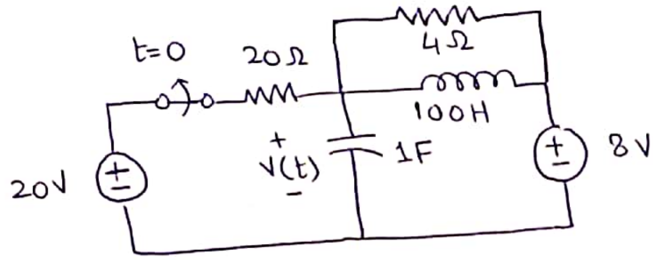
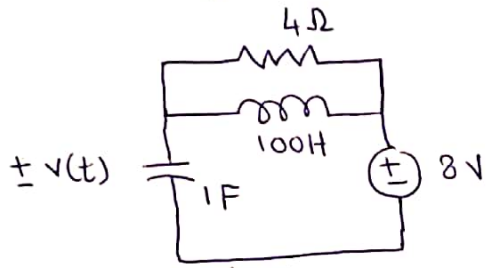
Assignment 05 Solutions

1.

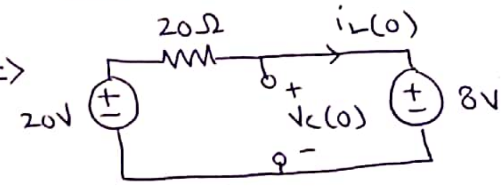


$\Downarrow$

t=0+



$\Rightarrow$ t=0−



$V_c(0) = 8V$

$i_L(0) = \dfrac{20-8}{20} = 0.6A$

$$\frac{dv(t)}{dt} + \overbrace{\frac{1}{100}\int (v(t)-8)\,dt}^{i_L(0)} + \overbrace{\frac{v(t)-8}{4}}^{0} = 0 \quad \Rightarrow \quad \frac{dv(t)}{dt} = -0.6$$

$$\frac{d^2v(t)}{dt^2} + 0.01(v(t)-8) + \frac{1}{4}\frac{dv(t)}{dt} = 0$$

$$\frac{d^2v(t)}{dt^2} + 0.25\frac{dv(t)}{dt} + 0.01v(t) = 0.08$$

char eq: $s^2 + 0.25s + 0.01 = 0$

$s_1 = -0.05, -0.2$

$V_c(t) = K_1e^{-0.05t} + K_2e^{-0.2t}$

$V_p(t) = A$

$V_p'(t) = V''_p(t) = 0 \quad \Rightarrow \quad 0.01A = 0.08$

$\boxed{A = 8}$

$V_p(t) = 8V$

$v(t) = K_1e^{-0.05t} + K_2e^{-0.2t} + 8$

$v(0) = K_1 + K_2 + 8 = 8$

$K_1 + K_2 = 0$ ①

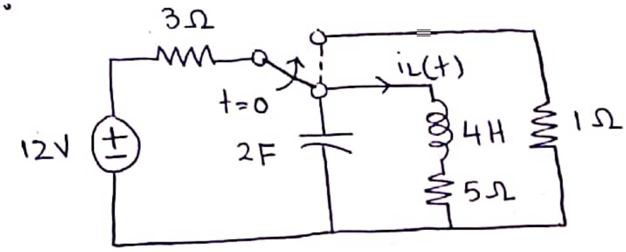
$\dfrac{dv(0)}{dt} = -0.05K_1 - 0.2K_2 = -0.6$ ②

solving ① & ② $\Rightarrow$ $K_1 = -4$

$K_2 = 4$

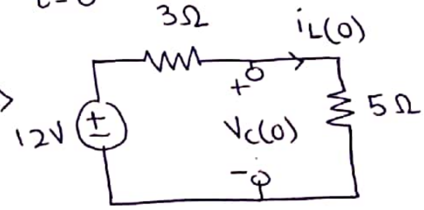
$$\boxed{v(t) = -4e^{-0.05t} + 4e^{-0.2t} + 8 \quad V \qquad t>0}$$ ANS

2.



