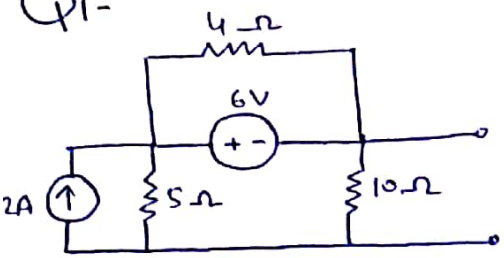
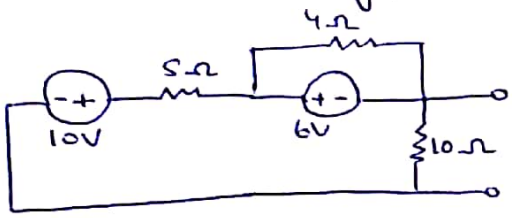
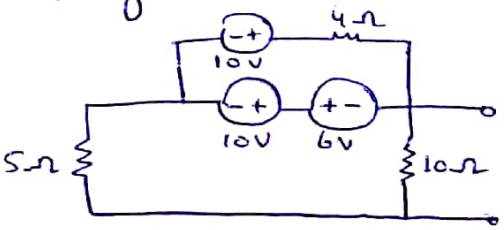
Q1-



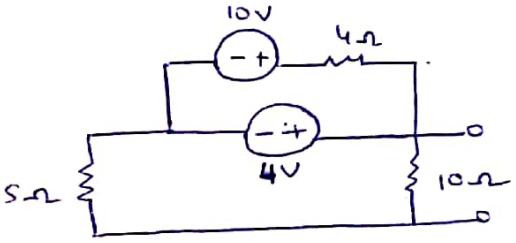
Source transforming:



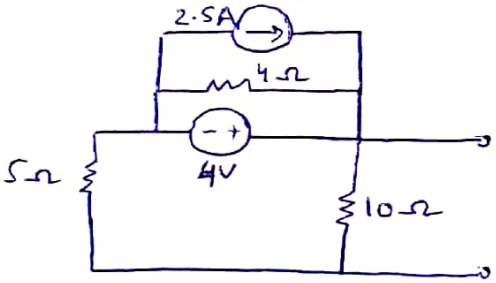
Shifting sources:



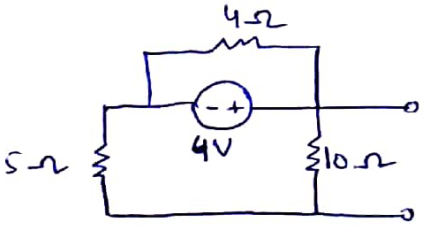
⇒



Source transformation:



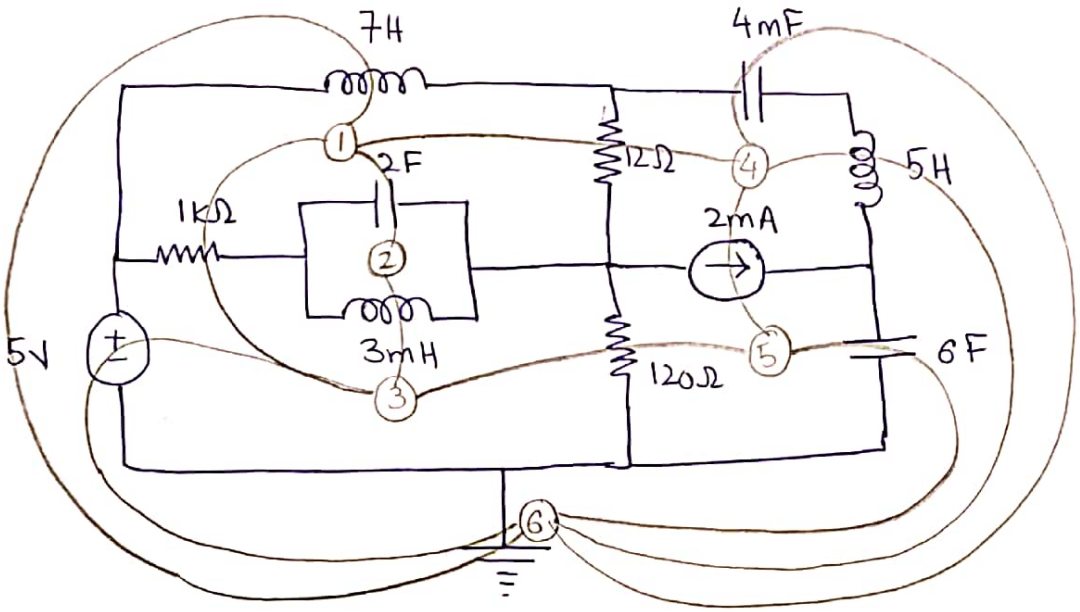
The current source is redundant. Simplifying:

