LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 08 - Section 2 (Solutions)

Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 20 minutes

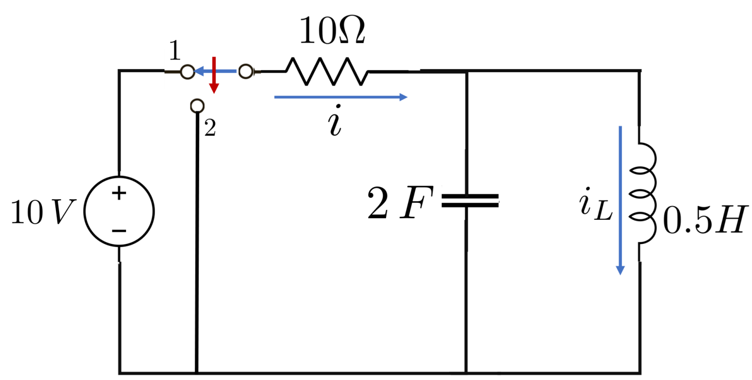
Question 1 (10 marks)

In the following circuit, the switch is moved from position 1 to position 2 at $t = 0$.

(a) [6 marks] Determine the current $i_L(t)$ for all times.

(b) [3 marks] Determine the current $i(t)$ for all times.

(c) [1 mark] What is the nature (over-damped, under-damped, critically damped or undamped) of the response?



Solution:

(a) **Initial Conditions:**

- Inductor short-circuited, which implies: $i_L(0^-) = i(0^-) = 1\,A, \quad v_c(0^-) = 0.$
- Voltage across inductor is $L di_L/dt$.
- $i_L(0^+) = 1\,A$, $\frac{di_L(0+)}{dt} = 0$ (Since voltage across capacitor is zero at $t = 0^+$).

Network Equation for $t \geq 0$

Using KCL, we have

$$LC\frac{d^2 i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L = \frac{d^2 i_L}{dt^2} + 0.05\frac{di_L}{dt} + i_L = 0.$$

The roots of the characteristic equation $s^2 + 0.05s + 1$ are $s_1, s_2 = -0.025 \pm j0.9997$. Therefore the solution is

$$i_L(t) = e^{-0.025\,t}\left(K_1\cos(0.9997t) + K_2\sin(0.9997t)\right).$$

Using initial conditions, we have $K_1 = 1$. Since $i_L'(t) = 0.9997e^{-0.025\,t}\left(-K_1\sin(0.9997t) + K_2\cos(0.9997t)\right) - 0.025e^{-0.025\,t}\left(K_1\cos(0.9997t) + K_2\sin(0.9997t)\right)$, we have $0.9997K_2 - 0.025K_1 = 0$ and $K_2 \approx 0.025$.

(b)

$$i(t) = -\frac{1}{R}(L\frac{di_L}{dt}) = -0.05\frac{di_L}{dt}, \quad t \geq 0$$

(c) Under-damped (Almost undamped).