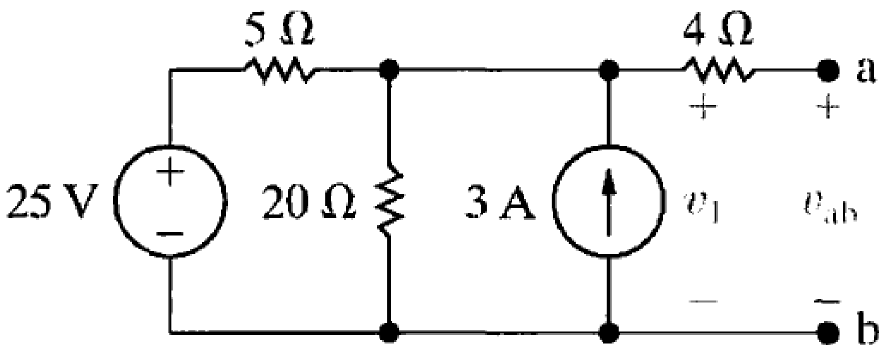
Thevenin's and Norton's Theorems

Problems – In class

Problem 1: Find the Thevenin's equivalent circuit for the following circuit.



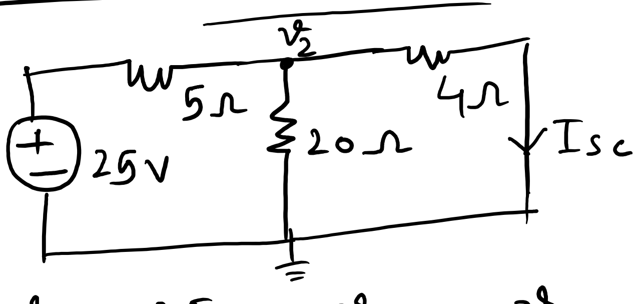
V_{th}:

* $V_{oc} = V_{ab} = v_1$

* $\frac{v_1 - 25}{5} + \frac{v_1}{20} = 3$

$\frac{v_1}{5} + \frac{v_1}{20} = 8 \Rightarrow \boxed{v_1 = 32V}$

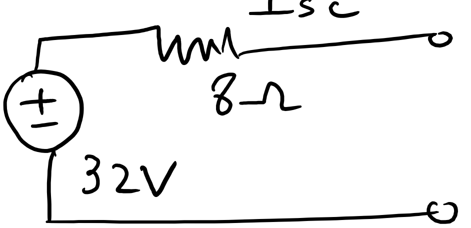
R_{th}: Find I_{sc}:



$\frac{v_2 - 25}{5} + \frac{v_2}{20} + \frac{v_2}{4} = 0$

$v_2 = 16V \Rightarrow \boxed{I_{sc} = 4A}$

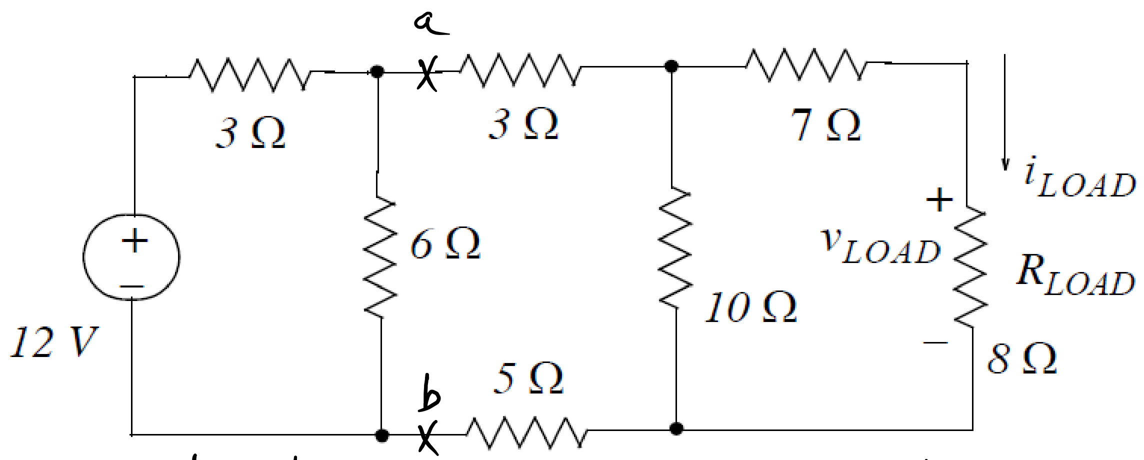
$R_{th} = \frac{v_{th}}{I_{sc}} = 8\Omega$