$\Rightarrow$

t=0⁻

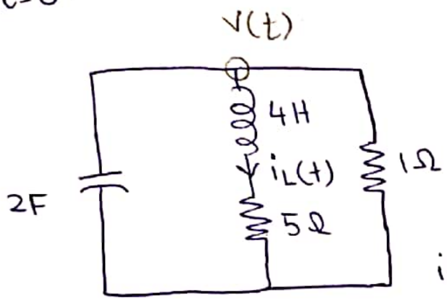


$i_L(0) = \frac{12}{8} = 1.5A$

$V_c(0) = \frac{5}{8} \times 12$
$= 7.5\,V$

$\Downarrow$

t=0⁺



$$\overbrace{V_c(t)}^{7.5} = 4\frac{di(t)}{dt} + \overbrace{5i(t)}^{5(1.5)} \qquad \frac{dV(t)}{dt} = 4\frac{d^2i(t)}{dt^2} + 5\frac{di(t)}{dt}$$

$$i(t) + \frac{V(t)}{1} + 2\frac{dV(t)}{dt} = 0$$

$$i(t) + 4\frac{di(t)}{dt} + 5i(t) + 8\frac{d^2i(t)}{dt^2} + 10\frac{di(t)}{dt} = 0$$

$$4\frac{di(0)}{dt} = 7.5 - 5(1.5)$$

$$\frac{di(0)}{dt} = 0\ As^{-1}$$

$$8\frac{d^2i(t)}{dt^2} + 14\frac{di(t)}{dt} + 6i(t) = 0$$

$$8s^2 + 14s + 6 = 0$$

$s_1 = -0.75$
$s_2 = -1$

$$i(t) = K_1e^{-0.75t} + K_2e^{-t}$$

$$i(0) = K_1 + K_2 = 1.5 \quad (1)$$

$$\frac{di(t)}{dt} = -0.75K_1e^{-0.75t} - K_2e^{-t}$$

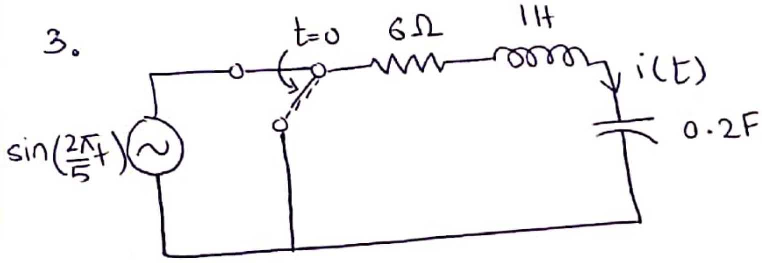
$$\frac{di(0)}{dt} = -0.75K_1 - K_2 = 0 \quad (2)$$

solving (1) & (2)

$K_1 = 6$
$K_2 = -4.5$

$$\boxed{i(t) = 6e^{-0.75t} - 4.5e^{-t}\ A \qquad t>0}$$ ANS

3.



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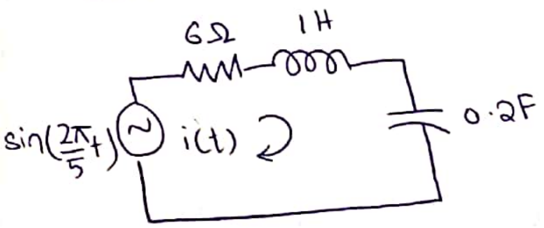
$t = 0^-$



$$6i(t) + \frac{di(t)}{dt} + 5\int i(t) = \sin\left(\frac{2\pi}{5}t\right)$$

$$6\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} + 5i(t) = \frac{2\pi}{5}\cos\left(\frac{2\pi}{5}t\right)$$