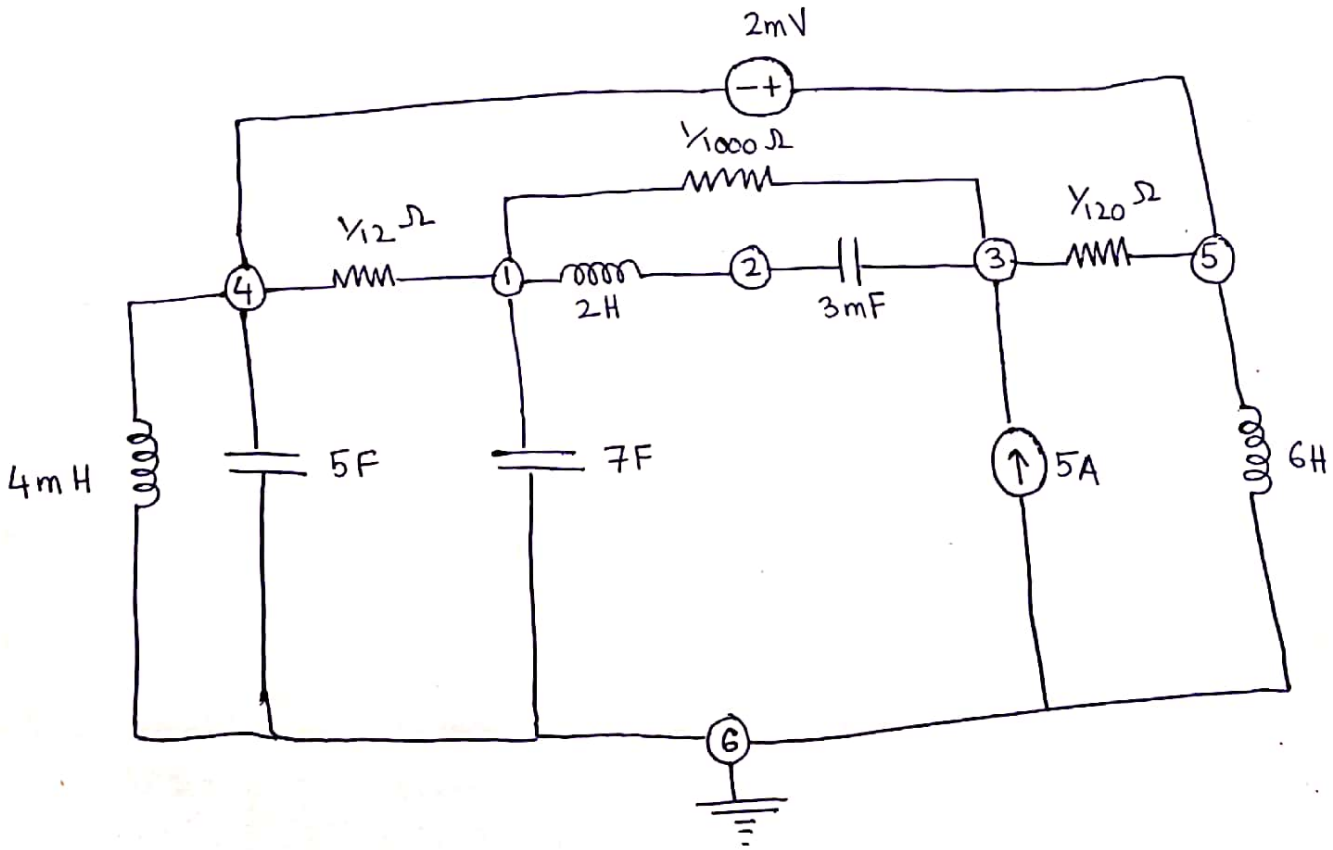


$\therefore V_o = 4V$

2.

a)