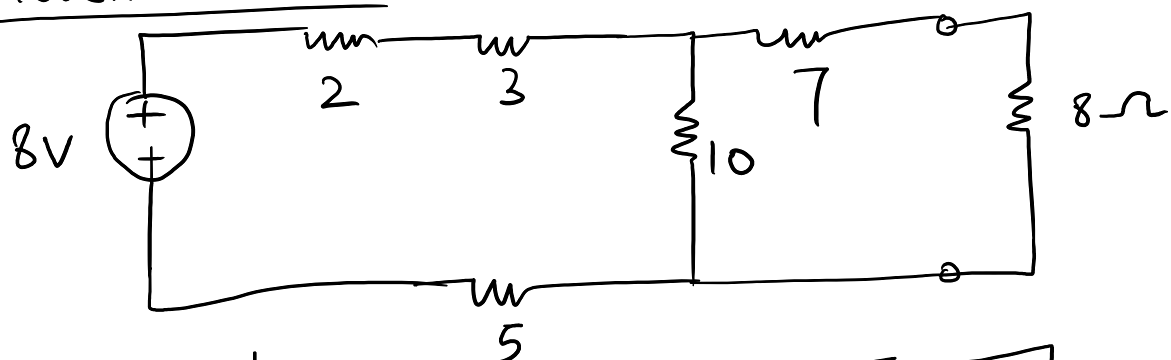
Thevenin's and Norton's Theorems

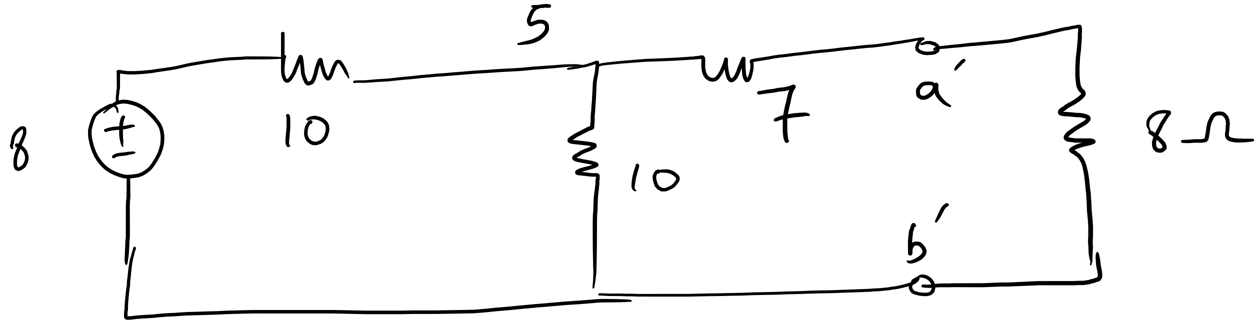
Problems – In class

Problem 2: Find i_{LOAD} through R_{LOAD} using Thevenin's theorem

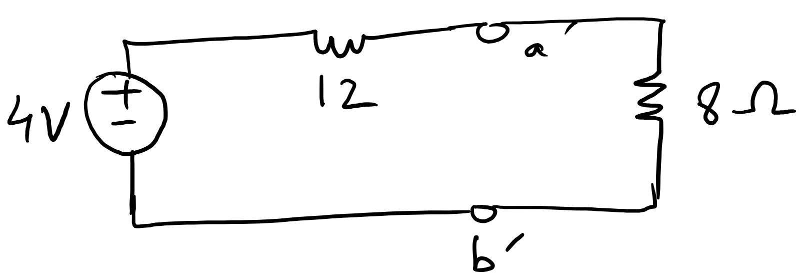


Thevenin at ab





Thevenin at a′ – b′

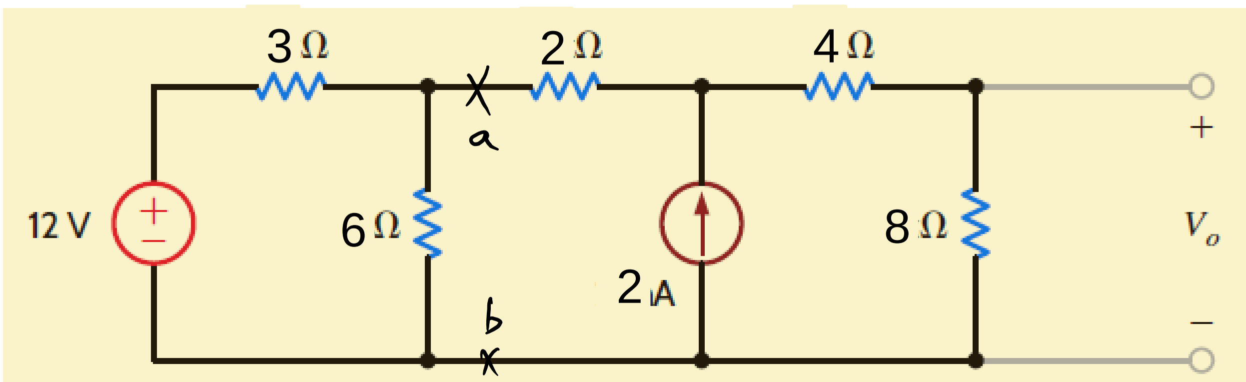


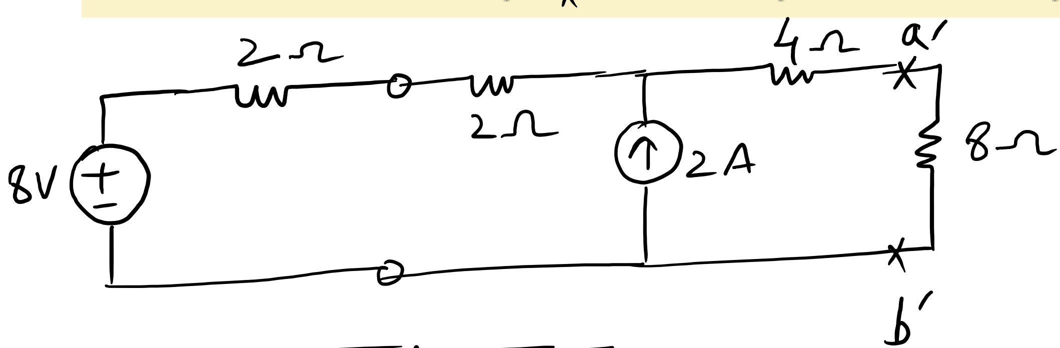
$$i_{LOAD} = \frac{4}{20} = 0.2A$$

