

# EE240: Circuits I

## ASSIGNMENT 5

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Due Date: 06 Dec 2019

- Submissions will only be accepted on A4 sized papers.
  - Write your name and roll number clearly at the top of the assignment.
  - You are required to show working where necessary; answers will not be graded if working is not shown.
  - The assignment is worth **100** marks, you are advised to start early as it is long.
  - Feel free to contact the instructor or teaching assistants if you have any questions.
  - Any instance of plagiarism will be **severely** dealt with; such cases would be subjected to **disciplinary action** in accordance with university rules and regulations.
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**Problem 1 (10 marks)**

Determine  $v(t)$  for  $t > 0$  in the following circuit.

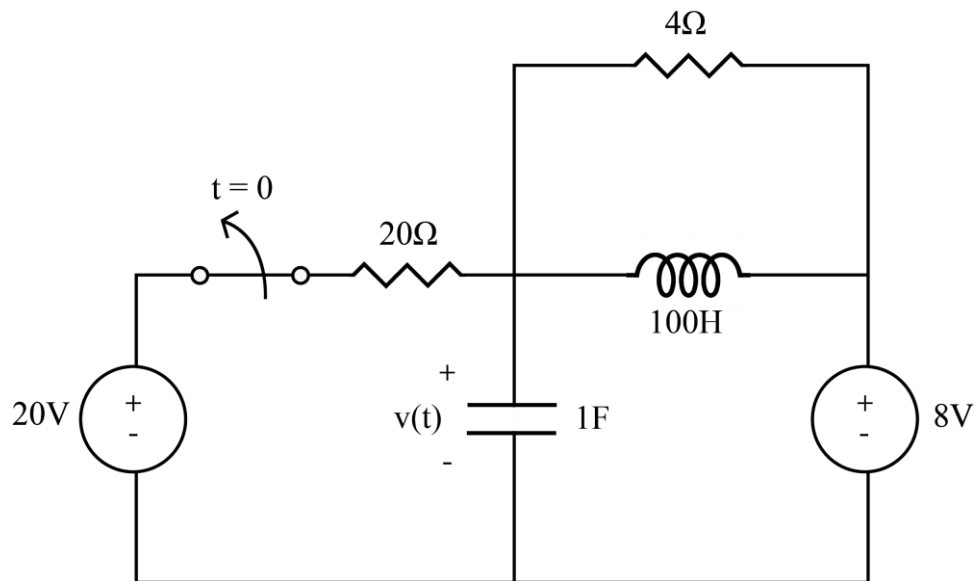


Figure 1

**Problem 2 (15 marks)**

Derive an expression for  $i(t)$  for  $t > 0$  in the circuit given below.

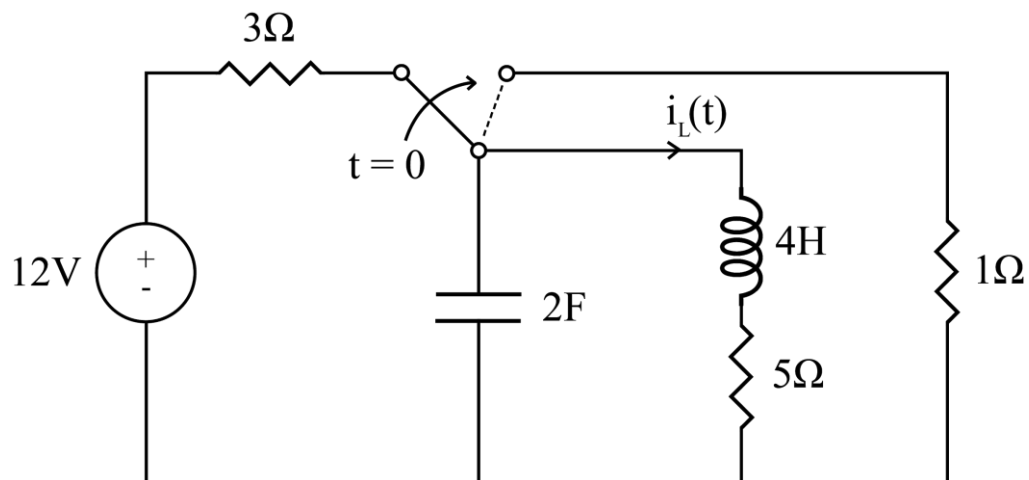


Figure 2

**Problem 3 (15 marks)**

Determine  $i(t)$  for  $t > 0$  in the network shown below.

