EE240 – Circuits I

Mid Examination (Fall 2018)

November 6, 2018 06:00 pm–08:30 pm

Student ID ______________________________ Name ______________________________

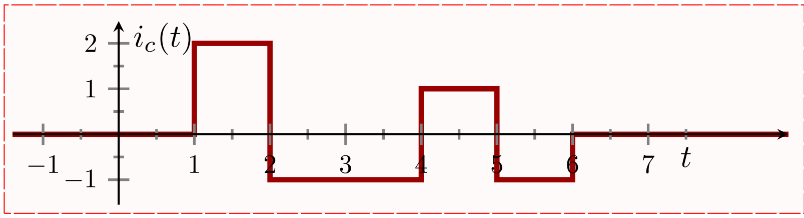
Signature ______________________

INSTRUCTIONS:

- Do not flip this page over until told to do so.
- **The exam needs to be solved on this book and not on blue book.**
- If you need the blue book for rough work, please ask the exam staff.
- The exam is closed book and notes. You are allowed to bring calculator and one A4 sheet with you with *hand-written* notes on both sides.
- Read all the questions before you start working on the exam.
- You cannot keep your mobile phone(s) with you (even on silent mode or switched off).

Part 1: Sources and I-V Characteristics of R, L, C

Problem 1. (**10 pts**) The current $i_c(t)$ through the capacitor of capacitance $\frac{1}{2}F$ is shown in Figure 1 below.



(a) (**1 pts**) Express $i_c(t)$ as piecewise function of time.

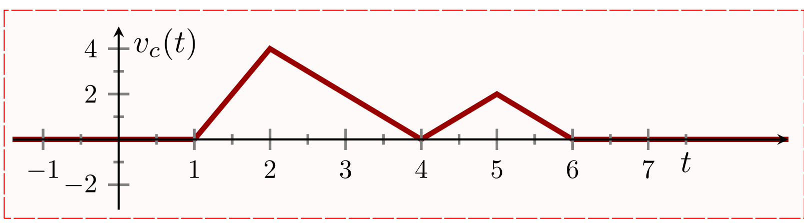
$$i_c(t) = \begin{cases} 0 & t < 1 \\ 2 & 1 \leq t < 2 \\ -1 & 2 \leq t < 4 \\ 1 & 4 \leq t < 5 \\ -1 & 5 \leq t < 6 \\ 0 & 6 \leq \end{cases}$$

(b) (**8 pts**) Assuming that the voltage is zero for times $t \leq -1$ seconds, determine the voltage across the capacitor and **plot** for $0 \leq t \leq 7$ seconds.

Let $v_c(t)$ be the voltage across capacitor.

$$v_c(t) = \frac{1}{C} \int_{-\infty}^{t} i_c(t)dt$$

$$v_c(t) = 2 \begin{cases} 0 & t < 1 \\ 2t - 2 & 1 \leq t < 2 \\ -t + 4 & 2 \leq t < 4 \\ t - 4 & 4 \leq t < 5 \\ -t + 6 & 5 \leq t < 6 \\ 0 & 6 \leq \end{cases}$$

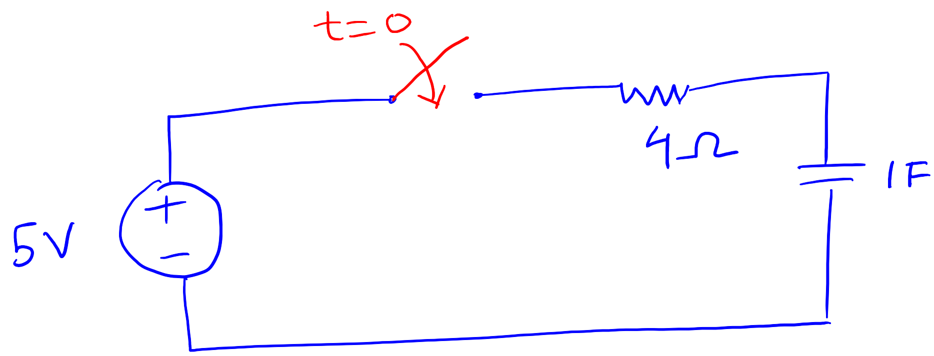


(c) (**1 pts**) Determine the energy stored in the capacitor at $t = 3.5$ seconds.

