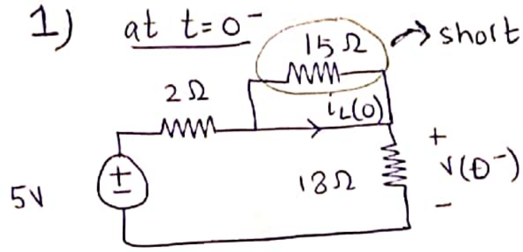
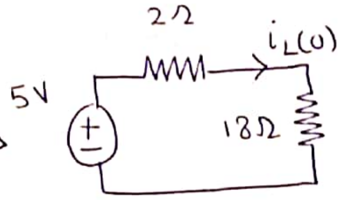
1) at $t=0^-$



$\Rightarrow$



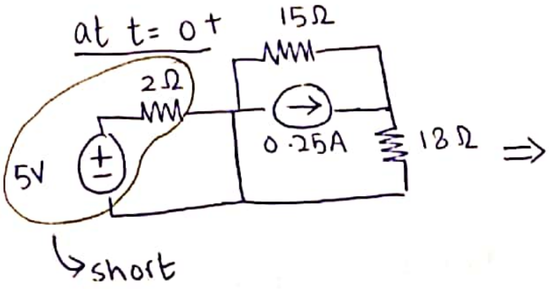
$i_L(0^-) = i_L(0^+) = 5/20$

$= 0.25A$

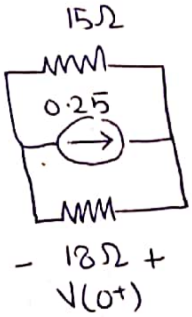
$V(0^-) = \frac{18}{20} \times 5 = 4.5V$

$\boxed{V(0^-) = 4.5V}$

at $t = 0^+$



$\Rightarrow$

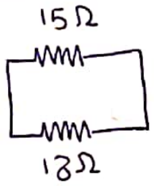


$V(0^+) = \frac{15}{15+18} \times 0.25 \times 18$

$= 45/22$

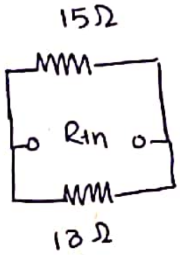
$\boxed{V(0^+) = 2.04V}$

at $t = \infty$



$\boxed{V(\infty) = 0V}$

R_{th}:



$R_{th} = \frac{15 \times 18}{15+18}$

$= 90/11$

$\boxed{R_{th} = 8.18\Omega}$

$\tau = L/R = \frac{500 \times 10^{-3}}{8.18}$

$= 0.0611 s$

$1/\tau = 16.4 s^{-1}$

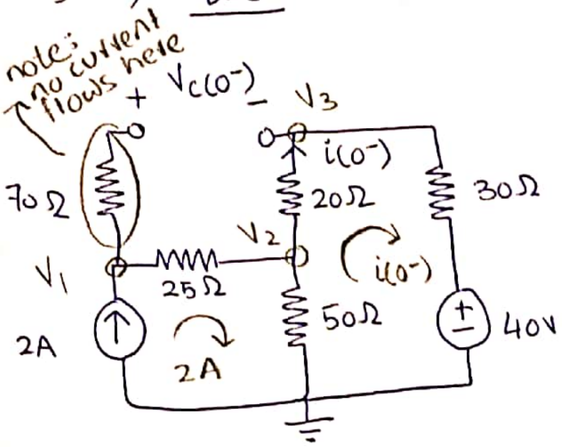
$K_1 = 0$

$K_2 = V(0^+) = 2.04$

$$\boxed{\begin{aligned} V(0^-) &= 4.5V \\ V(t) &= 2.04e^{-16.4t} \quad t>0 \end{aligned}}$$