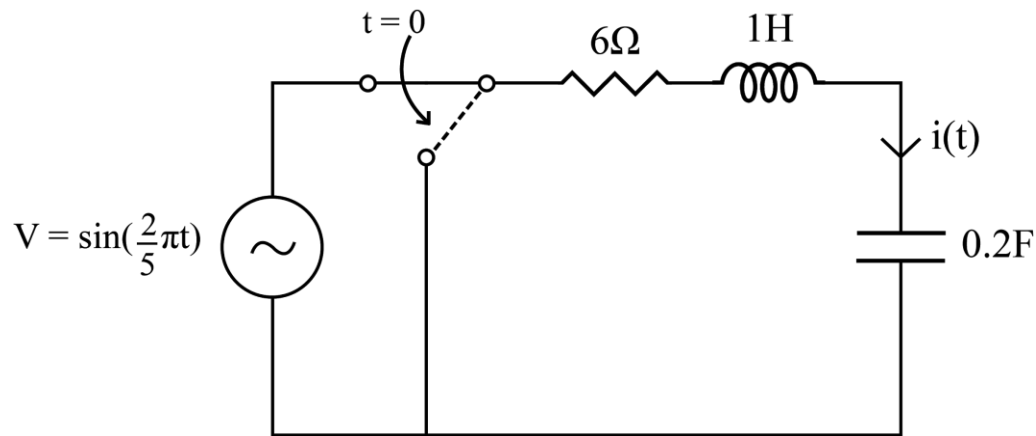


Figure 3

**Problem 4 (10 marks)**

The following network shows a combination of capacitors and resistors connected to a sinusoidal voltage source. Determine  $i(t)$  for  $t > 0$ .

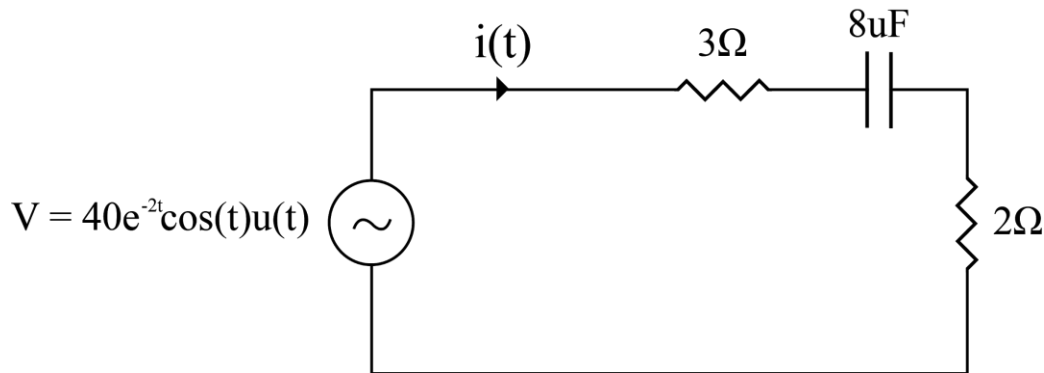


Figure 4

**Problem 5 (10 marks)**

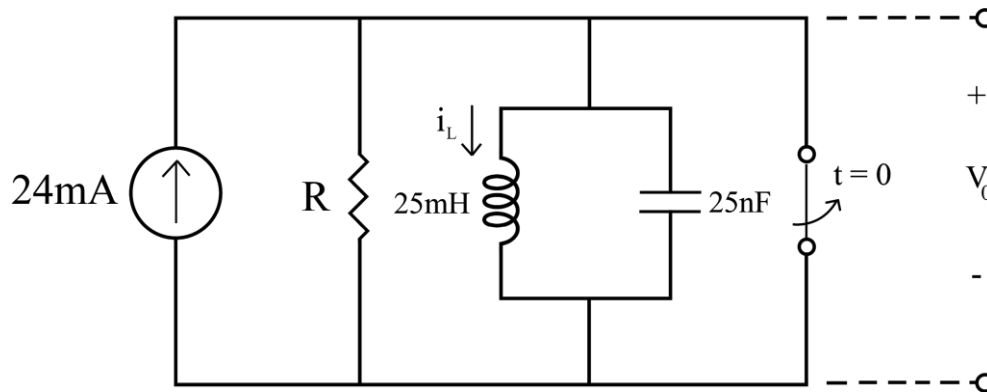
A parallel RLC circuit has the following component values:  $R = 2\text{k}\Omega$ ,  $L = 250\text{mH}$ , and  $C = 10\text{nF}$ .