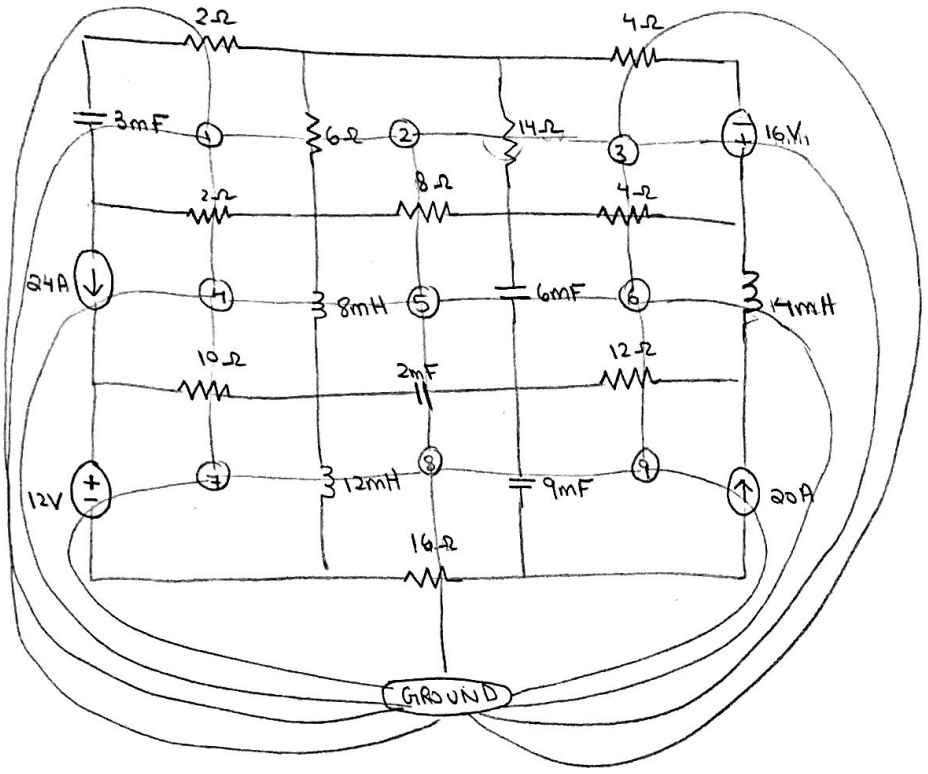

7H
4mF
1
2F
12Ω
4
5H
1kΩ
2mA
2
5V
3mH
120Ω
5
6F
3
6


2mV
1/1000 Ω
1/12 Ω
1/120 Ω
4
1
2H
2
3mF
3
5
4mH
5F
7F
5A
6H
6

EE240 ASSIGNMENT 3

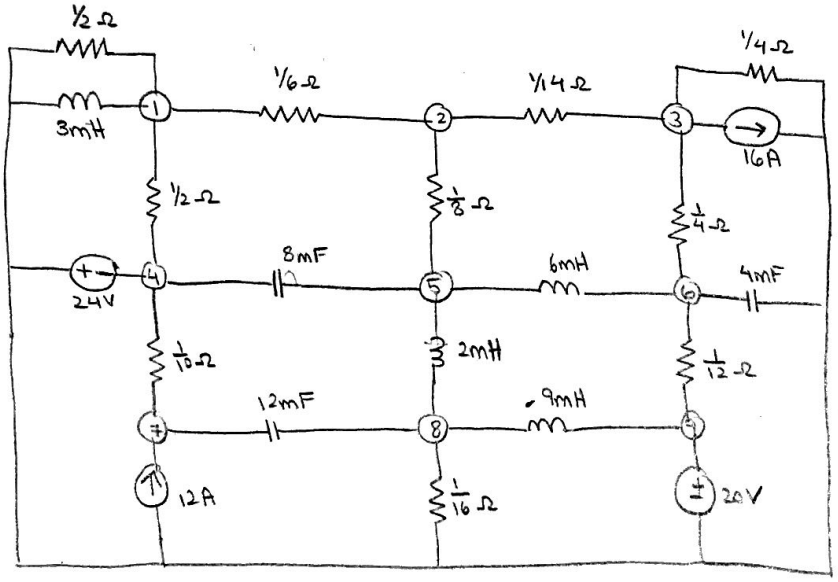
QUESTION # 2

PART B



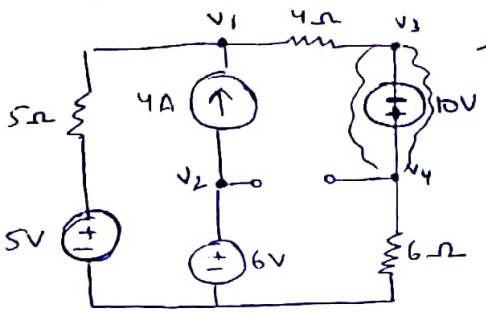
SOLUTION

1) Create a node inside each mesh
2) Create connections from node to node or node to ground that pass through each element.
3) Redraw Nodes & connections, taking the dual of each element



Q3.

Finding V_{th} :



$V_4 - V_3 = 10$

$V_4 = V_3 + 10$ — (A)

(V1) $\frac{V_1 - 5}{5} + \frac{V_1 - V_3}{4} = 4 \Rightarrow 9V_1 - 5V_3 = 100$ — (1)

(V2) $\frac{V_2 - 6}{3} + 4 = 0 \Rightarrow V_2 = -6\ V$

(V3) $\frac{V_3 - V_1}{4} + \frac{V_4}{6} = 0$ (Supernode eq.)

$\Rightarrow 6V_3 - 6V_1 + 4V_4 = 0$

Using (A) :

$6V_3 - 6V_1 + 4(V_3 + 10) = 0 \Rightarrow -6V_1 + 10V_3 = -40$ — (3)

Solving (1) and (3) simultaneously :

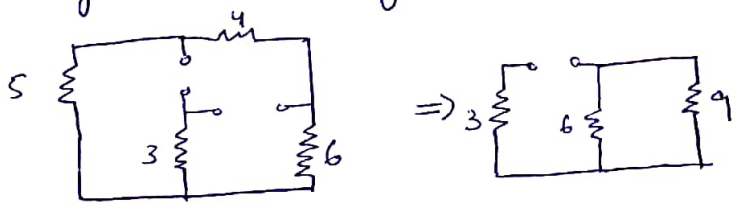
$V_1 = \frac{40}{3}$, $V_3 = 4$

$V_4 = V_3 + 10$

$= 14\ V$

$\therefore V_{th} = 14 - (-6) = 20\ V$

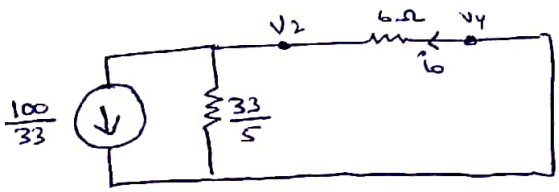
Finding R_{th}, switching off sources :



$R_{th} = 3 + (6 \| 9)$

$= \frac{33}{5}$

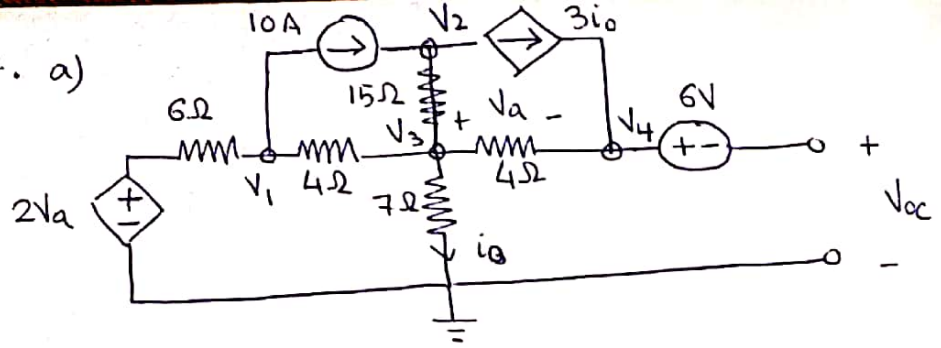
Norton eq:



$i_o = \frac{100}{33}\left(\frac{33/5}{\frac{33}{5} + 6}\right)$ (using current division rule)

$i_o = 1.587\ A$

4. a)



$$V_{oc} = V_4 - 6$$

$V_a = V_3 - V_4$

$i_o = V_3/7$

$$\frac{V_1 - 2V_a}{6} + \frac{V_1 - V_3}{4} + 10 = 0$$

$$\frac{V_1 - 2(V_3 - V_4)}{6} + \frac{V_1}{4} - \frac{V_3}{4} + 10 = 0$$

$$\frac{V_1}{6} + \frac{V_1}{4} - \frac{V_3}{3} - \frac{V_3}{4} + \frac{V_4}{3} = -10$$

$$\boxed{\frac{5V_1}{12} - \frac{7V_3}{12} + \frac{V_4}{3} = -10} \quad (1)$$

$$\frac{V_2 - V_3}{15} + 3i_o = 10$$

$$\frac{V_2}{15} - \frac{V_3}{15} + \frac{3V_3}{7} = 10$$

$$\boxed{\frac{V_2}{15} + \frac{38V_3}{105} = 10} \quad (2)$$

$$\frac{V_3 - V_4}{4} + \frac{V_3}{7} + \frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{15} = 0$$

$$\frac{V_3}{4} + \frac{V_3}{4} + \frac{V_3}{7} + \frac{V_3}{15} - \frac{V_1}{4} - \frac{V_2}{15} - \frac{V_4}{4} = 0$$

$$\boxed{-\frac{V_1}{4} - \frac{V_2}{15} + \frac{149}{210}V_3 - \frac{V_4}{4} = 0} \quad (3)$$

$$\frac{V_4 - V_3}{4} = \frac{3V_3}{7}$$

$$-\frac{V_3}{4} - \frac{3V_3}{7} + \frac{V_4}{4} = 0$$

$$\boxed{-\frac{19V_3}{28} + \frac{V_4}{4} = 0} \quad (4)$$

solving for V_4:

$V_1 = -29.3V$

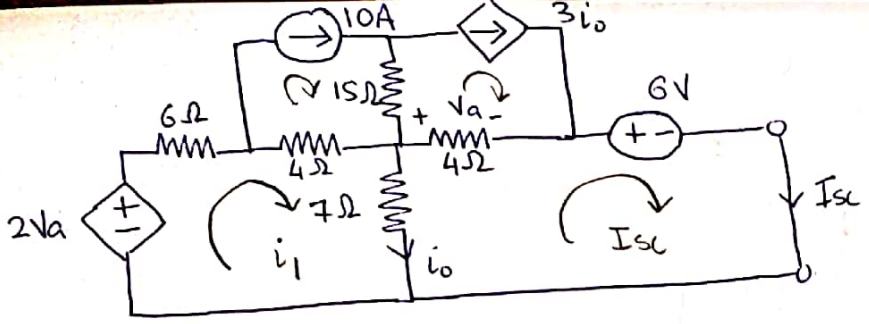
$V_2 = 112.9\,V$

$V_3 = 6.83V$

$\Rightarrow \boxed{V_4 = 18.5V}$

$V_{oc} = 18.5 - 6$

$\boxed{V_{oc} = 12.5V}$ ANS



$i_o = i_1 - I_{sc}$

$V_a = 4(I_{sc} - 3i_o)$

$= 4I_{sc} - 12i_o$

$= 4I_{sc} - 12i_1 + 12I_{sc}$

$V_a = 16I_{sc} - 12i_1$

$2V_a = 6i_1 + 4i_1 + 7i_1 - 4(10) - 7I_{sc}$

$32I_{sc} - 24i_1 = 17i_1 - 40 - 7I_{sc}$

$$41i_1 - 39I_{sc} = 40 \quad (1)$$

$4I_{sc} + 7I_{sc} - 4(3i_o) = -6$

$11I_{sc} - 12i_1 + 12I_{sc} = -6$

$$23I_{sc} - 12i_1 = -6 \quad (2)$$

$i_1 = 1.44A$

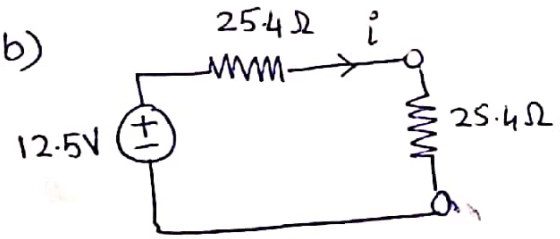
$$I_{sc} = 0.493A$$ ANS

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{12.5}{0.493}$$

$$R_{th} = 25.4\Omega$$ ANS

$$R_L = 25.4\Omega$$

b)



$i = \frac{12.5}{2(25.4)}$

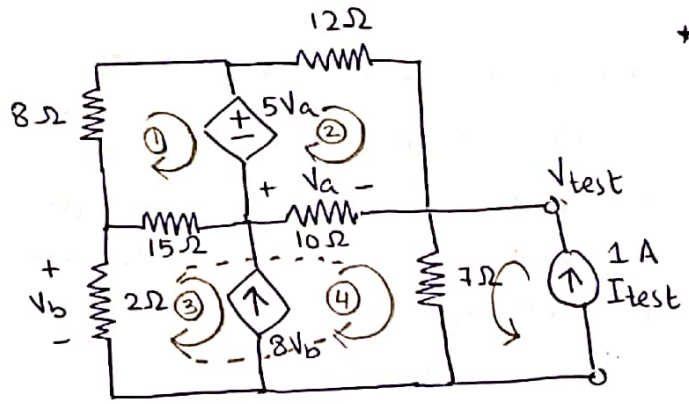
$i = 0.246A$

power dissipated across $R_L \Rightarrow P = i^2R$

$= (0.246)^2 \times 25.4$

$$P_{max} = 1.54W$$ ANS

5.