ANS

2) a) $t=0^-$



$\cancel{50i(0^-)+30i}$

$50\,i(0^-) - 50(2) + 20\,i(0^-) + 30\,i(0^-) = -40$

$100\,i(0^-) = -40 + 100$

$100\,i(0^-) = 60$

$\boxed{i(0^-) = 0.6A}$

$\frac{V_1 - V_2}{25} = 2 \Rightarrow V_1 - V_2 = 50$ ①

$\frac{V_2 - V_1}{25} + \frac{V_2}{50} + \frac{V_2 - V_3}{20} = 0$ ②

substituting ① in ②

$\frac{-50}{25} + \frac{V_2}{50} + \frac{V_2 - V_3}{20} = 0$

$\boxed{\frac{V_2}{50} + \frac{V_2}{20} - \frac{V_3}{20} = 2}$

$\frac{V_3 - V_2}{20} + \frac{V_3 - 40}{30} = 0$ ③

$\boxed{\frac{-V_2}{20} + \frac{V_3}{20} + \frac{V_3}{30} = 4/3}$

solving simutaneously

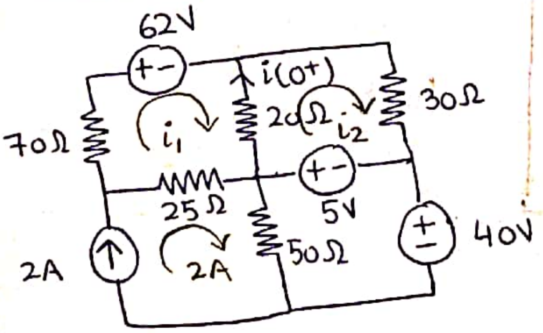
$\Rightarrow$ $V_2 = 70\,V$
$V_3 = 58\,V$

$V_1 = 50 + V_2$
$= 120\,V$

$V_c(0^-) = V_1 - V_3$
$= 120 - 58$

$\boxed{V_c(0^-) = 62V}$

at $t=0^+$



$70i_1 + 25i_1 - 25(2) + 20i_1 - 20i_2 = -62$

$115i_1 - 20i_2 = -62 + 50$

$115i_1 - 20i_2 = -12$ ①

$20i_2 - 20i_1 + 30i_2 = 5$

$50i_2 - 20i_1 = 5$ ②

solving ① & ② simultaneously $\Rightarrow$

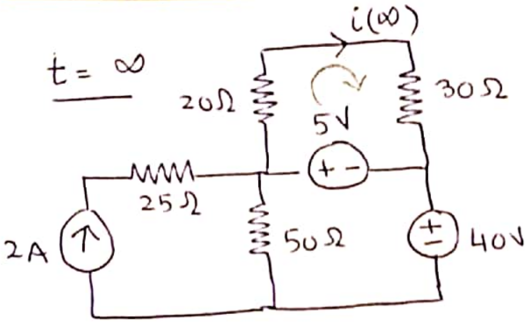
$i_1 = -10/107$
$= -0.0934A$

$i_2 = \frac{67}{1070}$
$= 0.0626A$

$i(0^+) = i_2 - i_1$
$= 0.0626 + 0.0934$

$\boxed{i(0^+) = 0.156A}$

$t = \infty$

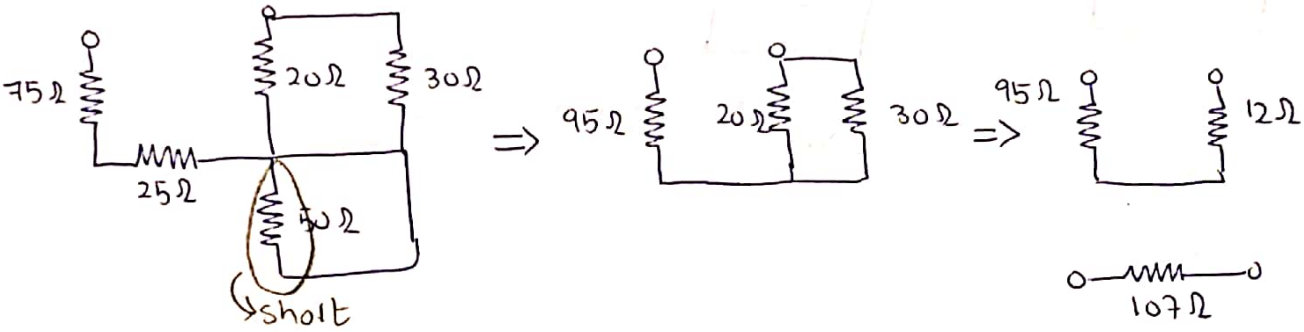


$20\,i(\infty) + 30\,i(\infty) = 5$

$i(\infty) = 5/50$

$\boxed{i(\infty) = 0.1\,A}$

R_{th}:



$\boxed{R_{th} = 107\,\Omega}$

$\tau = RC$

$= 107 \times 4.4 \times 10^{-3}$

$= 0.47\,s$

$1/\tau = 2.12\,s^{-1}$

$K_1 + K_2 = 0.156$

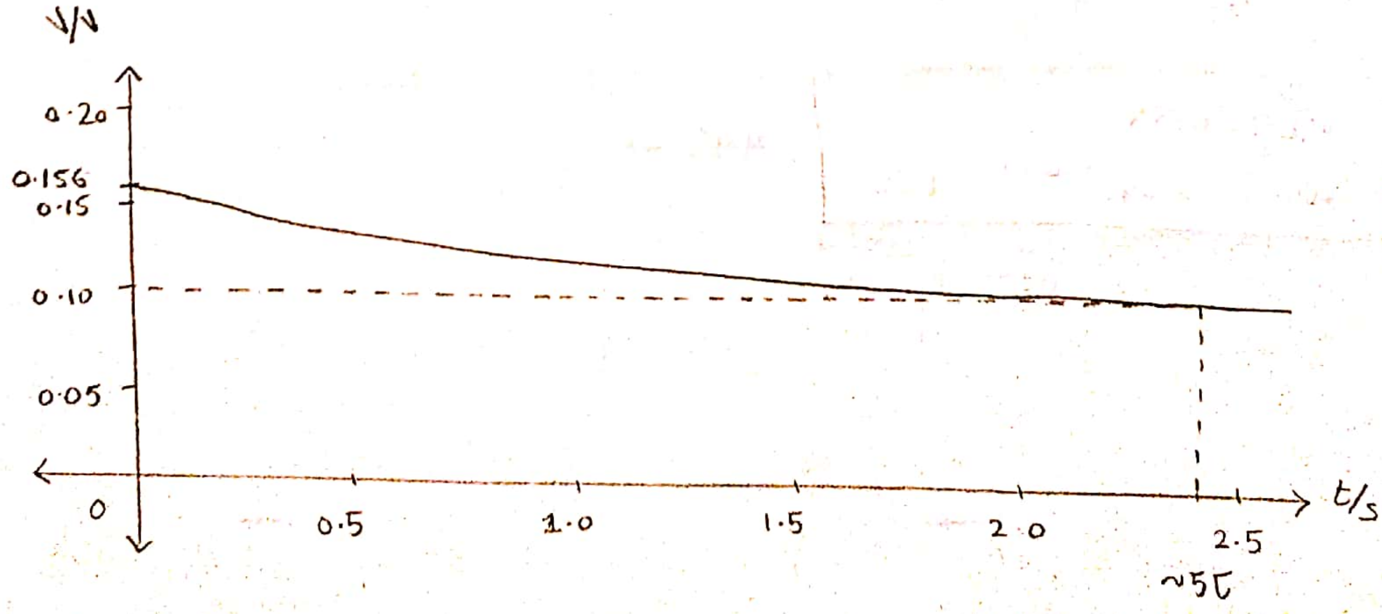
$K_1 = 0.1$

$K_2 = 0.056$

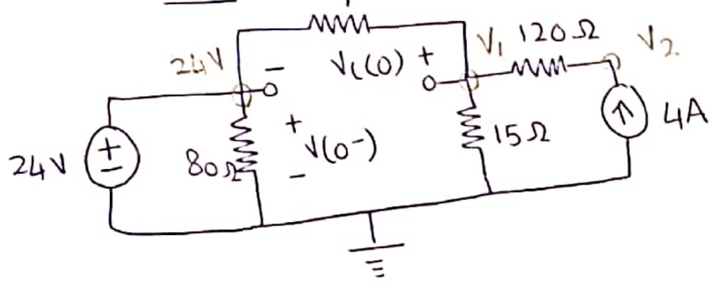
$$\boxed{i(t) = \begin{cases} 0.6\,A & t < 0 \\ 0.1 + 0.056e^{-2.12t}\,A & t > 0 \end{cases}}$$

ANS

b)