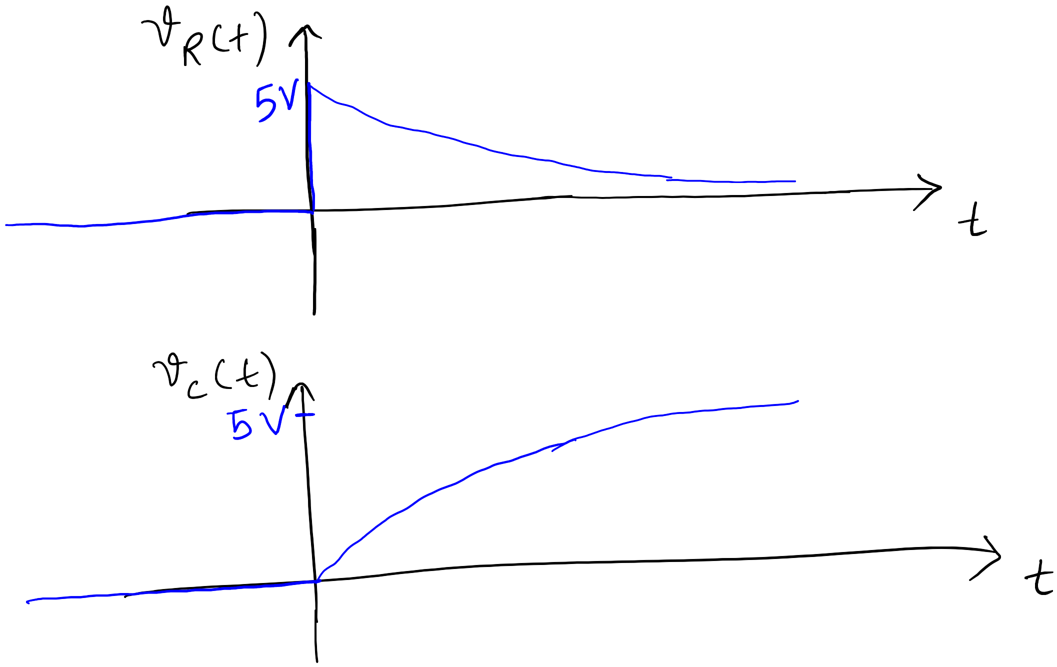
$w_c(t) = \frac{1}{2}C\big(v_c(t)\big)^2 \Rightarrow w_c(3.5) = \frac{1}{4}\big(v_c(3.5)\big)^2 = 0.25J.$

Problem 2. (**6 pts**) Consider a circuit where the DC voltage source of $5V$ is connected to a series combination of $4\ \Omega$ resistor and $1F$ capacitor through the switch. Assume that the switch is initially open and is closed at $t = 0$ and the capacitor is uncharged before the switch is closed, that is, the capacitor voltage $v_c(t) = 0$ for all $t < 0$.

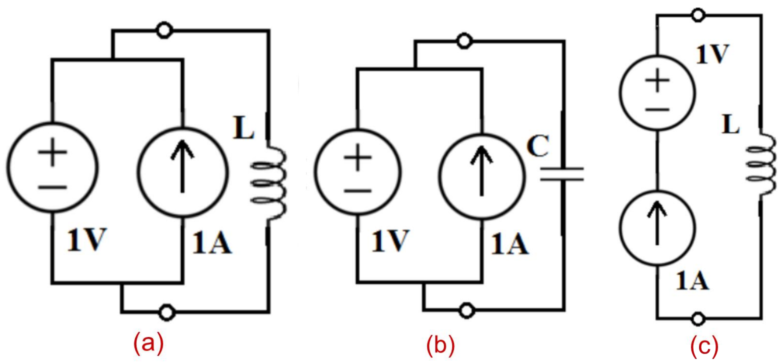
(a) (**1 pts**) Draw the circuit and indicate the current $i(t)$ through the circuit and the voltages $v_R(t)$ and $v_C(t)$ across the resistor and the capacitor respectively.