* apply test source and calculate corresponding current or voltage

note that: since there are no independent sources we can only use test source method

$(V_{th} = 0)$

use any value for Itest, e.g Itest = 1A here

$V_{test} = 7(I_{test} + i_4)$

$V_{test} = 7(1 + i_4)$

$V_a = 10(i_4 - i_2)$

$V_b = -2i_3$

$8i_1 + 15i_1 - 15i_3 = -5V_a$

$23i_1 - 15i_3 = -50i_4 + 50i_2$

$$\boxed{23i_1 - 50i_2 - 15i_3 + 50i_4 = 0} \quad (1)$$

$12i_2 + 10i_2 - 10i_4 = 50(i_4 - i_2)$

$22i_2 + 50i_2 - 10i_4 - 50i_4 = 0$

$$\boxed{72i_2 - 60i_4 = 0} \quad (2)$$

* ③ & ④ form supermesh

$i_4 - i_3 = 8V_b$

$i_4 - i_3 = 8(-2i_3)$

$i_4 - i_3 = -16i_3$

$$\boxed{15i_3 + i_4 = 0} \quad (3)$$

$2i_3 + 15i_3 - 15i_1 + 10i_4 + 7i_4 - 10i_2 + 7(1) = 0$

$$\boxed{-15i_1 - 10i_2 + 17i_3 + 17i_4 = -7} \quad (4)$$

solving equations for i_4:

$i_1 = 0.208A$

$i_2 = -0.428A$

$i_3 = 0.0343A$

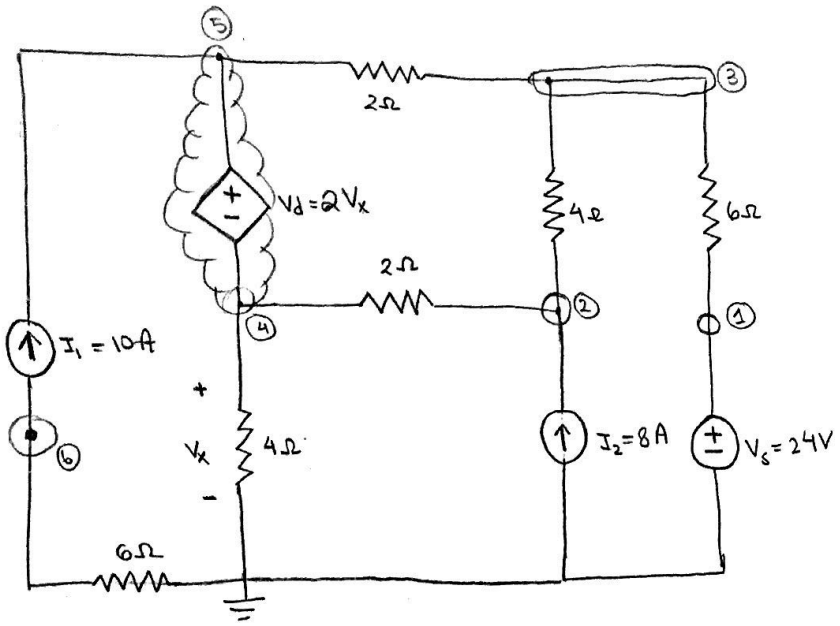
$$\boxed{i_4 = -0.514\,A} \Rightarrow \boxed{V_{test} = 7(1 - 0.514)} \quad \boxed{V_{test} = 3.40V}$$

$$R_{th} = \frac{V_{test}}{I_{test}}$$

$$\boxed{R_{th} = 3.40\Omega}$$ ANS

QUESTION # 6

EE240 ASSIGNMENT 3



STEP 1: Identify Nodes & Any Super Nodes

STEP 2: Formulate Nodal Equations

① $V_1 = V_s = 24V$

② $\frac{V_2 - V_3}{4} + \frac{V_2 - V_4}{2} = I_2$

③ $\frac{V_3 - V_1}{6} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_5}{2} = 0$

Super Node: $V_5 - V_4 = V_d = 2V_x$

④&⑤ $\frac{V_5 - V_3}{2} + \frac{V_4 - V_2}{2} + \frac{V_4}{4} = I_1$

⑥ $I_1 + \frac{V_6}{6} = 0$

STEP 3: Simplify Equations
* Substitute Knowns
* Apply Super Node

① $V_1 = 24$

② $\frac{V_2 - V_3}{4} + \frac{V_2 - V_4}{2} = 8$

$V_2 - V_3 + 2V_2 - 2V_4 = 32$

$3V_2 - V_3 - 2V_4 = 32$

③ $\frac{V_3 - V_1}{6} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_5}{2} = 0$