3) at $t=0^-$



$$\frac{V_1-V_2}{120}+\frac{V_1}{15}+\frac{V_1-24}{45}=0 \quad (1)$$

$$\frac{V_2-V_1}{120}=4 \quad (2)$$ substituting (2) in (1)

$$\frac{V_1}{15}+\frac{V_1-24}{45}=4$$

$$3V_1+V_1-24=4\times45$$

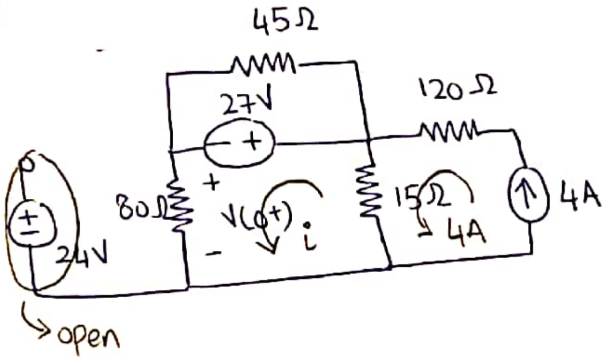
$$4V_1=180+24$$

$$V_1=51V$$

$$V_c(0)=51-24$$

$$\boxed{V_c(0)=27V}$$

at $t=0^+$



$$15i-15(4)+80i=-27$$

$$95i=-27+60$$

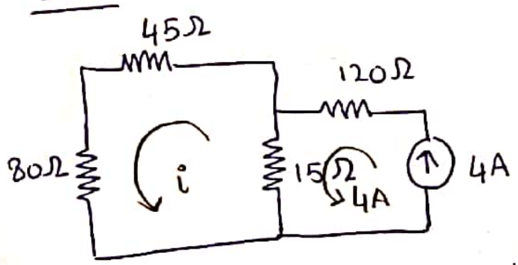
$$i=33/95$$

$$i=0.347A$$

$$V(0^+)=0.347\times80$$

$$\boxed{V(0^+)=27.8V}$$

$t=\infty$



$$15i-15(4)+45i+80i=0$$

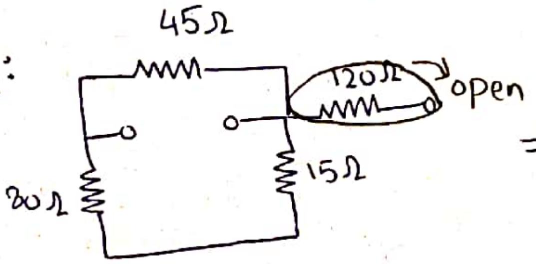
$$140i=60$$

$$i=3/7A$$

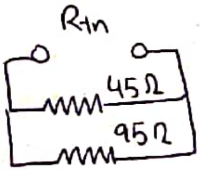
$$V(\infty)=80\times i$$

$$\boxed{V(\infty)=34.3V}$$

R_{th}:



⇒



$$R_{th}=\frac{45\times95}{45+95}$$

$$\boxed{R_{th}=30.5}$$

$\tau = RC$

$= 20.8\,s$

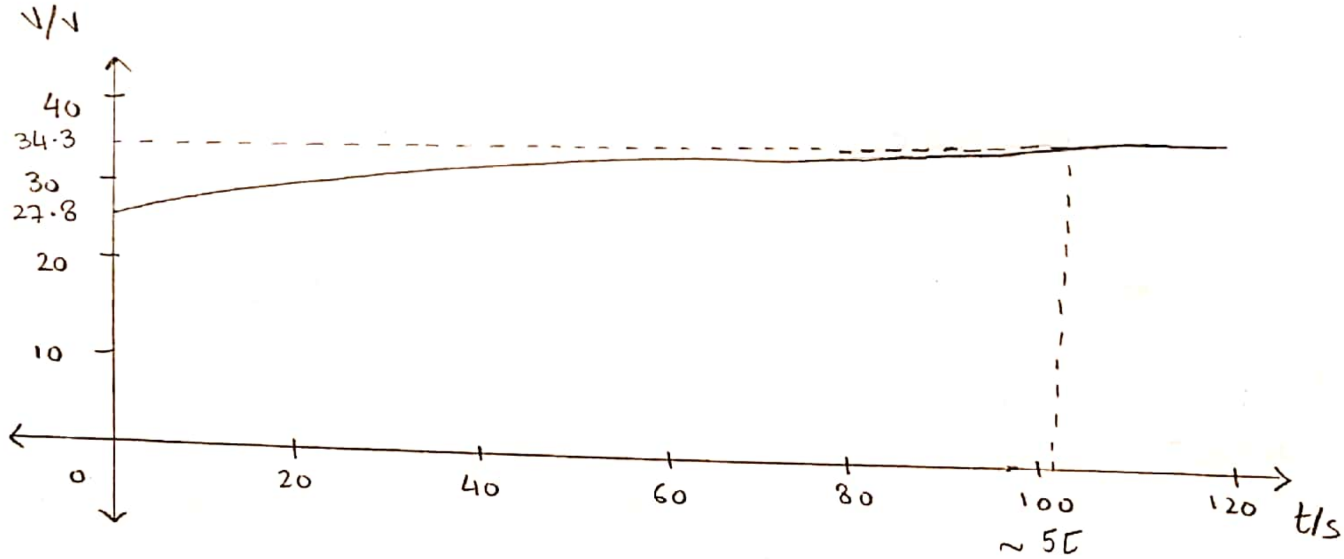
$1/\tau = 0.048\,s^{-1}$

$K_1 = 34.3$

$K_1 + K_2 = 27.8$

$K_2 = -6.5$

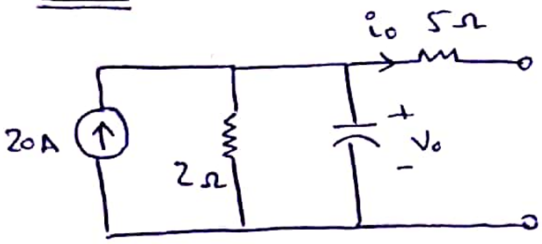
$$\boxed{v(t) = 34.3 - 6.5e^{-0.048t} \quad \forall\, t>0}$$



Q4.

a)

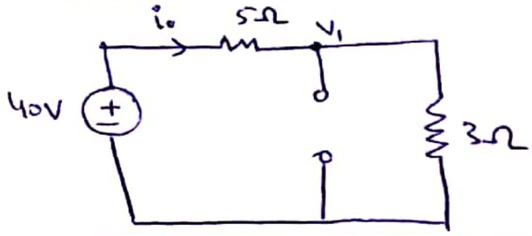
$\underline{\underline{t=0^-}}$



$$\boxed{\begin{array}{l} V_o(0^-) = V_c(0^-) = 40\,V \\ i_o = 0A \text{ (since open circuit)} \end{array}}$$

$i_L = 0A$

$\underline{\underline{t=0^+}}$

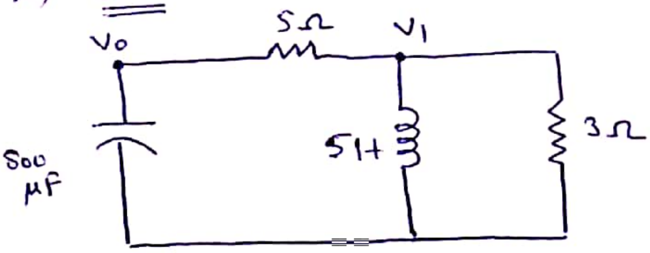


$$\boxed{\begin{array}{l} V_c(0^+) = V_o(0^+) = V_o(0^-) = 40\,V \\ i_o = 40/8 = 5A \end{array}}$$

$V_1(0^+) = 5 \times 3 = 15V$

$i_L(0^+) = 0A = i_L(0^-)$

b) $\underline{\underline{t \geq 0}}$



Node equations:

(V_o) $C\dfrac{dv_o}{dt} + \dfrac{V_o - V_1}{5} = 0$

(V_1) $\dfrac{V_1 - V_o}{5} + \underbrace{\dfrac{1}{L}\int v_L\, dt}_{\hookrightarrow\, i_L(0^+) = 0} + \dfrac{V_1}{3} = 0$

$\Rightarrow \dfrac{V_1 - V_o}{5} + \dfrac{V_1}{3} = 0 \quad \Rightarrow 8V_1 - 5V_o = 0 \quad \Rightarrow V_o = \dfrac{8}{5}V_1$

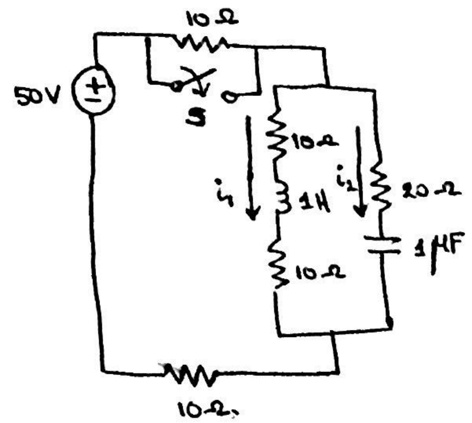
V_o : $C\dfrac{dv_o}{dt} + \dfrac{\left(\frac{8}{5}v_1\right) - v_1}{S} = 0$