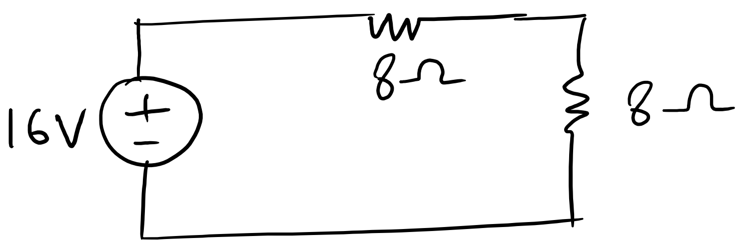
Thevenin's and Norton's Theorems

Problems – In class

Problem 3: Find V_o using Thevenin's theorem



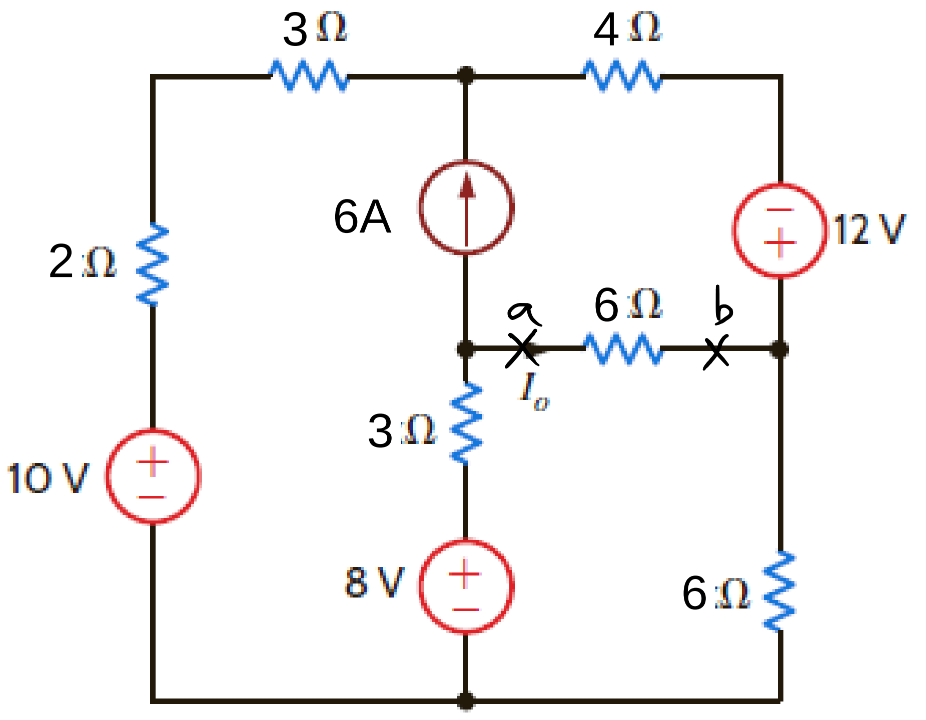




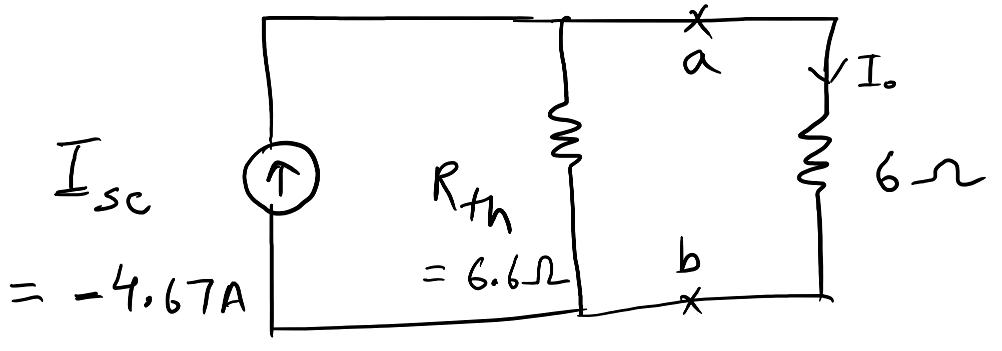
$$V_o = 16 \times \frac{8}{16} = \boxed{8V}$$

Problems – In class

Problem 4: Find I_o using Thevenin’s or Norton’s theorem



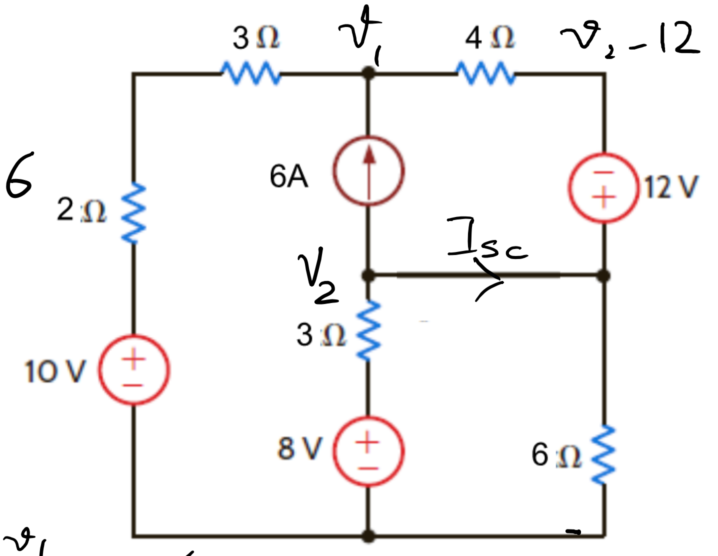
Equivalent: (computed on next page)