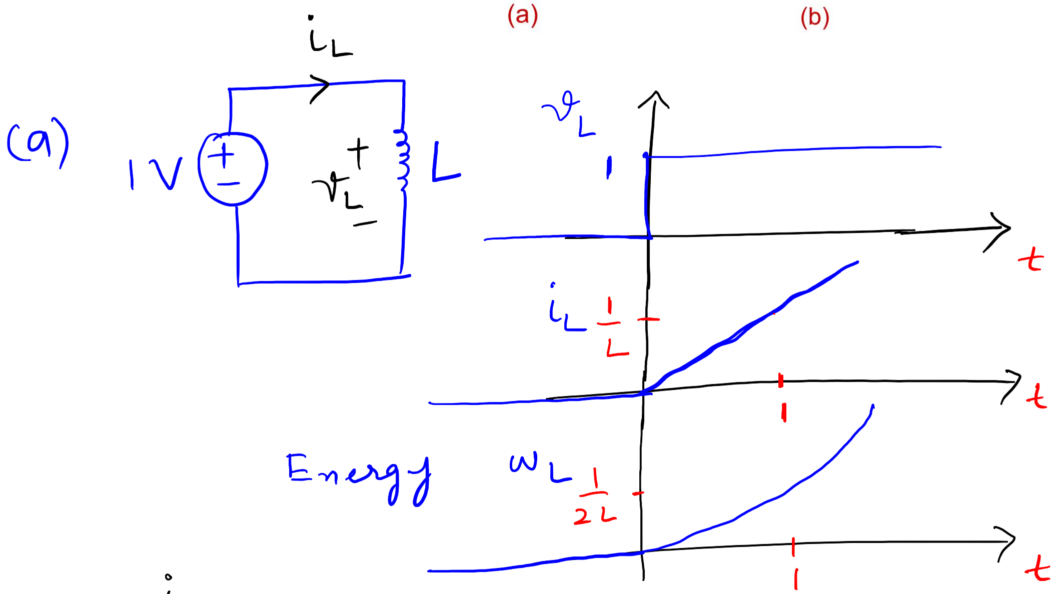


(b) (**5 pts**) Plot the waveforms (not to the scale) of the voltages $v_R(t)$ and $v_C(t)$.



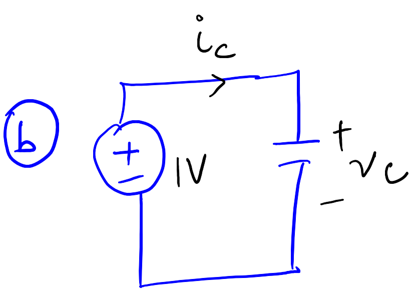
Problem 3. (**9 pts**) Consider the following four circuits. Assuming that the sources are switched on at $t = 0$ and the elements do not carry any current or voltage before the sources are turned-on, draw the voltage, current and energy waveform for $0 \le t \le 1$ seconds for each circuit (element). Total of 9 waveforms.

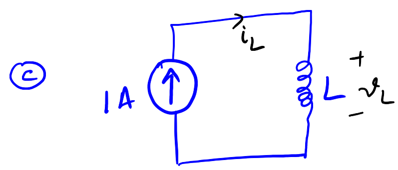


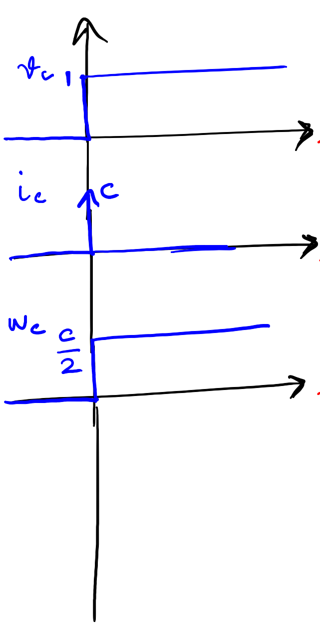


$$i_L = \frac{1}{L}\int_{-\infty}^{t} v\,dt = \begin{cases} \frac{t}{L} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$w_L = \frac{1}{2} L i^2 = \begin{cases} \frac{1}{2}\frac{t^2}{L} & t \geq 0 \\ 0 & t < 0 \end{cases}$$