$$SC\frac{dv_o}{dt} + \frac{3}{5}v_1 = 0$$

$$\frac{dv_o}{dt}(0^+) = -\frac{3}{5}v_1 \times \frac{1}{SC}$$

$v_1(0^+) = 15V$ and $C = 800\,\mu F$

$$\boxed{\therefore \frac{dv_o}{dt}(0^+) = -2250 \text{ V/sec}}$$



PROBLEM # 5

* Steady State Achieved w/ switch "S" initially open
* Switch "S" is closed at time $t = 0$

i) Produce the first order differential equations that govern the circuit

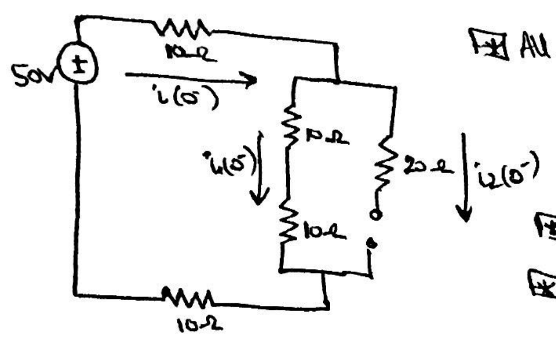
ii) Determine $V_c(0^-)$, the voltage across the capacitor before the switch is closed. Also find its polarity.

iii) Find $i_1(0^+)$ & $i_2(0^+)$

iv) Find $di_1/dt\,(0^+)$ & $di_2/dt\,(0^+)$

v) Find $di_1/dt\,(\infty)$

STEP 1: Analyze the circuit at $t = 0^-$



$\boxed{*}$ All of $i(0^-)$ is passed through the route of $i_1(0^-)$, hence

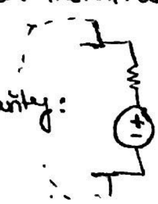
$i(0^-) = i_1(0^-) = i_1(0^+)$ because inductor doesn't permit instantaneous change in current.

Because $V = IR$, $i_1(0^-) = \frac{V}{40\,\Omega} = \frac{50\,V}{40\,\Omega} = \frac{5}{4}$ Amps

$\boxed{*}$ $i_2(0^-) = 0$ Amps due to open circuit

$\boxed{*}$ $V_c(0^-) = V_c(0^+)$ because capacitor doesn't permit instantaneous change in voltage

$$\frac{50V\,(20\,\Omega)}{40\,\Omega} = \frac{1000}{40} = \frac{100}{4} = 25\,V \longrightarrow \text{Polarity:}$$



i) At $t > 0$: Circuit Equations

① — $V = 20 i_1 + L\frac{di_1(t)}{dt} + 10 = 20 i_1 + \frac{di_1(t)}{dt} + 10$

② — $V = 20 i_2(t) + \frac{1}{C}\int_{-\infty}^{t} i_2(t)\,dt + 10 = 20 i_2(t) + 1000000\int_{-\infty}^{t} i_2(t)\,dt + 10$

ii) $V_c(0^-) = 25V$ as determined above in $t = 0^-$ analysis. Polarity $\longrightarrow$

iii) $i_1(0^+) = i_1(0^-)$ because capacitor prohibits instantaneous change in current $= \frac{5}{4}$ Amps

$i_2(0^+) \longrightarrow$

$V = 20\, i_2(0^+) + V_c(0^+) + 10$

$50 = 20\, i_2(0^+) + 35 \longrightarrow i_2(0^+) = \frac{15}{20}$ Amps $= 0.75$ A

iv) Eq 1: – $V = i_1(t)(20) + L\frac{di_1}{dt} + 10 \rightarrow 50 = 20 i_1(0^+) + \frac{di_1}{dt}(0^+) + 10 = 25 + \frac{di_1}{dt}(0^+)$

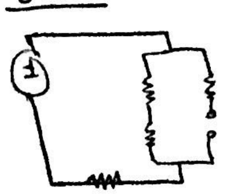
$\frac{di_1}{dt}(0^+) = 25$ Amp/s

Eq 2: – $V = 20 i_2(t) + \frac{1}{C}\int_{-\infty}^{\infty} i_2(t)\,dt + 10$

↳ differentiate

$0 = 20\frac{di_2(t)}{dt} + 1\times10^6\, i_2(t) \longrightarrow \frac{di_2(0^+)}{dt} = \frac{-10^6\left[\frac{V - V_c(0^+)}{20}\right]}{20} = -6.25\times10^4$ Amp/s

v) $t = \infty$



$V_L(t) = L\frac{di_1(t)}{dt}$

$V_L(\infty) = L\frac{di_1(\infty)}{dt}$

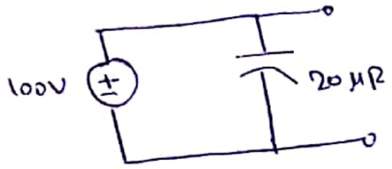
$V_L(\infty) = 0$ due to short circuit $\rightarrow$ Hence, $\frac{di_1}{dt}(\infty) = 0$.