$2V_3 - 2V_1 + 3V_3 - 3V_2 + 6V_3 - 6V_5 = 0$

$-2V_1 - 3V_2 + 11V_3 - 6V_5 = 0$

$-2(24) - 3V_2 + 11V_3 - 6V_5 = 0$

$-3V_2 + 11V_3 - 6(3V_4) = 48$

$-3V_2 + 11V_3 - 18V_4 = 48$

Super Node: $V_5 - V_4 = 2V_x$

$V_5 - V_4 = 2(V_4)$

$V_5 = 3V_4$

④&⑤ $\frac{V_5 - V_3}{2} + \frac{V_4 - V_2}{2} + \frac{V_4}{4} = 10$

$2V_5 - 2V_3 + 2V_4 - 2V_2 + V_4 = 40$

$-2V_2 - 2V_3 + 3V_4 + 2(3V_4) = 40$

$-2V_2 - 2V_3 + 9V_4 = 40$

⑥ $10 + \frac{V_6}{6} = 0$

$\frac{V_6}{6} = -10$

$V_6 = -60$

STEP 4: FORMULATE MATRIX & SOLVE

$$\begin{bmatrix} 3 & -1 & -2 \\ -3 & 11 & -18 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 32 \\ 48 \\ 40 \end{bmatrix}$$

$V_1 = 24V$

$V_2 = 424/7 \approx 60.571\,V$

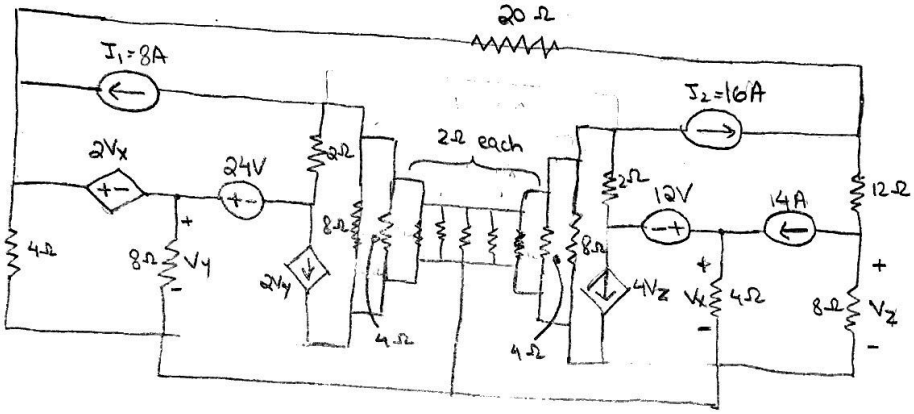
$V_3 = 552/7 \approx 78.857V$

$V_4 = 248/7 \approx 35.429V$ $\boxed{= V_x}$

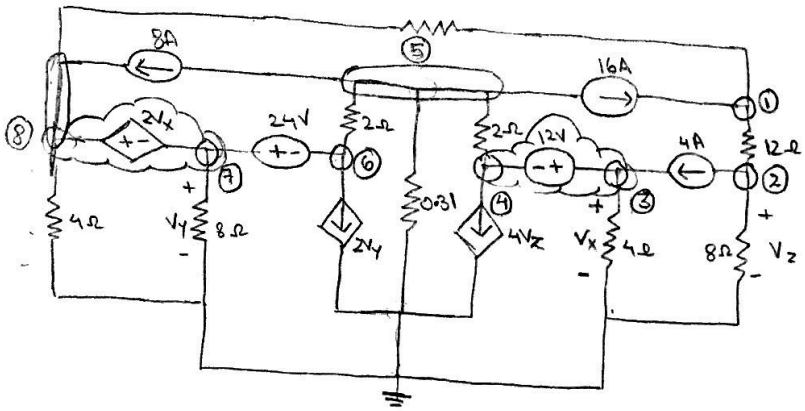
$V_5 = 3V_4 = 3(248/7) = 744/7 \approx 106.286V$

$V_6 = -60V$

QUESTION # 7



STEP 1: Simplify the circuit
* combine resistors



STEP 2: Identify Node & Super Nodes
(marked on circuit above)

STEP 3: Formulate Nodal Equations

① $\frac{V_1 - V_8}{5} + \frac{V_1 - V_2}{12} = 16$

② $\frac{V_2 - V_1}{12} + \frac{V_2}{8} + 4 = 0$

③/④ $\frac{V_3}{4} + \frac{V_4 - V_5}{2} + 4V_z = 4$

Super Node: $V_3 - V_4 = 12$

$V_3 = 12 + V_4$

⑤ $\frac{V_5 - V_4}{2} + \frac{V_5}{0.31} + \frac{V_5 - V_6}{2} + 8 + 16 = 0$

⑥&⑦ & ⑧ $\frac{V_6 - V_5}{2} + 2V_4 + \frac{V_7}{8} + \frac{V_8}{4} + \frac{V_8 - V_1}{5} = 8$

Super Node: $V_7 - V_6 = 24$

$V_7 = 24 + V_6$

Super Node: $V_8 - V_7 = 2V_x$

$V_8 = 2V_x + V_7 \rightarrow (V_x = V_3) \rightarrow 2V_3 + 24 + V_6 \rightarrow 2(12 + V_4) + 24 + V_6 = 48 + 2V_4 + V_6$ ($V_3 = 12 + V_4$)

STEP 4: Simplify Nodal Equations
* Apply Super Nodes

① $\frac{V_1 - V_8}{5} + \frac{V_1 - V_2}{12} = 16$

$12V_1 - 12V_8 + 5V_1 - 5V_2 = 960$

$12V_1 - 12(48 + 2V_4 + V_6) + 5V_1 - 5V_2 = 960 \rightarrow 17V_1 - 5V_2 - 24V_4 - 12V_6 = 1536$

