LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 09 - Section 2 (Solutions)

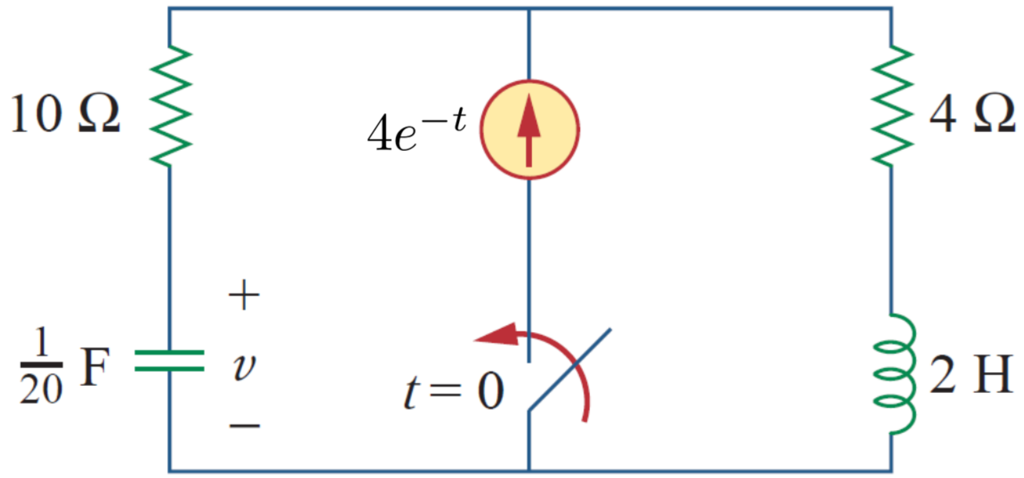
Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 20 minutes

Question 1 (10 marks)

In the following circuit, the switch is closed at $t = 0$. Determine $v(t)$ for all times.



Solution:

The current through the capacitor is $C\frac{dv}{dt} = \frac{1}{20}\frac{dv}{dt}$. Writing the outer-loop equation

$$(10)\frac{1}{20}\frac{dv}{dt} + v + 2\frac{d}{dt}\left(\frac{1}{20}\frac{dv}{dt} - 4e^{-t}\right) + 4\left(\frac{1}{20}\frac{dv}{dt} - 4e^{-t}\right) = 0 \qquad (1)$$

$$\frac{1}{10}\frac{d^2v}{dt^2} + (14)\frac{1}{20}\frac{dv}{dt} + v - 8e^{-t} = 0, \Rightarrow \quad \frac{d^2v}{dt^2} + 7\frac{dv}{dt} + 10v = 80e^{-t}. \qquad (2)$$

The solution for $v(t)$ has two parts: $v(t) = v_p(t) + v_c(t)$. To find $v_c(t)$, formulate the characteristic equation $s^2 + 7s + 10 = 0$ and determine the roots $s_1, s_2 = -2, -5$. Consequently, $v_c(t)$ is given by

$$v_c(t) = K_1e^{-2t} + K_2e^{-5t}, \quad t \geq 0.$$

To determine $v_p(t)$, we assume

$$v_p(t) = Ae^{-t} = 20e^{-t}.$$

where the constants A is obtained by substituting the assumed form of $v_p(t)$ in the differential equation such that $A - 7A + 10A = 80$.

The overall solution is

$$v(t) = K_1e^{-2t} + K_2e^{-5t} + 20e^{-t}$$

Initial Conditions:
- $i_L(0^-) = i_L(0^+) = 0\,A, \quad v(0^-) = v(0^+) = 0\,V.$
- To find $\frac{dv(0+)}{dt}$, we use the fact that inductor acts as open circuit and capacitor acts as short circuit. At $t = 0^+$, the current through the capacitor is $\frac{4}{10} = 0.4\ A$. Since $\frac{1}{20}\frac{dv}{dt}(0+) = 0.4$, we have $\frac{dv}{dt}(0+) = 8\,V/s$.

Constants of Integration:
We solve

$$K_1 + K_2 = -20, \quad -2K_1 - 5K_2 = 28$$

to obtain $K_1 = -24$ and $K_2 = 4$.