Q6.

Let $20\mu F \rightarrow C_1$

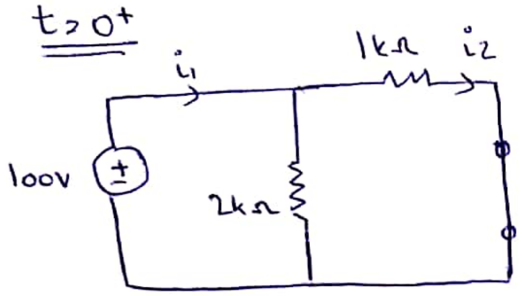
$10\mu F \rightarrow C_2$

$t = 0^-$



$$V_{C1} = 100V, \quad V_{C2} = 0V, \quad i_1 = i_2 = 0A$$

$t = 0^+$



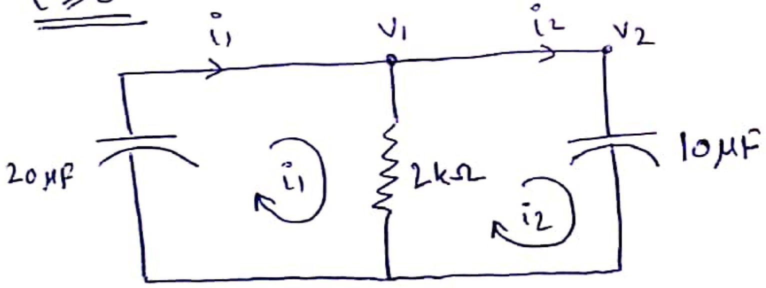
$$V_{C1}(0^+) = V_{C1}(0^-) = 100V$$

$$V_{C2}(0^+) = V_{C2}(0^-) = 0V$$

$$i_2 = \frac{100}{1000} = 0.1A$$

$$i_1 = 0.1 + \left(\frac{100}{2000}\right) = 0.15A$$

$t \geq 0$



Loop eq.s :

(i1) $\frac{1}{C_1}\int i_1\,dt + 2000(i_1 - i_2) = 0$

$$\Rightarrow 2000(i_2 - i_1) = \frac{1}{C_1}\int i_1\,dt \quad \text{——— (1)}$$

(i2) $\frac{1}{C_2}\int i_2\,dt + 1000\,i_2 + 2000(i_2 - i_1) = 0$

Substitute (1) in (i2)

$$\frac{1}{C_2}\int i_2\,dt + 1000 i_2 + \frac{1}{C_1}\int i_1\,dt = 0$$

Derivate wrt 't':

$$\frac{i_2}{C_2} + 1000\frac{di_2}{dt} + \frac{i_1}{C_1} = 0 \quad \text{——— (2)}$$

$$\frac{di_2}{dt} = \frac{1}{1000}\left(-\frac{i_1}{C_1} - \frac{i_2}{C_2}\right)$$

Putting in values $\rightarrow$ $\boxed{\frac{di_2}{dt} = -17.5 \text{ A/sec}}$

(1): $\frac{1}{C_1}\int i_1\,dt + 2000(i_1 - i_2) = 0$

Derivate wrt 't':

$$\frac{i_1}{C_1} + 2000\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = 0$$

$$\frac{di_1}{dt} = \frac{-i_1}{2000C_1} + \frac{di_2}{dt}$$

Putting in values of i_1, C_1, $\frac{di_2}{dt}$

$\Rightarrow$ $\boxed{\frac{di_1}{dt} = -21.25 \text{ A/sec}}$

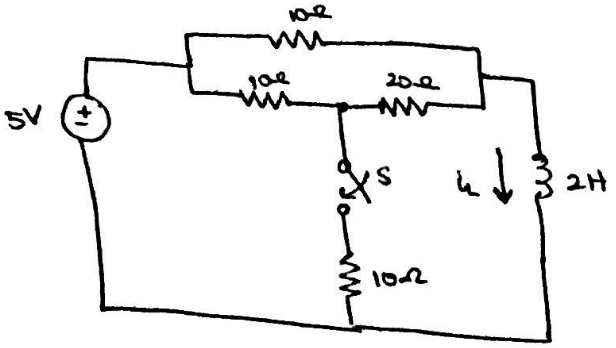
Derivate (2) wrt 't':