$$I_o = \frac{6.6}{12.6} \times (-4.67) = \boxed{-2.44A}$$

Thevenin's and Norton's Theorems

Problems – In class



$$\frac{v_1 - 10}{5} + \frac{v_1 - (v_2 - 12)}{4} = 6$$

$$\boxed{9v_1 - 5v_2 = 100}$$

$$\frac{v_2 - 8}{3} + \frac{v_2}{6} + \frac{v_2 - 12 - v_1}{4} = -6$$

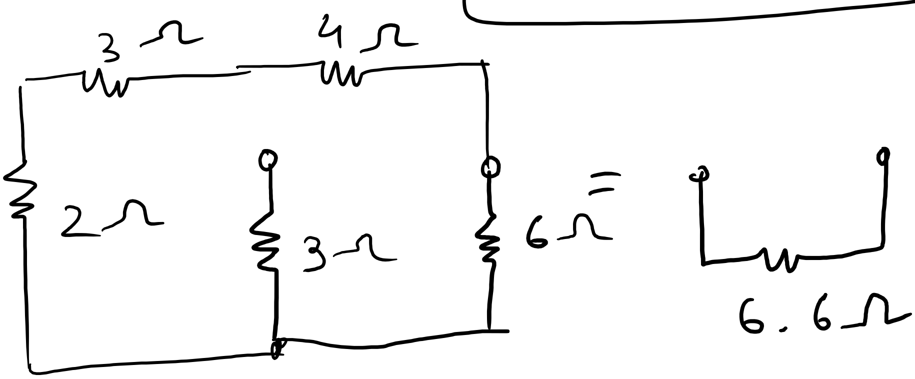
$$\Rightarrow \frac{4v_2 - 24 + 2v_2 + 3v_2 - 36 - 3v_1}{12} = -6$$

$$\boxed{-3v_1 + 9v_2 = -4} \Rightarrow -9v_1 + 27v_2 = -12$$

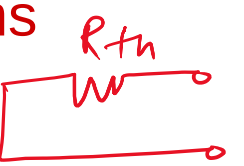
$$\Rightarrow 22v_2 = 88 \Rightarrow \boxed{v_2 = 4V}$$

$$\frac{v_2 - 8}{3} + 6 + I_{sc} = 0 \Rightarrow \boxed{I_{sc} = -4.667\,A}$$

Rth :

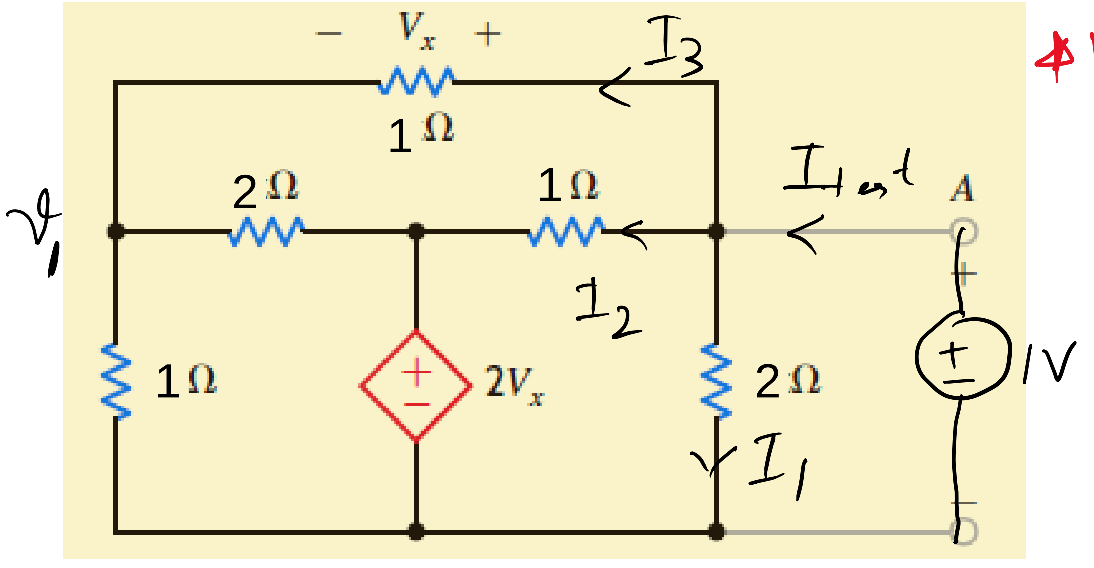


Thevenin's and Norton's Theorems



Problems – In class

Problem 5: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – Example 5.8)



$V_{th} = 0V$

We have

$$v_x + v_1 = 1V \Rightarrow v_1 = 1 - v_x$$

KCL;