- [2 marks]** Formulate the characteristic equation for the voltage response of the circuit and calculate its roots.
- [2 marks]** Determine whether the voltage response is underdamped, critically damped, or overdamped.
- [2 marks]** Find the value of the resistor (in ohms) for which the system will have a damped frequency of  $12\text{k rad/s}$ .
- [2 marks]** Formulate the characteristic equation for the voltage response with the resistance value from part (c) and calculate its roots.

(e) [2 marks] Find the value of the resistor (in ohms) for which the voltage response will become critically damped.

### Problem 6 (15 marks)

You are given the parallel RLC circuit shown in the figure below.



The initial energy stored in the circuit is zero. At time  $t = 0$ , a DC current source is applied to the circuit as shown.

(a) [5 marks] Suppose that the resistor has a value of  $400\Omega$ . Find the following:

- (i)  $i_L(0)$
- (ii)  $di_L(0^+)/dt$
- (iii) Roots of the characteristic equation
- (iv) Mathematical expression for  $i_L(t)$  for  $t \geq 0$ . Sketch the graph on a plot ranging from 0 to  $220\mu s$  (time axis).

(b) [4 marks] Suppose that the value of the resistor has been changed to  $625\Omega$ . Find the expression for  $i_L(t)$  for  $t \geq 0$ . Sketch its graph on the same plot from part (a)(iv).

(c) [4 marks] Suppose that the value of the resistor has been changed yet again to  $500\Omega$ . Find the expression for  $i_L(t)$  for  $t \geq 0$  and plot its graph on the same plot from part (a)(iv).

(d) [2 marks] Looking at the plots of  $i_L(t)$  for  $t \geq 0$  for the three resistor values:  $400\Omega$ ,  $625\Omega$ , and  $500\Omega$ , identify the time it takes for each plot to reach 90% of its final output value. Based on your observations, which value of  $R$  (ohms) creates a system design that puts a premium on reaching 90% of its final output value in the shortest time?

*Note: You may use Matlab to plot your graphs*

### Problem 7 (10 marks)

Solve the differential equations given below:

(a) [6 marks]

$$d^2i/dt^2 + 6(di/dt) + 8i = 5\cos(7t)$$

$$\text{where } i(0^+) = 3.94 \text{ and } di(0^+)/dt = -9.57$$

(b) [4 marks]

$$d^3i/dt^3 + 10(d^2i/dt^2) + 31(di/dt) + 30i = 0$$

$$\text{where } i(0^+) = 5, \quad di(0^+)/dt = -6, \quad \text{and } d^2i(0^+)/dt^2 = 30$$

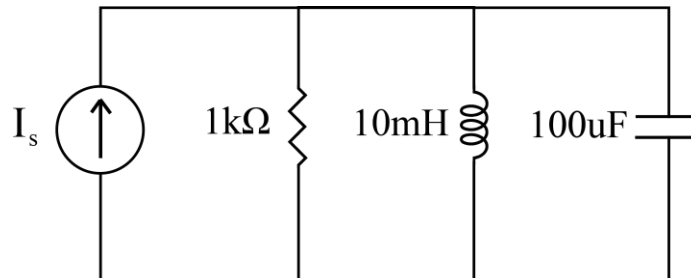
### Problem 8 (5 marks)

Given the following parallel RLC circuit powered by a current source, compute its:

(a) [2 marks] Resonant Frequency

(b) [2 marks] Bandwidth

(c) [1 mark] Quality Factor



### Problem 9 (10 marks)

The current flowing through a series RLC circuit is given by:

$$i(t) = Xe^{-4t} - 25.5e^{-6t} - 2.5e^{-2t} + 30e^{-5t}$$

where  $X$  is an unknown constant, the voltage source is given by:

$$V_s(t) = (5e^{-2t} + 3e^{-5t})u(t)$$

Where  $u(t)$  is the unit step function. The inductance is given by  $L = 0.5\text{H}$ .

(a) [4 marks] Find the corresponding values of  $R$  and  $C$  and draw the circuit.

(b) [4 marks] Determine the value of  $X$ .

(c) [2 marks] Derive an expression for voltage across inductor  $v_L(t)$  and find  $v_L(1)$ , i.e. inductor voltage at  $t = 1\text{s}$ .