$$\frac{1}{C_2}\cdot\frac{di_2}{dt} + 1000\frac{d^2 i_2}{dt^2} + \frac{di_1}{dt}\cdot\frac{1}{C_1} = 0$$

$$\frac{d^2 i_2}{dt^2} = \frac{-1}{1000}\left(\frac{1}{C_1}\cdot\frac{di_1}{dt} + \frac{1}{C_2}\cdot\frac{di_2}{dt}\right)$$

Putting in values:

$$\boxed{\frac{d^2 i_2}{dt^2} = 2812.5 \text{ A/sec}^2}$$



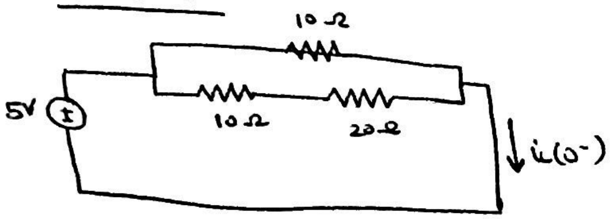
PROBLEM #7

* steady state Achieved w/ switch "S" open
* switch "S" is closed at time $t = 0s$.

i) Derive & sketch the current i_L through the inductor for $t > 0$

ii) Find the time constant

At $t = 0^-$



$V = i_L(0^-)\, R_{eq}$

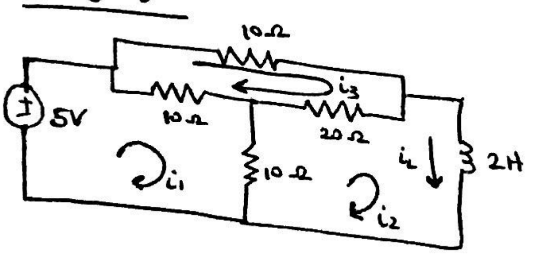
$5 = i_L(0^-)\left[\frac{(30)(10)}{30+10}\right] = i_L(0^-)\left(\frac{300}{40}\right)$

$i_L(0^-) = \frac{2}{3}$ Amps $= 0.667$ Amps

[*] saturated Inductor = short circuit

[*] $i_L(0^-)$ represents total current in circuit

At $t = 0^+$



Loop Equations

$-5 + 10(i_1 - i_3) + 10(i_1 - i_2) = 0$

$20i_1 - 10i_2 - 10i_3 = 5$ —— $\boxed{1}$

$10i_3 + 20(i_3 - i_2) + 10(i_3 - i_1) = 0$

$i_1 + 2i_2 - 4i_3 = 0$ —— $\boxed{3}$

Substituting $\boxed{3}$ in $\boxed{1}$

$\boxed{3}$ becomes $\rightarrow$ $i_3 = \frac{i_1 + 2i_2}{4}$ $\rightarrow$ substitute into $\boxed{1}$

$\boxed{1}$ becomes: $20i_1 - 10i_2 - 10\left[\frac{i_1 + 2i_2}{4}\right] = 5$

$7i_1 - 6i_2 = 2$

$i_1 = \frac{2 + 6i_2}{7}$

Loop 2: $20(i_2 - i_3) + 2\frac{di_2}{dt} + 10(i_2 - i_1) = 0$

$20i_2 - 20\left[\frac{i_1 + 2i_2}{4}\right] + 2\frac{di_2}{dt} + 10i_2 - 10i_1 = 0$

$2\frac{di_2}{dt} + 20i_2 = 15i_1 = 15\left[\frac{2 + 6i_2}{7}\right]$

$\frac{di_2}{dt} + 3.5714\, i_2 = 2.1429$

$i_2(t) = \frac{2.1429}{3.5714} + Ke^{-3.57t}$

$i_2(t) = 0.6 + Ke^{-3.57t}$

$i_2(0) = 0.6 + K = 0.667$

$K = 0.067$

$i_2(t) = 0.6 + 0.067e^{-3.57t}$