$$\frac{v_1}{1} + \frac{v_1 - 2v_x}{2} + \frac{v_1 - 1}{2} = 0$$

$$\Rightarrow 1 - v_x + \frac{1 - 3v_x}{2} - v_x = 0$$

$$\Rightarrow 3 = 7v_x \Rightarrow \boxed{v_x = \frac{3}{7} V}$$

$$I_1 = \frac{1}{2}A, \quad I_2 = \frac{1 - 2(3/7)}{1} = 1/7\, A$$

$$I_3 = 3/7\, A$$

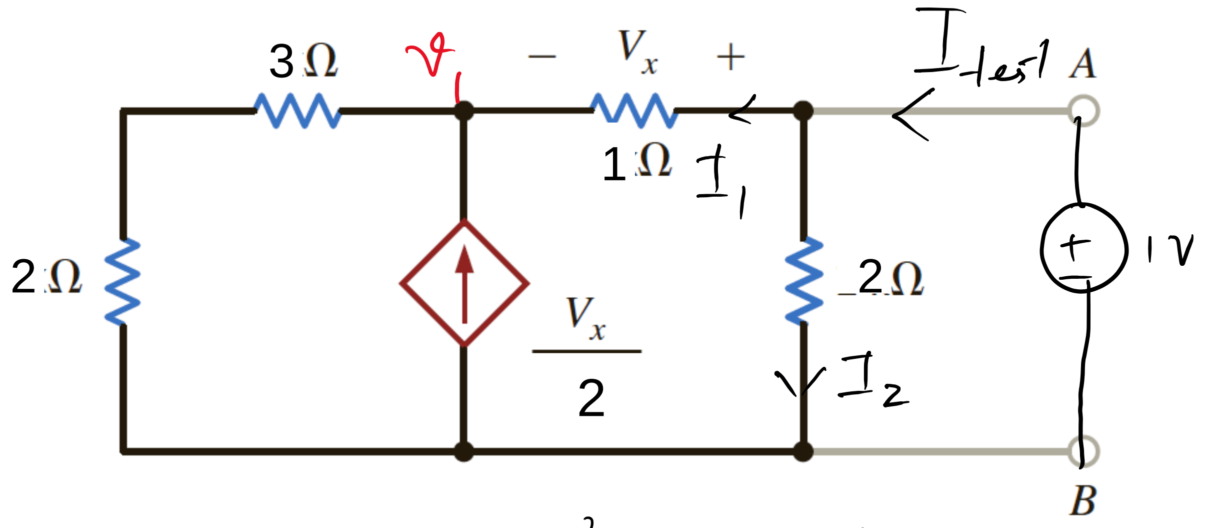
$$I_{test} = I_1 + I_2 + I_3 = \boxed{\frac{15}{14} A}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{14}{15}\,\Omega$$

Thevenin’s and Norton's Theorems

Problems – In class

Problem 6: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – E 5.13)



$$\frac{v_1}{5} + \frac{v_1 - 1}{1} = \frac{V_x}{2}$$

$$v_x + v_1 = 1 \Rightarrow v_x = 1 - v_1$$

$$\Rightarrow \boxed{v_1 = \frac{15}{17}\ V}$$

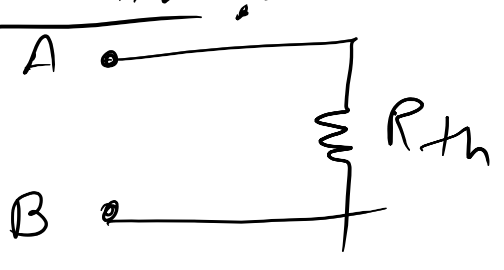
$$I_{test} = I_1 + I_2 = \frac{1}{2} + \frac{2}{17} = \frac{21}{34}\ A$$

$$I_1 = \frac{1 - v_1}{1} = \frac{2}{17}\ A$$

$$I_2 = \frac{1}{2}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{34}{21}\ \Omega$$

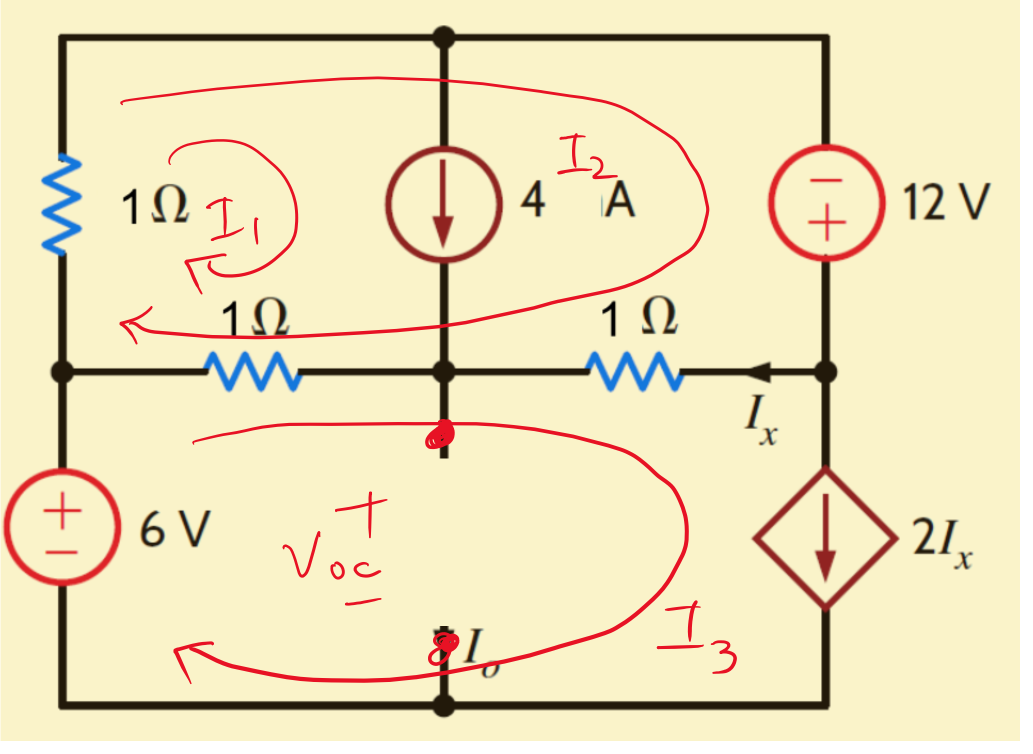
Equivalent :-



Thevenin's and Norton's Theorems

Problems – In class

Problem 7: Find I_o using Thevenin's theorem (See problem sheet for problems)



