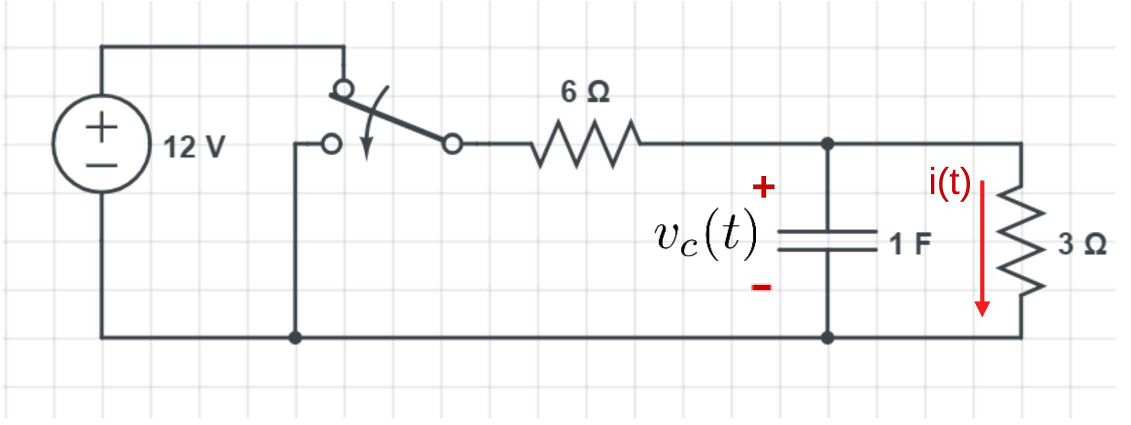
First Order Circuits

Problems – In class

Problem 1: In the following circuit, the switch is operated at t=0. Determine the current i(t) through 3Ω resistor for all times.



Analysis at $t = 0^-$

- Capacitor is open circuit.
- $v_c(0^-) = \frac{3}{9} \times 12 = 4V$

Analysis at $t = 0^+$

- $v_c(0^+) = v_c(0^-) = 4V$
- $i(0^+) = v_c(0^+)/3 = \frac{4}{3}\ A$

Analysis at $t = \infty$

- No source in the circuit - $i(\infty) = 0$

Circuit Time Constant τ

- Find $R_{eq} = R_{th}$. - Equivalent resistance across capacitor terminals is $6||3 = 2\ \Omega$.
- $\tau = C\,R_{eq} = 2\ s$

Solution Formulation

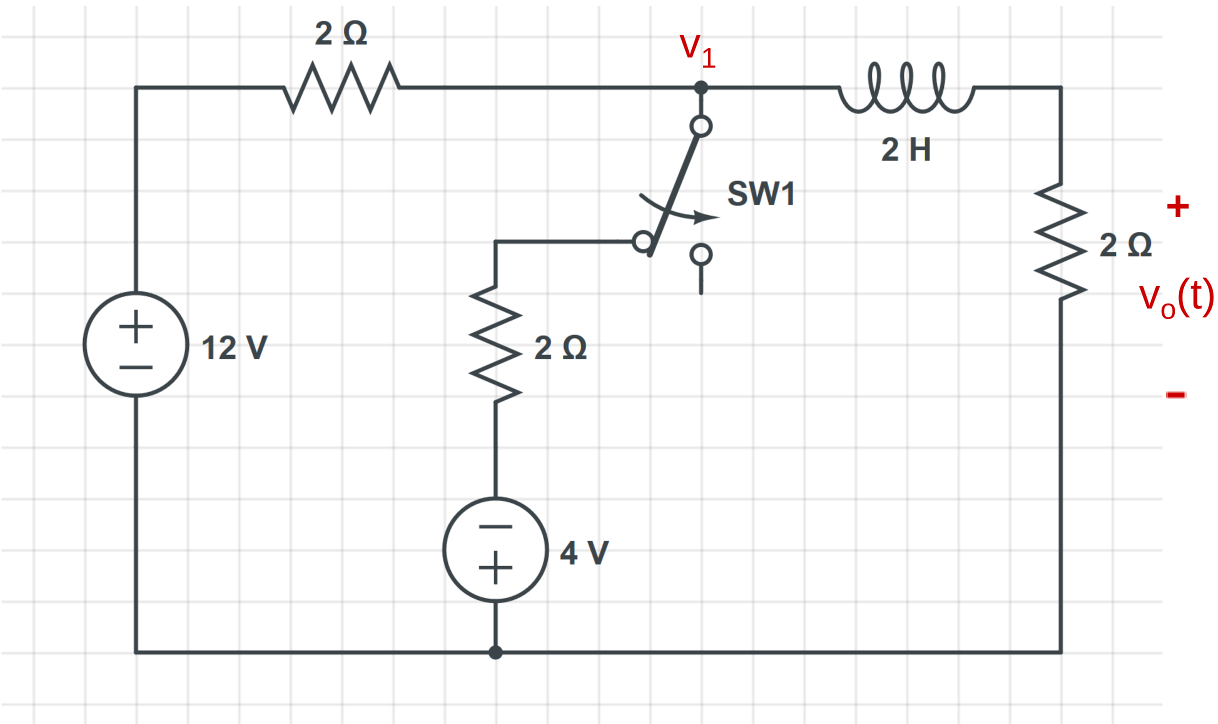
$$i(t) = K_1 + K_2 e^{-t/\tau}, \quad K_1 = i(\infty) = 0, \quad K_2 = i(0^+) - i(\infty) = \frac{4}{3}\ A$$