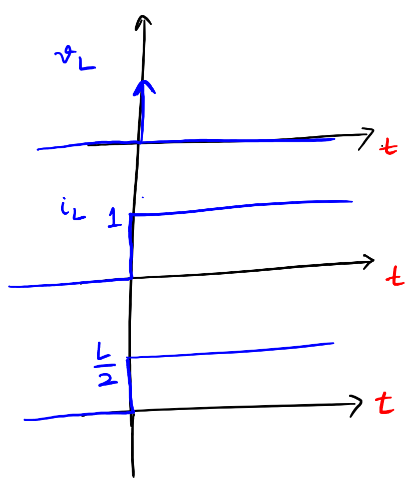


$i_c = C\frac{dv_c}{dt}$

impulse

$i_c = C\,\delta(t)$

$w_c = \frac{1}{2} C v_c^2$



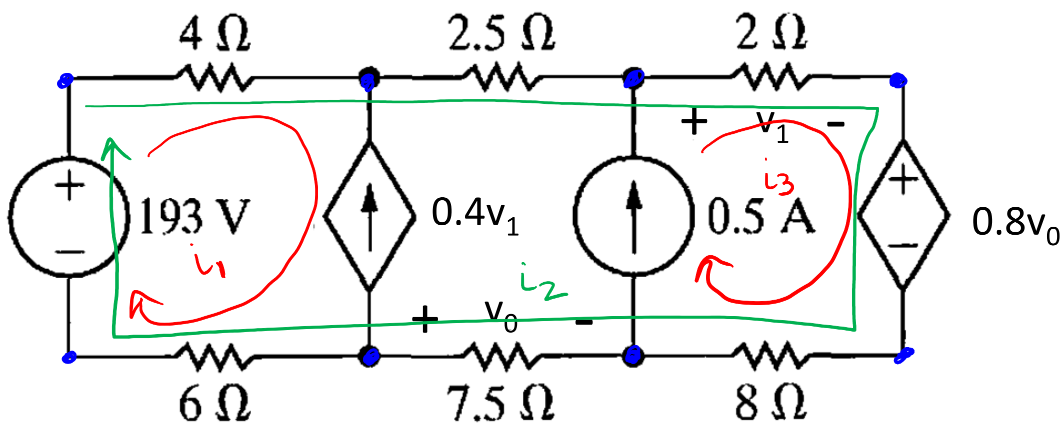
$v_L = L\frac{di_L}{dt} = L\,\delta(t)$

$w_L = \frac{1}{2} L i_L^2$

Part 2: Network Topology, Network Equations and Equivalent Circuits

Problem 4. (**20 pts**)

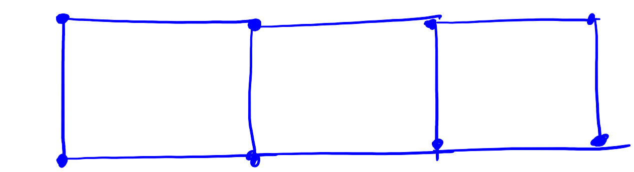
Consider the circuit given below.



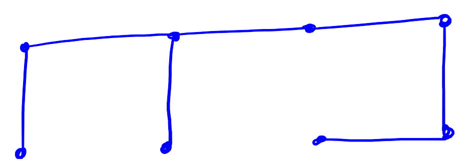
Nodes : Blue dots

(a) (**5 pts**) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in a circuit.

Graph



Tree (one possibility)



no. of nodes = n = 8

no. of branches = b = 10

(b) (**2 pts**) Determine the number of network equations required for carrying out i) nodal analysis and ii) loop analysis.

Nodal Analysis : No. of equations = $n-1 = 7$

Loop Analysis : No. of equations = $b-(n-1) = 3$

(c) (**9 pts**) Carry out the loop analysis, that is, identify and determine the loop currents.

Controlled Sources

$v_o = -7.5 i_2$

$v_1 = (i_2 + i_3)(2)$

$\boxed{v_1 = 2 i_2 + 1}$

Currents indicated on circuit

$i_1 = -0.4 v_1 = -0.8 i_2 - 0.4$

$\boxed{i_3 = 0.5A}$

Green loop :