$I_1 = 4A, \quad I_3 = 2I_x$

Loop 2

$$-12 + 1(I_2 - I_3) + 1(I_1 + I_2 - I_3) + (I_1 + I_2)1 = 0$$

$$I_x = I_2 - I_3 \Rightarrow \boxed{3I_x = I_2}$$

Solving

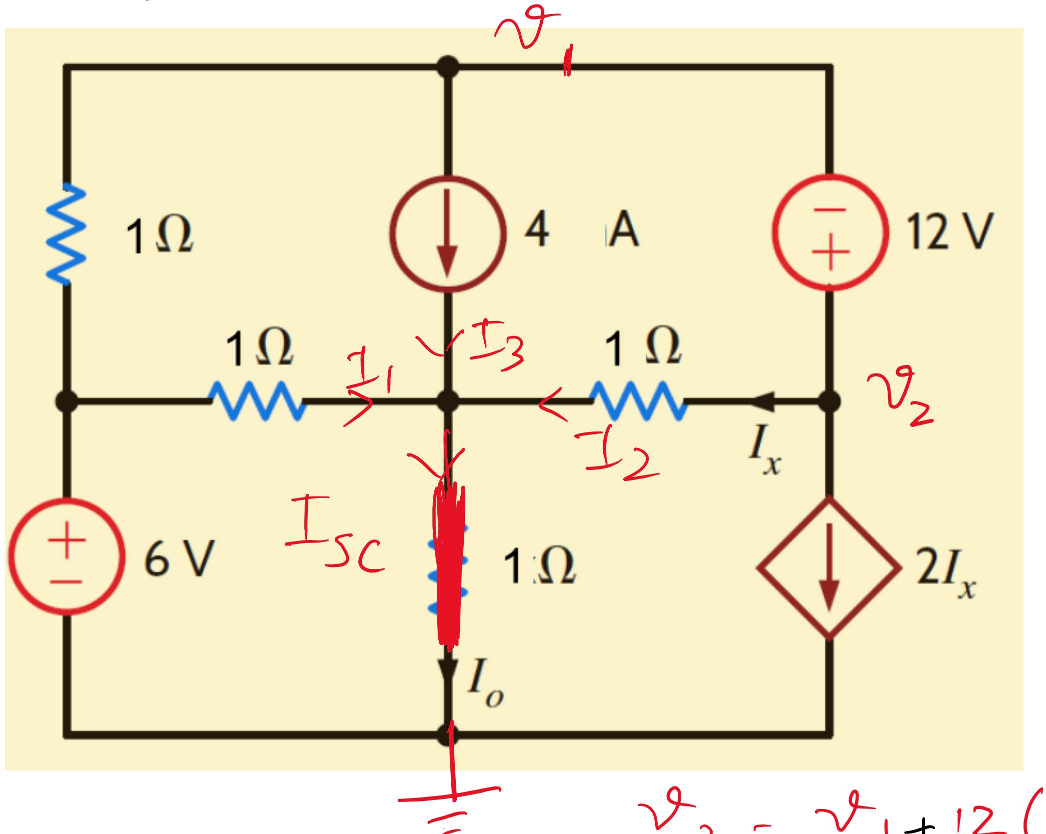
$$I_3 = \frac{8}{5}A, \quad I_2 = \frac{12}{5}A, \quad I_x = \frac{4}{5}A$$

$$V_{oc} = 6 + (I_1 + I_2 - I_3)1 = \frac{54}{5}V$$

Thevenin's and Norton's Theorems

Problems – In class

Problem 7: Find I_o using Thevenin's theorem



$v_2 = v_1 + 12$ (Super Node)

$$\frac{v_1 - 6}{1} + 4 + \frac{v_2}{1} + 2I_x = 0 \qquad I_x = \frac{v_2}{1}$$

$$\Rightarrow v_1 - 6 + 4 + 3v_1 + 36 = 0$$

$$\Rightarrow v_1 = -\frac{17}{2}\,V, \quad v_2 = 7/2$$

$$I_{sc} = \frac{6}{1} + \frac{7}{2} + 4 = \frac{27}{2}\,A$$

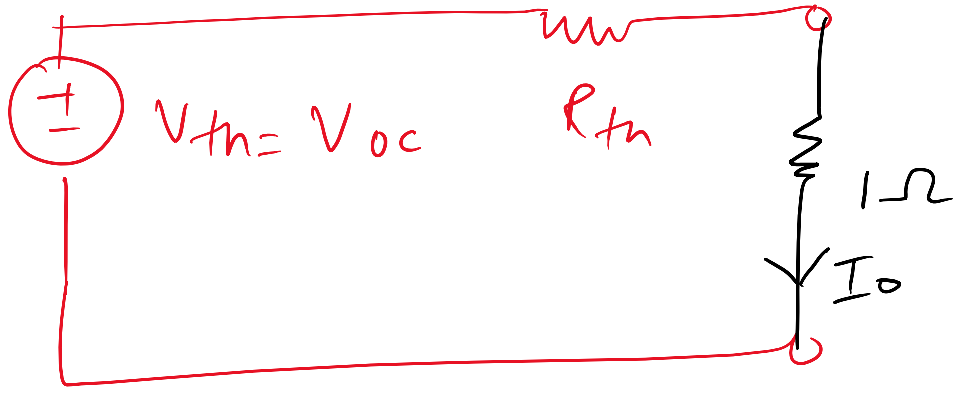
$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{54/5}{27/2} = \frac{4}{5}\,\Omega$$