$$i(t) = \frac{4}{3} e^{-t/2}\ \text{(A)}$$

First Order Circuits

Problems – In class

Problem 2: In the following circuit, the switch is operated at t=0. Determine the voltage $v_o(t)$ for all times.

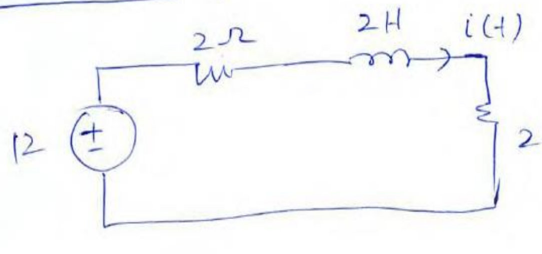


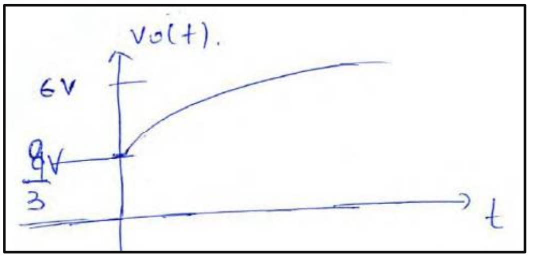
Apply nodal analysis for v_1:

$$\frac{v_1 - 12}{2} + \frac{v_1 + 4}{2} + \frac{v_1}{2} = 0 \quad \Rightarrow v_1 = \frac{8}{3}V.$$

Current through inductor: $i_L(0^-) = i_L(0^+) = \frac{4}{3}A, \quad \Rightarrow v_o(0^+) = \frac{8}{3}V$

At $t = 0^+$; 4V and 2Ω (do not have any contribution)





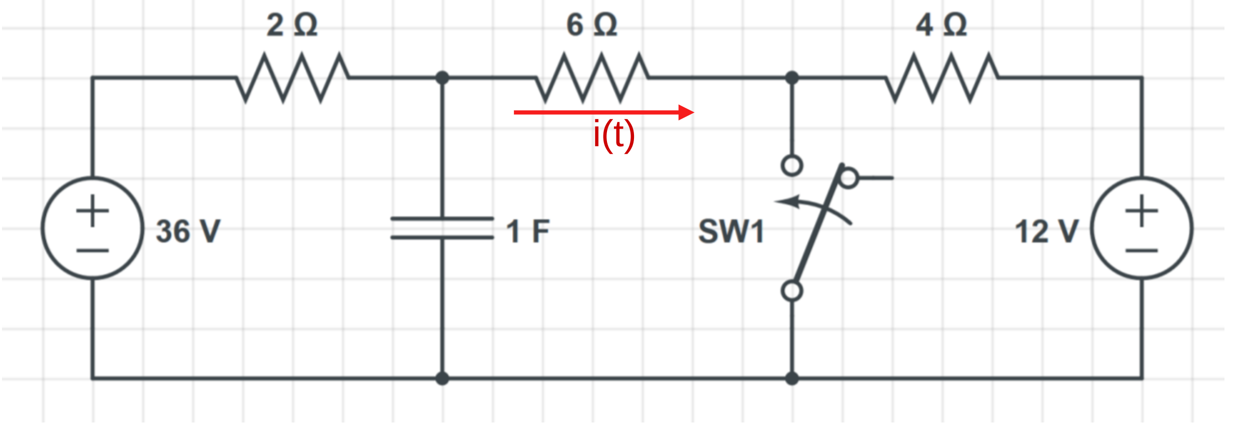
$$v_o(t) = K_1 + K_2 e^{-t/\tau}, \quad K_1 = v_o(\infty) = 6V, \quad K_2 = v_o(0^+) - v_o(\infty) = \frac{10}{3}V$$