② $\frac{V_2 - V_1}{12} + \frac{V_2}{8} + 4 = 0$

$2V_2 - 2V_1 + 3V_2 + 96 = 0$

$-2V_1 + 5V_2 = -96$

③&④ $\frac{V_3}{4} + \frac{V_4 - V_5}{2} + 4V_2 = 4$

$V_3 + 2V_4 - 2V_5 + 16V_2 = 16$

$(12 + V_4) + 2V_4 - 2V_5 + 16V_2 = 16 \rightarrow V_2 = V_2$

$16V_2 + 3V_4 - 2V_5 = 4$

⑤ $\frac{V_5 - V_4}{2} + \frac{V_5}{0.31} + \frac{V_5 - V_6}{2} + 8 + 16 = 0$

$0.31V_5 - 0.31V_4 + 2V_5 + 0.31V_5 - 0.31V_6 = -24$

$-0.31V_4 + 2.62V_5 - 0.31V_6 = -24$

⑥&⑦&⑧ $\frac{V_6 - V_5}{2} + 2V_4 + \frac{V_7}{8} + \frac{V_8}{4} + \frac{V_8 - V_1}{5} = 8$

$20V_6 - 20V_5 + 80V_4 + 5V_7 + 10V_8 + 8V_8 - 8V_1 = 320 \rightarrow V_4 = V_7$

$-8V_1 - 20V_5 + 20V_6 + 85V_7 + 18V_8 = 320$

$V_7 = 24 + V_6$ & $V_8 = 48 + 2V_4 + V_6$

$-8V_1 - 20V_5 + 20V_6 + 85(24 + V_6) + 18(48 + 2V_4 + V_6) = 320$

$-8V_1 + 36V_4 - 20V_5 + 123V_6 = -2584$

STEP 5: FORMULATE MATRIX & SOLVE

	①	②	④	⑤	⑥
	17	-5	-24	0	-12
	-2	5	0	0	0
	0	16	3	-2	0
	0	0	-0.31	2.62	-0.31
	-8	0	36	-20	123

$\begin{bmatrix} V_1 \\ V_2 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 1536 \\ -96 \\ 4 \\ -24 \\ -2584 \end{bmatrix} \rightarrow$

$V_1 = 53.661$

$V_2 = 2.265$ [$= V_2$]

$V_3 = 12 + V_4 = 12 + (-19.488) = -7.488$ [$= V_x$]

$V_4 = -19.488$

$V_5 = -13.116$

$V_6 = -13.947$

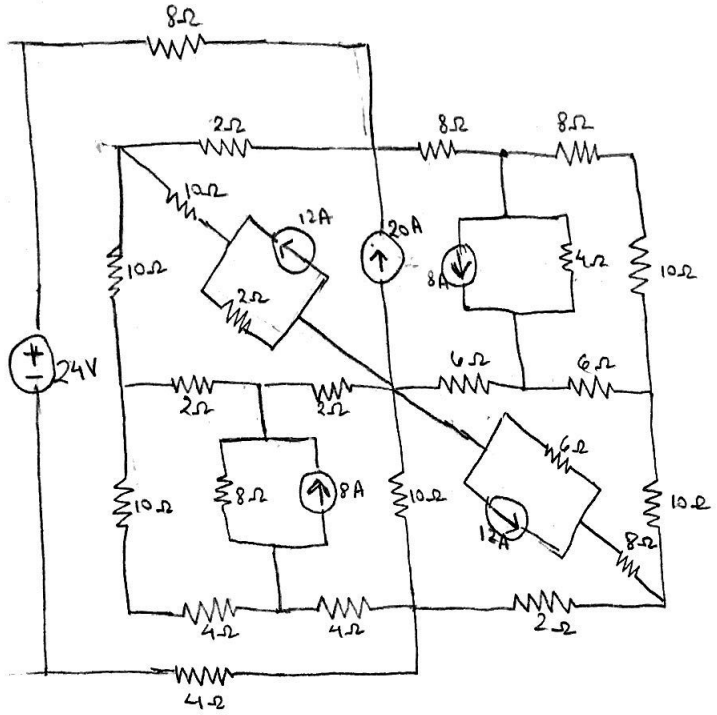
$V_7 = 24 + V_6 = 24 - 13.947 = 10.053$ [$= V_4$]

$V_8 = 48 + 2V_4 + V_6 = -4.923$

QUESTION # 8

EE240 ASSIGNMENT 3

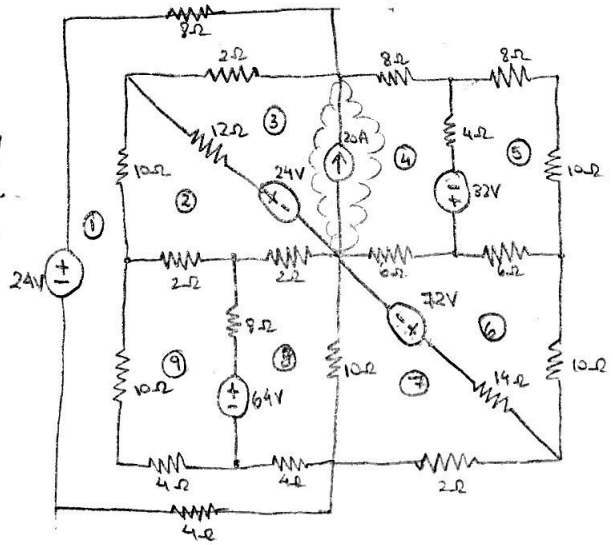
Given the following circuit, produce the complete set of loop equations, formulate the equation matrix, and calculate all loop currents by performing Gaussian Elimination.