Thevenin's and Norton's Theorems

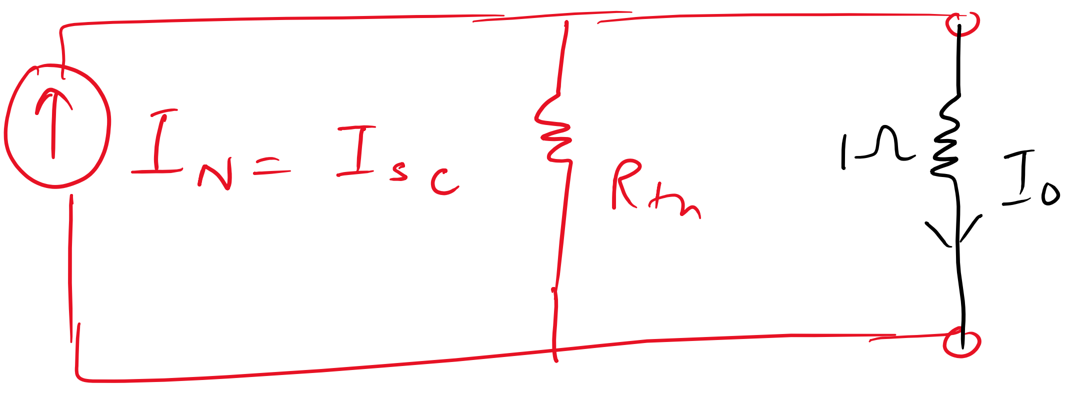
Problems – In class

Problem 7: Find I_o using Thevenin's theorem

Thevenin Equivalent



Norton Equivalent

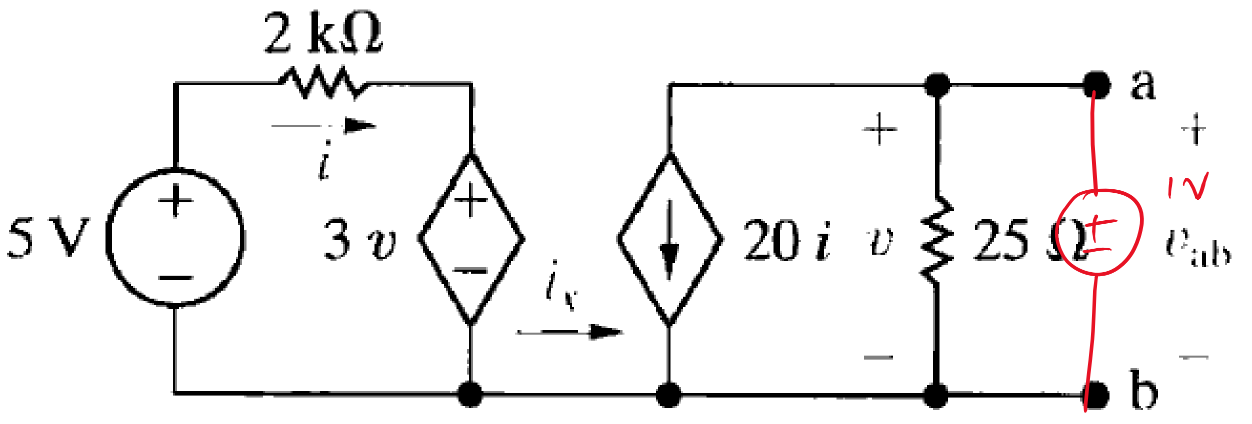


$$I_o = \frac{54/5}{1 + \frac{4}{5}} = \boxed{6\ A}$$

Thevenin's and Norton's Theorems

Problems – In class

Problem 8: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



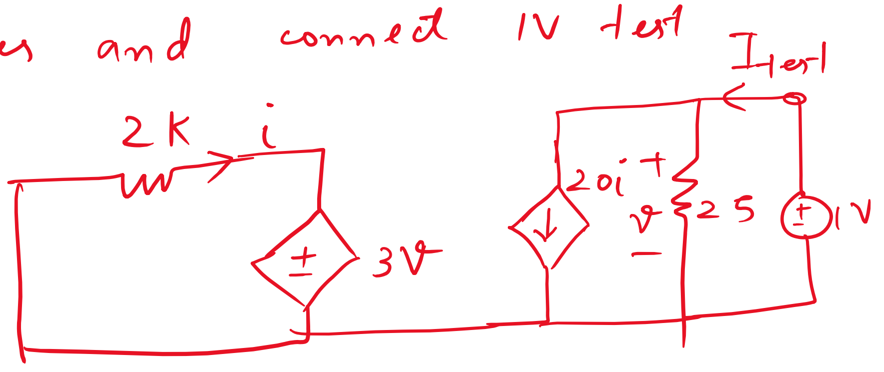
First find R_{th} :-

Switch off sources and connect 1V test source

$v = 1V$

$3v = 3V$

$i = -3/2k$



$\Rightarrow \quad I_{test} = \frac{1}{25} - (20)(i) = 0.04 - 0.03 = 0.01A$

$\Rightarrow \quad \boxed{R_{th} = 100\Omega} \leftarrow \frac{1}{I_{sc}}$

V_{th} :- $V_{ab} = v = v_{th}$

$V_{th} = -20i \times 25$ — ①

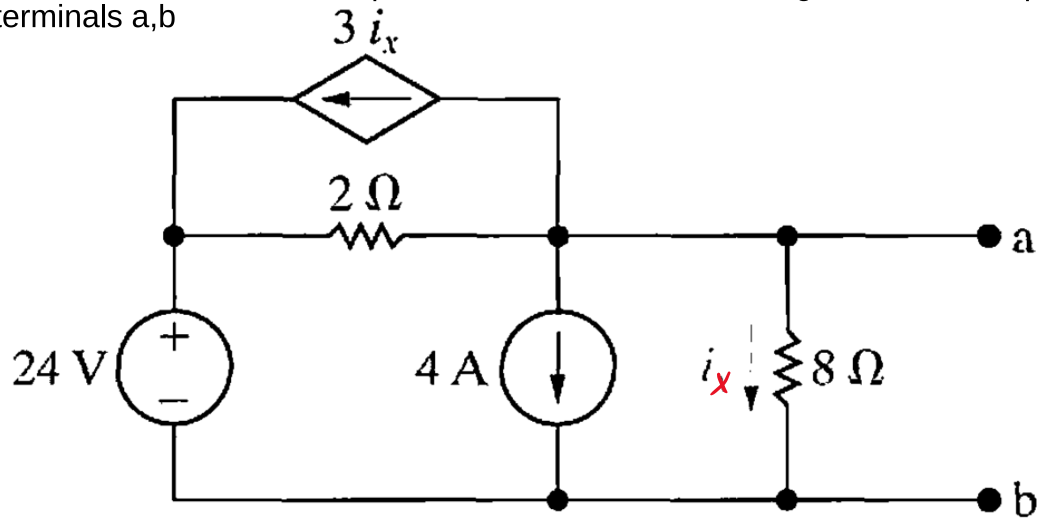
$i = \frac{25 - v_{th}}{2k}$ — ②

①+② $\Rightarrow \quad \boxed{V_{th} = -5V}$