$$\tau = \frac{L}{R}, \quad L = 2H, \quad R = 4\Omega \Rightarrow \tau = 0.5 \text{ seconds.}$$

First Order Circuits

Problems – In class

Problem 3: In the following circuit, the switch is operated at t=0. Determine the current i(t) for all times.



$$v_c(0^-) = v_c(0^+) = 32V$$

$$i(0+) = \frac{32}{6} = \frac{16}{3}A$$

$$i(\infty) = \frac{36}{8} = 4.5A$$

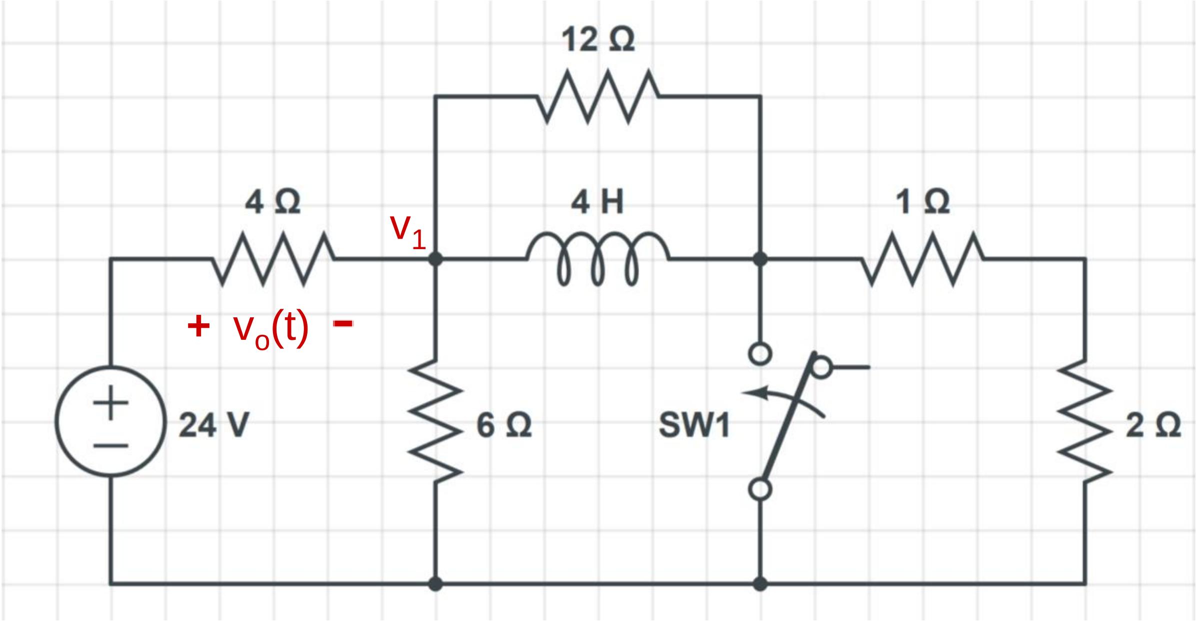
$$i(t) = K_1 + K_2 e^{-t/\tau}, \quad K_1 = i(\infty) = 4.5A, \quad K_2 = i(0^+) - i(\infty) = 5/6A$$

$\tau = R_{eq}C, \quad c = 1F, \quad R_{eq} = \frac{4}{3}\Omega \Rightarrow \tau = 1.33$ seconds. Here R_{eq} is the equivalent resistance that appears across capacitor, that is, the parallel combination of 6 and 2 Ohms.