$10 i_1 + 30 i_2 + 10 i_3 + 0.8 v_o = 193$

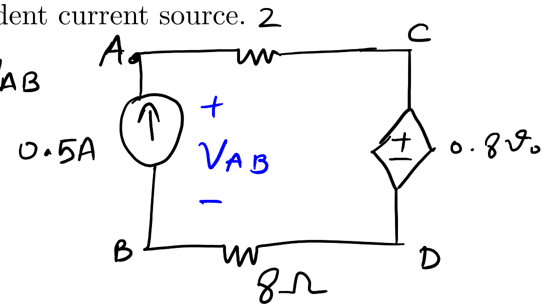
$10(-0.8 i_2 - 0.4) + 30 i_2 + 5 + (0.8)(-7.5 i_2) = 193$

$16 i_2 = 192 \Rightarrow \boxed{i_2 = 12A}$

$i_1 = -0.8 i_2 - 0.4$

$\boxed{i_1 = -10A}$

(d) (**4 pts**) Determine the power delivered by the independent current source. 2



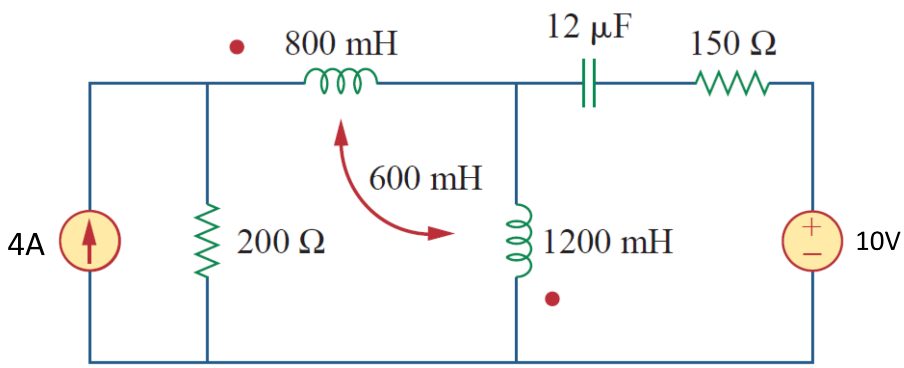
We find the voltage across current source as V_{AB}

$v_{AB} = v_{AC} + v_{CD} + v_{DA}$

$= 2(i_2 + i_3) + (0.8)(-7.5 i_2) + 8(i_2 + i_3)$

$= 25 - 72 + 10$

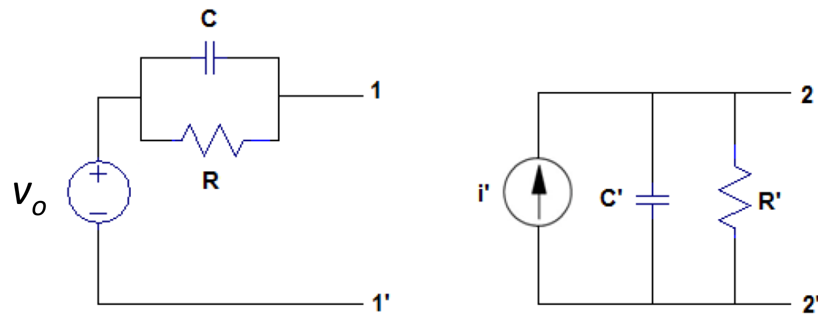
Problem 5. (**10 pts**) Consider the circuit given below.



(a) (**5 pts**) Formulate the network equations using loop analysis.

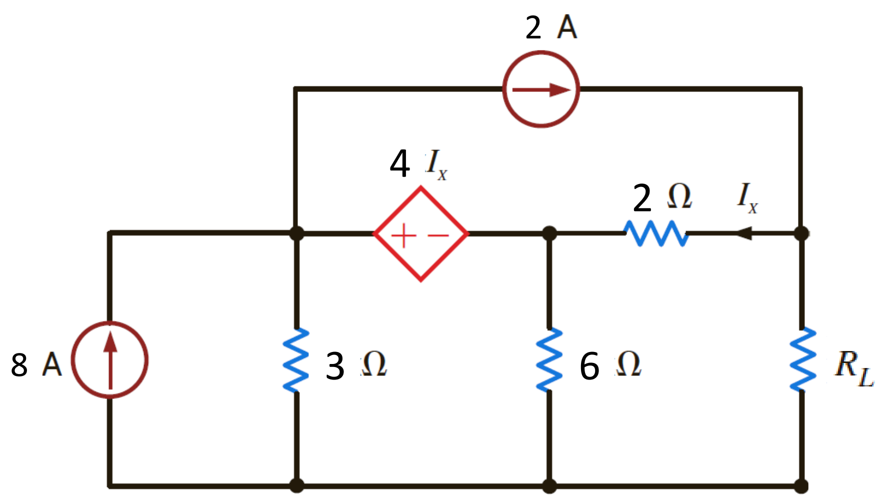
(b) (**4 pts**) Find an equivalent circuit where each coupled inductor is replaced with an uncoupled inductor and a controlled source. Draw the equivalent circuit.