Thevenin's and Norton's Theorems

Problems – In class

Problem 9: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



* Circuit contains both dependent and Independt Sources; we can use either of the following techniques

1) Determine V_{ab} and I_{sc}
2) Determine V_{ab} ; Determine R_{Th} by switching off independent sources and applying test current (or voltage) source at a-b.

Let's apply 2)

Using KCL;

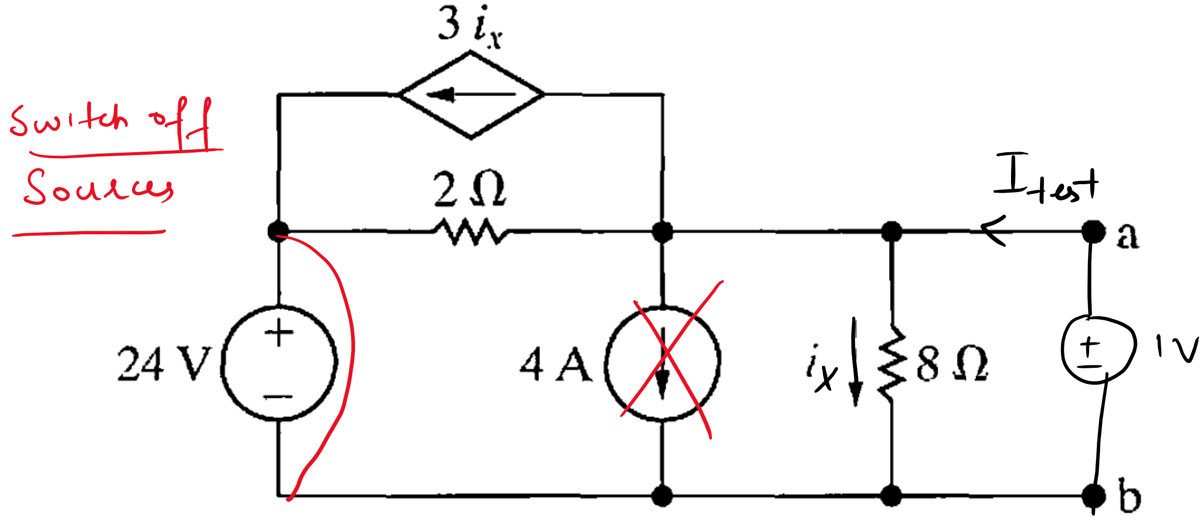
V_{ab} :

$$\frac{V_{ab}}{8} + \frac{V_{ab}-24}{2} + 3\,i_x + 4 = 0$$

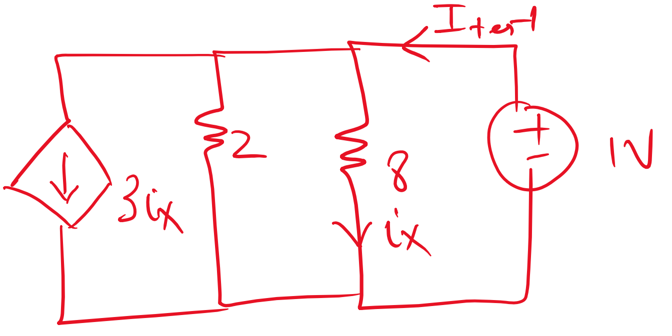
where $i_x = \dfrac{V_{ab}}{8} \Rightarrow$ $\boxed{V_{ab} = 8\,i_x}$

Solving $\boxed{V_{ab} = 8V}$

R_{th} : Apply 1V voltage source :



$i_x = \frac{1}{8}$ A



$I_{test} = \frac{1}{8} + \frac{1}{2} + \frac{3}{8}$

$= 8/8 = 1A$

$\Rightarrow R_{th} = \frac{1V}{I_{test}} = \boxed{1\,\Omega}$