$$i_p(t) = A\cos\left(\frac{2\pi}{5}t\right) + B\sin\left(\frac{2\pi}{5}t\right)$$

$$i_p'(t) = -\frac{2\pi}{5}A\sin\left(\frac{2\pi}{5}t\right) + \frac{2\pi}{5}B\cos\left(\frac{2\pi}{5}t\right)$$

$$i_p''(t) = -\frac{4\pi^2}{25}A\cos\left(\frac{2\pi}{5}t\right) - \frac{4\pi^2}{25}B\sin\left(\frac{2\pi}{5}t\right)$$

$$-\frac{12\pi}{5}A\sin\left(\frac{2\pi}{5}t\right) + \frac{12\pi}{5}B\cos\left(\frac{2\pi}{5}t\right) - \frac{4\pi^2}{25}A\cos\left(\frac{2\pi}{5}t\right) - \frac{4\pi^2}{25}B\sin\left(\frac{2\pi}{5}t\right) + 5A\cos\left(\frac{2\pi}{5}t\right)$$
$$+ 5B\sin\left(\frac{2\pi}{5}t\right) = \frac{2\pi}{5}\cos\left(\frac{2\pi}{5}t\right)$$

$$-\frac{12\pi}{5}A - \frac{4\pi^2}{25}B + 5B = 0$$

$$-\frac{12\pi}{5}A = \left(5 - \frac{4\pi^2}{25}\right)B$$

$$B = 2.204A$$

$$\frac{12\pi}{5}B - \frac{4\pi^2}{25}A + 5A = \frac{2\pi}{5}$$

$$16.62\cancel{B}\,A - \frac{4\pi^2}{25}A + 5A = \frac{2\pi}{5}$$

$$\boxed{\begin{array}{l} A = 0.0627 \\ B = 0.138 \end{array}}$$

$$i_p(t) = 0.0627\cos\left(\frac{2\pi}{5}t\right) + 0.138\sin\left(\frac{2\pi}{5}t\right)$$

$$i(0) = i_p(0) = 0.0627\text{ A}$$

$$\cancel{\frac{di(0)}{dt}} = \frac{di_p(0)}{dt} = 0.138 \times \frac{2\pi}{5} = 0.174\text{ As}^{-1}$$

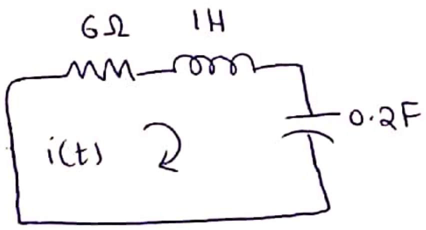
$\cancel{V_c(0)}$

$$6i(t) + \frac{di(t)}{dt} + \overbrace{5\int i(t)}^{V_c} = \cancel{\sin\left(\frac{2\pi}{5}t\right)}^{\,0}$$

$$-(6(0.0627) + 0.174) = V_c(0)$$

$$V_c(0) = -0.550\text{ V}$$

$t = 0^+$



$$6i(t) + \frac{di(t)}{dt} + \overbrace{5\int i(t)\,dt}^{V_c} = 0 \quad \Rightarrow \frac{di(0)}{dt} = 0.550 - 6(0.0627)$$

$$\frac{d^2 i(t)}{dt^2} + 6\frac{di(t)}{dt} + 5i(t) = 0 \qquad \frac{di(0)}{dt} = 0.174\,As^{-1}$$

$$s^2 + 6s + 5 = 0$$

$$s_1 = -1$$
$$s_2 = -5$$

$$i(t) = K_1 e^{-t} + K_2 e^{-5t} \qquad \frac{di(t)}{dt} = \cancel{K_1 e^{-t} - 5e^{-5t}} - K_1 e^{-t} - 5K_2 e^{-5t}$$

$$i(0) = K_1 + K_2 = 0.0627 \qquad \frac{di(0)}{dt} = -K_1 - 5K_2 = 0.174$$

solving simultaneously:

$K_1 = 0.122$

$K_2 = -0.060$

$$\boxed{i(t) = 0.122e^{-t} - 0.060e^{-5t}\ A \quad t > 0}$$ ANS

Q4-

$t = 0^-$

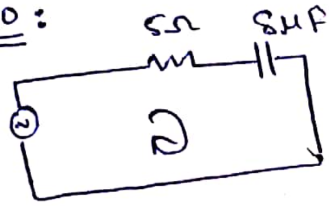
$i(0^-) = 0\,A$

$t = 0^+$

$i(0^+) = i(0^-) = 0\,A$

(inductor prevents instantaneous change in current)

For $t > 0$:



Loop eq:

$$5i(t) + \frac{1}{8\,\mu F}\int i(t)\,dt = 40e^{-2t}\cos(t)$$

Taking derivative and simplifying:

$$\frac{di(t)}{dt} + 25000\,i(t) = -8e^{-2t}\sin(t) - 16e^{-2t}\cos(t) \quad \text{— (1)}$$

Now, $i(t) = i_c(t) + i_p(t)$

1) for $i_c(t)$, characteristic eq. from (1):

$$s + 25000 = 0$$
$$s = -25000$$

$$i_c(t) = Ke^{-25000t}$$

2) For $i_p(t)$:

solution will be in the form:

$$i_p(t) = [A\cos(t) + B\sin(t)]e^{-2t}$$

$$\frac{di_p}{dt} = [A\cos(t) + B\sin(t)](-2e^{-2t}) + [-A\sin(t) + B\cos(t)]e^{-2t}$$

Putting these in (1):

$$[A\cos(t) + B\sin(t)](-2e^{-2t}) + [-A\sin(t) + B\cos(t)](e^{-2t})$$
$$+ 25000[(A\cos(t) + B\sin(t))e^{-2t}] = -8e^{-2t}\sin(t) - 16e^{-2t}\cos(t)$$

$$e^{-2t}\cos(t)\left[-2A+B+25000A\right]+e^{-2t}\sin(t)\left[-2B-A+25000B\right]$$

$$= -8e^{-2t}\sin(t) - 16e^{-2t}\cos(t)$$

By comparison :

$$24998A + B = -16$$

$$-A + 24998B = -8$$

Solving simultaneously :

$$A = -6.4\times10^{-4}$$

$$B = -3.2\times10^{-4}$$

$$\Rightarrow i_p = \left[(-6.4\times10^{-4})\cos(t) + (-3.2\times10^{-4})\sin(t)\right]e^{-2t}$$

$$\therefore i(t) = Ke^{-25000t} + \left[(-6.4\times10^{-4})\cos(t) + (-3.2\times10^{-4})\sin(t)\right]e^{-2t}$$

For K, using $i(0^+) = 0$

$$i(0^+) = Ke^{-25000(0)} + \left[(-6.4\times10^{-4})\cos(0) + (-3.2\times10^{-4})\sin(0)\right]e^{-2(0)} = 0$$

$$K - 6.4\times10^{-4} = 0$$

$$K = 6.4\times10^{-4}$$

$$\Rightarrow \boxed{i(t) = (6.4\times10^{-4})e^{-25000t} + \left[(-6.4\times10^{-4})\cos(t) + (-3.2\times10^{-4})\sin(t)\right]e^{-2t}}$$

QUESTION #5

Parallel RLC circuit

$R = 2k\Omega$

$L = 250mH$

$C = 10nF$

a) Voltage Response Characteristic Equation + roots.

neper frequency $\alpha = \frac{1}{2RC} = \frac{1}{(2)(2000)(10\times10^{-9})} = 25{,}000$ rad/s

resonant radian frequency $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.25)(10\times10^{-9})}} = 20{,}000$ rad/s