First Order Circuits

Problems – In class

Problem 4: In the following circuit, the switch is operated at t=0. Determine the voltage $v_o(t)$ for all times.



$i_L(0^-) = i_L(0^+) = \frac{8}{3}A$

$v_o(\infty) = 24V$

To find out $i(0+)$, we use KCL to find v_1 considering the bottom node as ground: (equation of the circuit at $t = 0^+$)

$$\frac{v_1 - 24}{4} + \frac{v_1}{6} + \frac{v_1}{12} + \frac{8}{3} = 0, \quad \Rightarrow v_1 = \frac{20}{3}V$$

$$v_o(0^+) = 24 - v_1 = \frac{52}{3}V$$

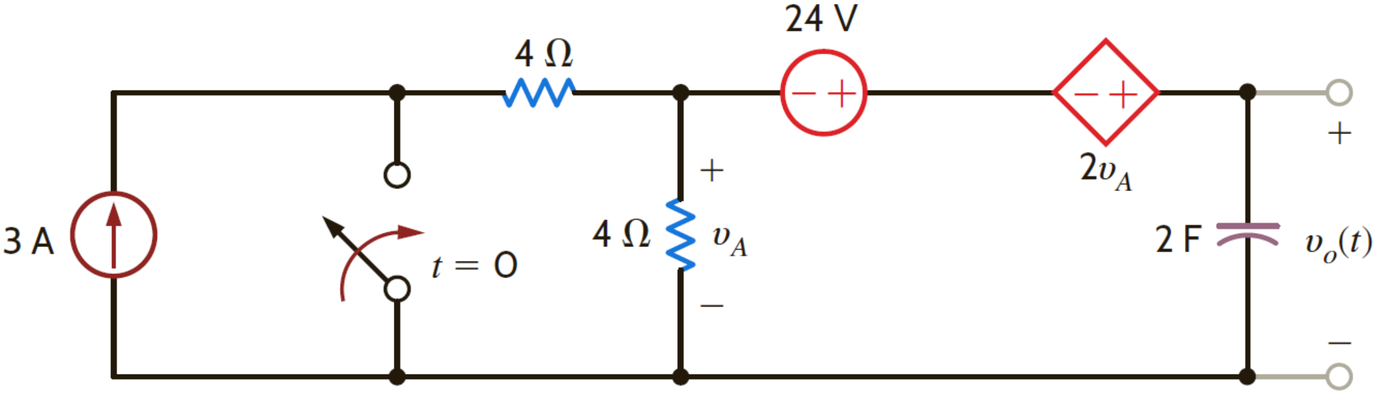
$v_o(t) = K_1 + K_2 e^{-t/\tau}, \quad K_1 = v_o(\infty) = 24V, \quad K_2 = v_o(0^+) - v_o(\infty) = -\frac{20}{3}V$

$\tau = \frac{L}{R_{eq}}, \quad L = 4H, \quad R_{eq} = 2\Omega \Rightarrow \tau = 2$ seconds. Here R_{eq} is the equivalent resistance that appears across inductor, that is, the parallel combination of 12, 6 and 4 Ohms.

First Order Circuits

Problems – In class

Problem 5: In the following circuit, the switch is operated at t=0. Determine the voltage $v_o(t)$ for all times.



Analysis at $t = 0^-$

- Capacitor is open circuit.
- $v_A = 4 \times 3 = 12V$.
- $v_o(0^-) = 2v_A + 24 + v_A = 60V$

Analysis at $t = 0^+$

- $v_o(0^+) = v_o(0^-) = 60V$

Analysis at $t = \infty$

- Capacitor is open circuit.
- $v_A = 0$ (no current).
- $v_o(\infty) = 24V$

Circuit Time Constant τ

- Find $R_{eq} = R_{th}$. Since we have dependent voltage source, use V_{th}/I_{SC} to find equivalent resistance across capacitor terminals
- $V_{th} = 24V$, $I_{SC} = 4A \Rightarrow R_{eq} = 6\,\Omega$
- $\tau = CR_{eq} = 12\ s$

Solution Formulation

$$v_o(t) = K_1 + K_2 e^{-t/\tau}, \quad K_1 = v_o(\infty) = 24V, \quad K_2 = v_o(0^+) - v_o(\infty) = 36V$$

$$v_o(t) = 24 + 36e^{-t/12} \text{ (Volts)}$$