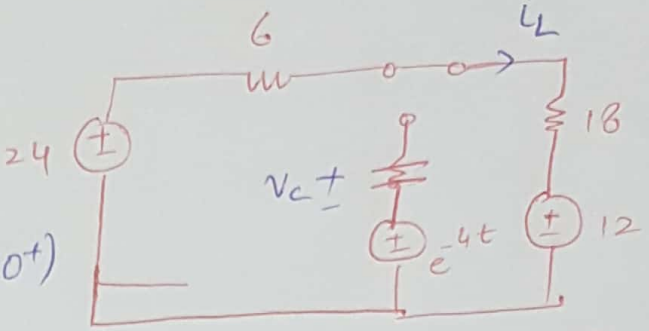
Quiz - 09

Section 01

Solution

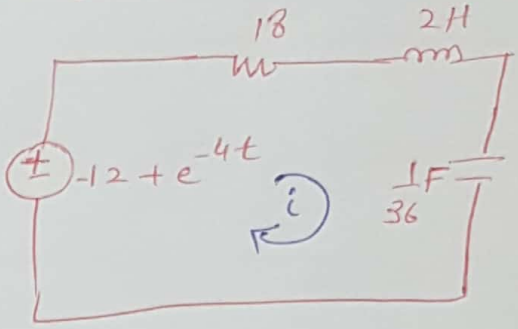


At $t=0^-$

Inductor sc

$$i_L(0^-) = \frac{24-12}{24} = 0.5A = i_L(0^+)$$

$$v_c(0^-) = v_c(0^+) = 0$$



At $t=0^+$ (series RLC network)

$$2\frac{di}{dt} + 18i + \frac{1}{C}\int i\,dt = e^{-4t} - 12 \quad (1)$$

$$\Rightarrow \boxed{\frac{d^2i}{dt^2} + 9\frac{di}{dt} + 18i = -2e^{-4t}} \quad (2)$$

$i(t) = i_c(t) + i_p(t)$

$i_c(t)$; Auxiliary equation

$s^2 + 9s + 18 = 0 \Rightarrow s_1 = -3$, $s_2 = -6$

$$\boxed{i_c(t) = K_1 e^{-3t} + K_2 e^{-6t}}$$

$$i_p(t) = Ae^{-4t}$$

$16Ae^{-4t} - 9Ae^{-4t} + 18Ae^{-4t} = -2e^{-4t}$

$$\Rightarrow \boxed{A = \frac{-2}{25}}$$

$$\boxed{i(t) = K_1 e^{-3t} + K_2 e^{-6t} - \frac{2}{25}e^{-4t}}$$

* Use initial conditions to find K_1 and K_2

$i(0) = i_L(0^+) = 0.5A = K_1 + K_2 - \frac{2}{25}$

$\frac{di}{dt}(0^+) = \frac{1 - 12 - 18i(0^+)}{2} = -10\,A/sec = -3K_1 - 6K_2 - \frac{8}{25}$

$$\Rightarrow \boxed{K_1 = -2.28,\quad K_2 = 2.86}$$

$$\boxed{v_0(t) = 18\,i(t) = -2.28e^{-3t} + 2.86e^{-6t} - \frac{2}{25}e^{-4t}, \quad t \geq 0}$$