Roots:
$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -25{,}000 + \sqrt{(25{,}000)^2 - (20{,}000)^2} = -10{,}000 \text{ rad/s}$$
$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -25{,}000 - \sqrt{(25{,}000) - (20{,}000)^2} = -40{,}000 \text{ rad/s}$$

b) $\omega_0^2 < \alpha^2$, therefore Voltage response is over-damped.

c) $\omega_d = \sqrt{\omega_0^2 - \alpha^2} \longrightarrow \alpha = \sqrt{\omega_0^2 - \omega_d^2} = \sqrt{20{,}000^2 - 12{,}000^2} = 16{,}000$ rad/s

$\alpha = \frac{1}{2RC} \longrightarrow R = \frac{1}{2\alpha C} = \frac{1}{(2)(16{,}000)(10\times10^{-9})} = 3.125\ k\Omega = 3125\ \Omega$

d) $\omega_0^2 > \alpha^2$, therefore voltage response is under damped

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{20{,}000^2 - 16{,}000^2} = 12{,}000$ rad/s

$S = -\alpha \pm j\omega_d$

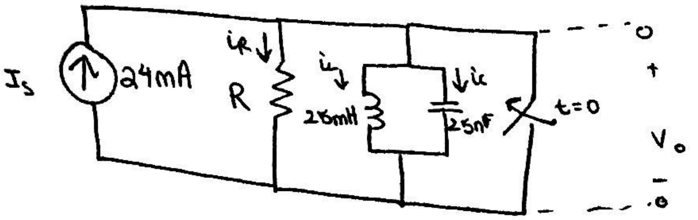
$S_1 = -\alpha + j\omega_d = -16{,}000 + j\,12{,}000$ rad/s

$S_2 = -\alpha - j\omega_d = -16{,}000 - j\,12{,}000$ rad/s

e) Critically damped occurs when $\omega_0^2 = \alpha^2 = 20{,}000$ rad/s

$R = \frac{1}{2\alpha C} = \frac{1}{(2)(20{,}000)(10\times10^{-9})} = 2500\ \Omega = 2.5k\Omega$

QUESTION #6



a) $R = 400\,\Omega$

i) Initial value i_L

ii) Initial value di/dt

iii) Char Eq * Roots

iv) expression $i_L(t)$ for $t \geq 0$. Plot 0–200μs

b) $R = 625\,\Omega$. Expression + plot

c) $R = 500\,\Omega$. Expression + plot

d) compare graphs. Time for 90% output, which R-system = fastest

a) i) No energy stored $t<0$. Current stored in inductor = 0 & inductor doesn't permit instantaneous change. $\underline{i_L(0) = 0}$

ii) Initial Voltage on Capacitor = 0. Same at 0^+. $V = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt}(0^+) = 0$

iii) $\omega_0^2 = \frac{1}{LC} = \frac{10^{12}}{(25)(25)} = 16 \times 10^8$

$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(400)(25)} = 5 \times 10^4$ rad/s ; $\alpha^2 = 25 \times 10^8$

$\omega_0^2 < \alpha^2$ $\therefore$ real & distinct roots.

$\boxed{\begin{aligned} S_1 &= (-5\times10^4) + (3\times10^4) \\ &= -20{,}000 \text{ rad/s} \\ S_2 &= (-5\times10^4) - (3\times10^4) \\ &= -80{,}000 \text{ rad/s} \end{aligned}}$

iv) Inductor Current Response = overdamped.

$i_L = I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$

$i_L(0) = I_f + A_1' + A_2' = 0$

$\frac{di_L(0)}{dt} = s_1 A_1' + s_2 A_2' = 0$

solve simultaneously $\Longrightarrow$ $A_1' = -32$ mA, $A_2' = 8$ mA

$i_L(t) = 24 - 32e^{-20{,}000t} + 8e^{-80{,}000t}$ mA for $t \geq 0$

b) L & C same $\therefore$ ω_0^2 same $= 16 \times 10^8$ } $\omega_0^2 > \alpha^2$ = complex roots. (underdamped)

