LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 07 - Section 2 (Solutions)

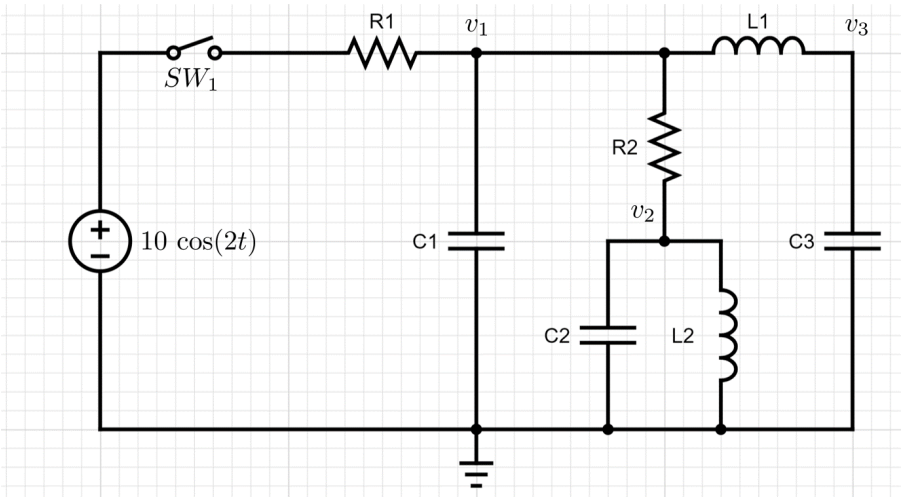
Name: _____

Campus ID: _____

Total Marks: 10
Time Duration: 15 minutes

Question 1 (10 marks)

In the following circuit, the switch SW_1 is closed at $t = 0$. Determine $v_1(0^+)$, $v_2(0^+)$, $v_3(0^+)$, $\frac{dv_1}{dt}(0^+)$, $\frac{dv_2}{dt}(0^+)$ and $\frac{d^2v_1}{dt^2}(0^+)$.



Solutions: Let $v(t) = 10\cos(2t)$.

Since the capacitors are initially uncharged, $v_1(0^+) = v_2(0^+) = v_3(0^+) = 0$

Furthermore, the current through inductors at $t = 0^+$ is zero.

Network Equations:

$$\text{Node 1}: \quad \frac{v_1 - v}{R_1} + C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_2} + \frac{1}{L_1}\int (v_1 - v_3)dt = 0 \tag{1}$$

$$\text{Node 2}: \quad \frac{v_2 - v_1}{R_1} + C_2\frac{dv_2}{dt} + \frac{1}{L_2}\int v_2 dt = 0 \tag{2}$$

$$\text{Node 3}: \quad C_3\frac{dv_3}{dt} + \frac{1}{L_1}\int (v_3 - v_1)dt = 0 \tag{3}$$

Using equation (1), $\frac{dv_1}{dt}(0^+) = \frac{v(0^+)}{R_1C_1} = \frac{10}{R_1C_1}$.

Using equation (2), $\frac{dv_2}{dt}(0^+) = 0$.

Taking derivative of equation (1), we have

$$\frac{1}{R_1}\frac{dv_1}{dt} - \frac{1}{R_1}\frac{dv}{dt} + C_1\frac{d^2v_1}{dt^2} + \frac{1}{R_2}\frac{dv_1}{dt} - \frac{1}{R_2}\frac{dv_2}{dt} + v_1 - v_3 = 0 \tag{4}$$

$$C_1\frac{d^2v_1}{dt^2}(0^+) = \frac{1}{R_1}\frac{dv}{dt}(0^+) - \frac{1}{R_1}\frac{dv_1}{dt}(0^+) - \frac{1}{R_2}\frac{dv_1}{dt}(0^+) = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{10}{R_1C_1}$$