STEP 1: Simplify the circuit.

* Apply source transformation to current sources || to resistors
* Combine resistances where possible

STEP 2: Identify & label loops. Identify any super loops. All loop currents are taken as clockwise.



STEP 3: Formulate Loop Equations

① $3i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) + 10(i_1 - i_9) + 4(i_1 - i_9) + 4(i_1 - i_8) + 4i_4 = 24$

② $10(i_2 - i_1) + 12(i_2 - i_3) + 2(i_2 - i_8) + 2(i_2 - i_9) = -24$

③/④ $12(i_3 - i_2) + 2(i_3 - i_1) + 8i_4 + 4(i_4 - i_5) + 6(i_4 - i_6) = 24 + 32$

Super Loop: $i_3 + 20 = i_4$

⑤ $4(i_5 - i_4) + 8i_5 + 10i_5 + 6(i_5 - i_6) = -32$

⑥ $6(i_6 - i_4) + 6(i_6 - i_5) + 10i_6 + 14(i_6 - i_7) = -72$

⑦ $10(i_7 - i_8) + 14(i_7 - i_6) + 2i_7 = 72$

⑧ $8(i_8 - i_9) + 2(i_8 - i_2) + 10(i_8 - i_7) + 4(i_8 - i_1) = 64$

⑨ $10(i_9 - i_1) + 2(i_9 - i_2) + 8(i_9 - i_8) + 4(i_9 - i_1) = -64$

STEP 4: Simplify Loop Equations & Substitute Super Loop

① $3i_1 + 2i_1 - 2i_3 + 10i_4 - 10i_2 + 10i_4 - 10i_9 + 4i_1 - 4i_9 + 4i_1 - 4i_8 + 4i_1 = 24$

$32i_1 - 10i_2 - 2i_3 - 4i_8 - 14i_9 = 24$

② $10i_2 - 10i_1 + 12i_2 - 12i_3 + 2i_2 - 2i_8 + 2i_2 - 2i_9 = -24$

$-10i_1 + 26i_2 - 12i_3 - 2i_8 - 2i_9 = -24$

③/④ $12i_3 - 12i_2 + 2i_3 - 2i_4 + 8i_4 + 4i_4 - 4i_5 + 6i_4 - 6i_6 = 24 + 32$

$-2i_4 - 12i_2 + 14i_3 + 18i_4 - 4i_5 - 6i_6 = 56$

$-2i_4 - 12i_2 + 14i_3 + 18(i_3+20) - 4i_5 - 6i_6 = 56$

$-2i_4 - 12i_2 + 14i_3 + 18i_3 + 360 - 4i_5 - 6i_6 = 56$

$-2i_4 - 12i_2 + 32i_3 - 4i_5 - 6i_6 = -304$

⑤ $4i_5 - 4i_4 + 8i_5 + 10i_5 + 6i_5 - 6i_6 = -32$

$-4(i_3+20) + 28i_5 - 6i_6 = -32$

$-4i_3 - 80 + 28i_5 - 6i_6 = -32$

$-4i_3 + 28i_5 - 6i_6 = 48$

⑥ $6i_6 - 6i_4 + 6i_6 - 6i_5 + 10i_6 + 14i_6 - 14i_7 = -72$

$-6i_4 - 6i_5 + 36i_6 - 14i_7 = -72$

$-6(i_3+20) - 6i_5 + 36i_6 - 14i_7 = -72$

$-6i_3 - 120 - 6i_5 + 36i_6 - 14i_7 = -72$

$-6i_3 - 6i_5 + 36i_6 - 14i_7 = 48$

⑦ $10i_7 - 10i_8 + 14i_7 - 14i_6 + 2i_7 = 72$

$-14i_6 + 26i_7 - 10i_8 = 72$

⑧ $8i_8 - 8i_9 + 2i_8 - 2i_2 + 10i_8 - 10i_7 + 4i_8 - 4i_1 = 64$

$-4i_1 - 2i_2 - 10i_7 + 20i_8 - 8i_9 = 64$

⑨ $10i_9 - 10i_1 + 2i_9 - 2i_2 + 8i_9 - 8i_8 + 4i_9 - 4i_1 = -64$

$-14i_1 - 2i_2 - 8i_8 + 24i_9 = -64$

STEP 5: Formulate Matrix

i_1	i_2	i_3	i_5	i_6	i_7	i_8	i_9			
32	-10	-2	0	0	0	-4	-14	i_1		24
-10	26	-12	0	0	0	-2	-2	i_2		-24
-2	-12	32	-4	-6	0	0	0	i_3		-304
0	0	-4	28	-6	0	0	0	i_5		48
0	0	-6	-6	36	-14	0	0	i_6	=	48
0	0	0	0	-14	26	-10	0	i_7		72
-4	-2	0	0	0	-10	20	-8	i_8		64
-14	-2	0	0	0	0	-8	24	i_9		-64

STEP 6: solve

$i_1 = \frac{-884015336}{101599551} \approx -8.701$

$i_2 = \frac{-1199386552}{101599551} \approx -11.805$

$i_3 = \frac{-1486886500}{101599551} \approx -14.635$

$i_5 = \frac{-52608836}{101599551} \approx -0.518$

$i_6 = \frac{-42913752675409 83}{6453187736343199} \approx -0.6600$

$i_7 = \frac{46331296}{33866517} \approx 1.368$

$i_8 = \frac{-27624892}{101599551} \approx -0.719$

$i_9 = \frac{-955306192}{101599551} \approx -9.403$

$i_4 = i_3 + 20 \approx 5.365$