R↑, so α↓. $\alpha = 3.2 \times 10^4$ rad/s

$S_1 = -3.2\times10^4 + j\,2.4\times10^4$ rad/s

$S_2 = -3.2\times10^4 - j\,2.4\times10^4$ rad/s

$\longrightarrow i_L(t) = I_f + B_1' e^{-\alpha t}\cos(\omega_d t) + B_2' e^{-\alpha t}\sin(\omega_d t)$

$\alpha = 32$k rad/s, $\omega_d = 24$k rad/s, $I_f = 24$ mA

$i_L(0) = I_f + B_1' = 0$

$\frac{di_L(0)}{dt} = \omega_d B_2' - \alpha B_1' = 0$

$\longrightarrow B_1' = -24$ mA, $B_2' = -32$ mA $\Longrightarrow$ $i_L(t) = 24 - 24e^{-32000t}\cos(24{,}000t) - 32e^{-32000t}\sin(24{,}000t)$ mA for $t \geq 0$

c) L & C same $\therefore$ ω_0^2 same $= 16\times10^8$

$R = 500\,\Omega$, $\alpha = 4\times10^4$ /s (critical damping)

$i_L(t) = I_f + C_1' te^{-\alpha t} + C_2' e^{-\alpha t}$

$i_L(0) = I_f + C_2' = 0$

$\frac{di_L(0)}{dt} = C_1' - \alpha C_2' = 0$

$\longrightarrow C_1' = -960{,}000$ mA/s, $C_2' = -24$ mA

$i_L(t) = 24 - 960{,}000te^{-40{,}000t} - 24e^{-40{,}000t}$ mA for $t \geq 0$

d) Final Value $i_L = 24$ mA, 90% = 21.6 mA

$R = 625\,\Omega$ (underdamped) $\longrightarrow$ $\boxed{t = 74\,\mu s}$ — underdamped response at $R = 625\,\Omega$ reaches 90% output fastest.

$R = 500\,\Omega$ (critically damped) $\longrightarrow$ $t = 97\,\mu s$

$R = 400\,\Omega$ (overdamped) $\longrightarrow$ $t = 130\,\mu s$

Q7.

a) $\frac{di^2}{dt^2} + 6\frac{di}{dt} + 8i = 5\cos(7t)$

1) $i_c(t) = ?$

Characteristic eq: $s^2+6s+8=0$

Solving: $(s+2)(s+4)=0$

$s=-2,-4$

$i_c(t) = K_1e^{-2t} + K_2e^{-4t}$

2) $i_p(t) = A\cos(7t) + B\sin(7t)$

$\frac{di_p}{dt} = -7A\sin(7t) + 7B\cos(7t)$

Plugging into original equation:

$-49A\cos(7t) - 49B\sin(7t) - 42A\sin(7t) + 42B\cos(7t)$
$+ 8A\cos(7t) + 8B\sin(7t) = 5\cos(7t)$

By comparison:

$-49A + 42B + 8A = 5$

$-41A + 42B = 5$ — ①

$-49B - 42A + 8B = 0$

$-42A - 41B = 0$ — ②

$\Rightarrow A = -0.0595,\ B = 0.0610$

$\Rightarrow i(t) = K_1e^{-2t} + K_2e^{-4t} - 0.0595\cos(7t) + 0.0610\sin(7t)$

Using conditions for K_1 and K_2:

1) $i(0^+) = K_1 + K_2 - 0.0595 = 3.94$

$\Rightarrow K_1 + K_2 = 4$ — ③

2) $\frac{di(0^+)}{dt} = -2K_1 - 4K_2 + 0.427 = -9.57$

$\Rightarrow 2K_1 + 4K_2 = 10$ — ④

$\Rightarrow K_1 = 3,\ K_2 = 1$

$$\boxed{i(t) = 3e^{-2t} + e^{-4t} - 0.0595\cos(7t) + 0.0610\sin(7t)}$$

b) $\frac{d^3 i}{dt^3} + 10\frac{d^2 i}{dt^2} + 31\frac{di}{dt} - 30i = 0$

This is a homogenous equation, hence only the Complementary solution exists for this equation.