Problem 6. (**6 pts**) Find the values of i', R' and C' in terms of R, C and v_o such that the networks shown below are equivalent at terminals 11 and 22.

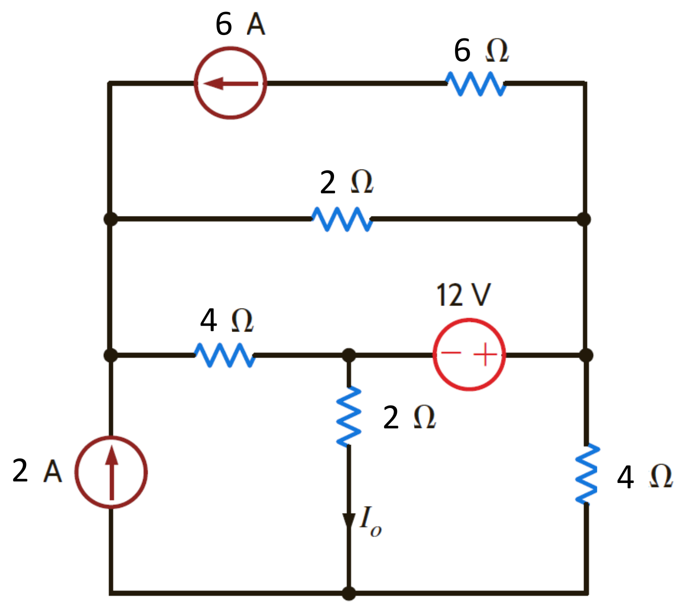


Part 3: Additional Analysis Techniques

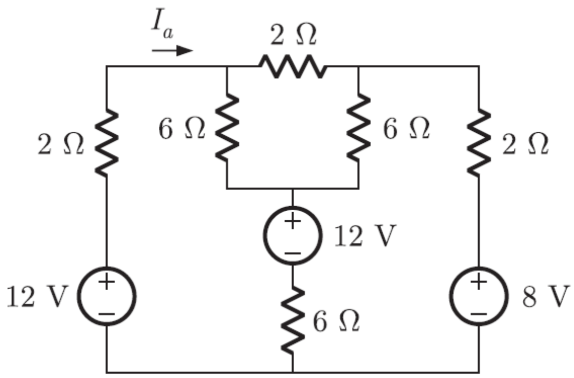
Problem 7. (**12 pts**) For the circuit given below, determine the value of R_L for maximum power transfer to R_L using Thevenin's Theorem.



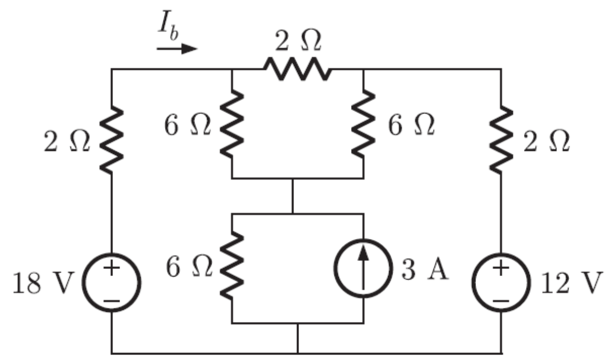
Problem 8. (**9 pts**) Determine I_o using the superposition theorem (principle) for the circuit given below.



Problem 9. (**4 pts**) Given the following two circuits, determine the relation between I_a and I_b. You must provide justification to support your answer.
(Hint: Use the concept of linearity.)



(a)



(b)

Problem 10. (**5 pts**) Consider the network shown below, where the rest of the circuit is pure resistive in nature. The voltage $V_R = 6V$ for $R = 4\Omega$ and the current $i_R = 2.4A$ for $R = 0\Omega$. Determine the voltage V_R in volts when $R = \infty$.