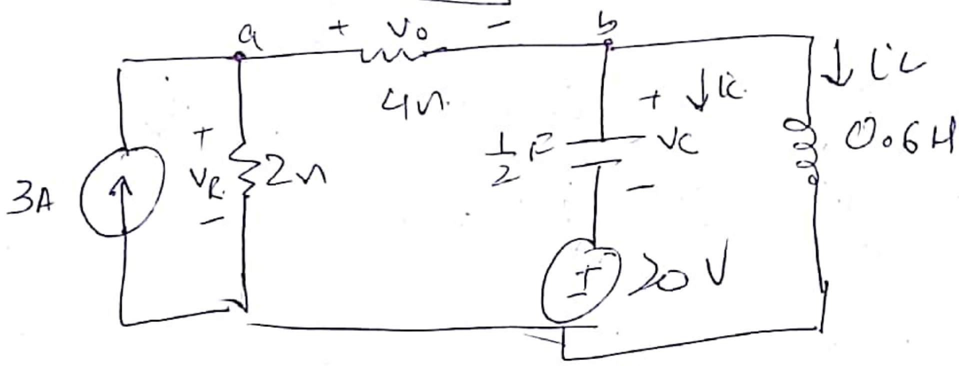
(a) For $t<0$, $3u(t)=0$; Inductor $\Rightarrow$ S.C,
Capacitor $\Rightarrow$ O.C

$i_L(0^-)=0$, $V_R(0^-)=0$ $\quad V_C(0^-)=-20V$



For $t>0$, $3u(t)=3$, $\Rightarrow$ so source is 3A
Since Inductor Current and Capacitor voltage cannot abruptly change hence:

$\boxed{i_L(0^+)=i_L(0^-)=0}$ $\quad \boxed{V_C(0^+)=V_C(0^-)=-20}$



KCL at node a:

$$3=\frac{V_R(0^+)}{2}+\frac{V_o(0^+)}{4}$$

Applying KVL to the middle mesh:

$$-V_R(0^+)+V_o(0^+)+V_c(0^+)+20=0$$

Since $V_c(0^+)=-20V$ so $V_R(0^+)=V_o(0^+)$

$$\boxed{V_R(0^+)=V_o(0^+)=4V}$$

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(b) We know $L \frac{di_L}{dt} = V_L$

hence

$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

KVL in Right Most Mesh :

$$V_L(0^+) = V_C(0^+) + 20 = 0$$

$$\therefore \boxed{\frac{di_L(0^+)}{dt} = 0}$$

$$C \frac{dV_C}{dt} = i_C \Rightarrow \frac{dV_C}{dt} = \frac{i_C}{C}$$

KVL at node b

$$\frac{V_0(0^+)}{4} = i_C(0^+) + i_L(0^+)$$

Since $V_0(0^+) = 4$, $i_L(0^+) = 0$, $i_C(0^+) = \frac{4}{4} = 1$ then:

$$\frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{1}{0.5} = \boxed{2 = \frac{dV_C(0^+)}{dt}}$$

$\frac{dV_R(0^+)}{dt} \Rightarrow$ apply KCL at node a.

$$3 = \frac{V_R}{2} + \frac{V_0}{4}$$

Take its derivative and setting $t = 0^+$

$$0 = 2\frac{dV_R(0^+)}{dt} + \frac{dV_0(0^+)}{dt}$$

KVL to middle mesh

$$-V_R + V_C + 20 + V_0 = 0$$

$$\frac{dV_R(0^+)}{dt} + \frac{dV_C(0^+)}{dt} + \frac{dV_0(0^+)}{dt} = 0$$

Substitute $\frac{dv_C}{dt}(0^+) = 2$

$$\frac{dv_R(0^+)}{dt} = 2 + \frac{dv_o(0^+)}{dt}.$$

hence, $\boxed{\frac{dv_R(0^+)}{dt} = \frac{2}{3} \text{ V/s}}$

(Q7)

Using KCL at node 'V_1'

For $t \geq 0$

$$\frac{V_1 - V(t)}{R_1} + C_1 \frac{dV_1}{dt} + \frac{(V_1 - V_2)}{R_2} + \frac{1}{L}\int (V_1 - V_3)\,dt = 0 \quad (i).$$

Using KCL at node 'V_2'

$$\frac{V_2 - V_1}{R_2} + C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt = 0 \quad (ii)$$

Using KCL at node 'V_3'

$$\cancel{\frac{1}{L_3}} \frac{1}{L_1}\int (V_3 - V_1)\,dt + C_3 \frac{dV_3}{dt} = 0 \quad (iii)$$

At $t = 0+$ C_1 becomes short circuit as a result of which:

$V_1(0^+) = 0V$; $V_2(0^+) = 0V$; $V_3(0^+) = 0V$.

At $t = 0+$

$$\frac{1}{L_1}\int (V_3 - V_1)\,dt + C_3 \frac{dV_3}{dt} = 0.$$

$$i_{L_1}(0^+) + C_3 \frac{dV_3(0^+)}{dt} = 0.$$

After Simplification:

$$\boxed{\frac{dV_3(0^+)}{dt} = 0}$$

Here $i_{L1}(0^-) = i_{L1}(0^+) = 0A$

At $t = 0+$

$$\frac{V_2(0^+) - V_1(0^+)}{R_2} + C_2 \frac{dV_2(0^+)}{dt} + i_{L2}(0^+) = 0 \quad \text{(ii)}$$

$i_{L2}(0-) = i_{L2}(0^+) = 0A$

after simplification

$$\boxed{\frac{dV_2(0^+)}{dt} = 0}$$

At $t = 0+$ eq (i):

$$\frac{V_1(0^+) - V(0^+)}{R_1} + C_1 \frac{dV_1(0^+)}{dt} + \frac{(V_1(0^+) - V_2(0^+))}{R_2} + \underbrace{\frac{1}{L}\int V_1(0^+) - V_3(0^+)\,dt}_{i_3(0^+)} = 0$$

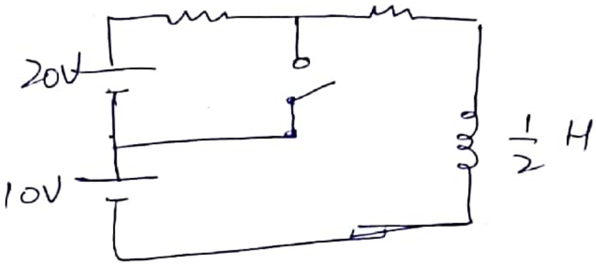
hence :

$$\boxed{\frac{dV_1(0^+)}{dt} = 0}$$

Q8

For $t<0$;

$-10-20+30i(0^-)+20i(0^-)+0=0$



$-30+50i(0^-)=0$

$i(0^-)=\frac{3}{5}=0.6$

For $t>0$;

$-10+20\,i(t)+\frac{1}{2}\frac{di(t)}{dt}=0$

$\frac{di(t)}{dt}+40\,i(t)=20$

$I.F=e^{40t}$

$\therefore\ i(t)\,e^{40t}=\int 20e^{40t}\,dt+K$

$i(t)\,e^{40t}=\frac{20e^{40t}}{40}+K$

$i(t)=0.5+Ke^{-40t}$

$i(0)=0.5+K=0.6$

$K=0.1$

$i(t)=0.5+0.1\,e^{-40t}$

$\frac{1}{T}=40 \qquad T=\frac{1}{40}$ s.