Char. eq:

$$s^3 + 10s^2 + 31s - 30 = 0$$

Solving:

$$s = -5, -2, -3$$

$\Rightarrow i(t) = K_1 e^{-5t} + K_2 e^{-2t} + K_3 e^{-3t}$

Using conditions to find K_1, K_2, K_3:

1) $i(0^+) = K_1 + K_2 + K_3 = 5$ — ①

2) $\frac{di}{dt}(0^+) = -5K_1 - 2K_2 - 3K_3 = -6$ — ②

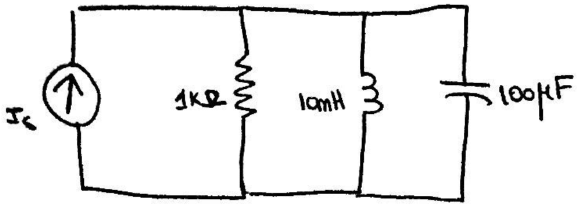
3) $\frac{d^2 i}{dt^2}(0^+) = 25K_1 + 4K_2 + 9K_3 = 30$ — ③

Solving simultaneously:

$\Rightarrow K_1 = 5,\ K_2 = 19,\ K_3 = -19$

$$\Rightarrow \boxed{i(t) = 5e^{-5t} + 19e^{-2t} - 19e^{-3t}}$$

QUESTION #8



a) Resonant Frequency

b) Bandwidth

c) Quality Factor

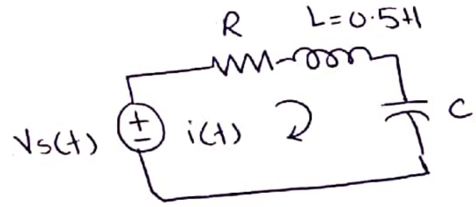
a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(10^{-2})(10^{-4})}} = 10^3 \text{ rad/s}$

b) $BW = \dfrac{1}{RC} = \dfrac{1}{(10^3)(100\times10^{-6})} = \dfrac{1}{0.1} = 10 \text{ rad/s}$

c) $QF = \dfrac{\omega_0}{BW} = \dfrac{\left(\frac{1}{\sqrt{LC}}\right)}{\left(\frac{1}{RC}\right)} = \dfrac{RC}{\sqrt{LC}} = R\sqrt{\dfrac{C}{L}} = 10^3\sqrt{\dfrac{(10^{-4})}{(10^{-2})}} = 100$

9. a) roots are $S_1 = -4, S_2 = -6$

char eq. $s^2 + 10s + 24$



$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V_s(t)$$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{V_s'(t)}{L}$$

$$\frac{d^2i(t)}{dt^2} + 2R\frac{di(t)}{dt} + \frac{2}{C}i(t) = 2V_s'(t)$$

$2R = 10$ $\qquad$ $2/C = 24$

$\boxed{R = 5\Omega}$ $\qquad$ $\boxed{C = 1/12 F}$

b) $i(0) = 0 A$

$$\frac{di(0)}{dt} = V_s(0)/L = 2\times 8 = 16\ As^{-1}$$

~~$i(t) = K_1 + K_2$~~

$$i(t) = K_1e^{-4t} + \overbrace{K_2e^{-6t}}^{-25.5} - 2.5e^{-2t} + 30e^{-5t}$$

$$i(0) = K_1 - 25.5 - 2.5 + 30 = 0$$

$\boxed{K_1 = -2}$

$\boxed{X = -2}$

c) $V_L(t) = L\frac{di(t)}{dt}$

$\boxed{V_L(t) = 4e^{-4t} + 76.5e^{-6t} + 2.5e^{-2t} - 75e^{-5t}}$

$\boxed{V_L(1) = 0.0959V}$