LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 09 - Section 1 (Solutions)

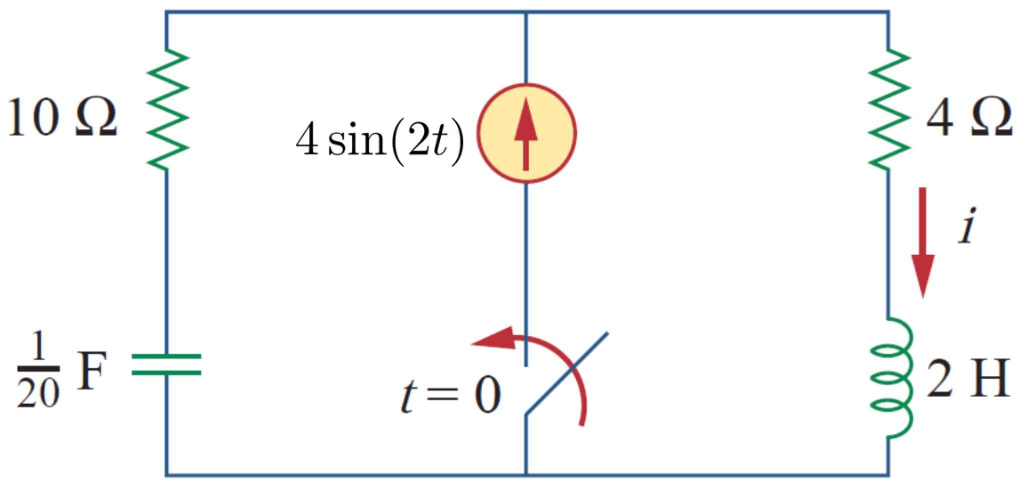
Name: ____________________

Campus ID: ____________________

Total Marks: 10
Time Duration: 20 minutes

Question 1 (10 marks)

In the following circuit, the switch is closed at $t = 0$. Determine $i(t)$ for all times.



Solution:

Writing the outer-loop equation

$$2\frac{di}{dt} + 4i + 20\int(i - 4\sin(2t))dt + 10(i - 4\sin(2t)) = \frac{di}{dt} + 2i + 10\int(i - 4\sin(2t))dt + 5(i - 4\sin(2t)) = 0 \quad (1)$$

$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 10i - 40\sin(2t) - 40\cos(2t)) = 0. \Rightarrow \frac{d^2i}{dt^2} + 7\frac{di}{dt} + 10i = 40\sin(2t) + 40\cos(2t).$$

The solution for $i(t)$ has two parts: $i(t) = i_p(t) + i_c(t)$. To find $i_c(t)$, formulate the characteristic equation $s^2 + 7s + 10 = 0$ and determine the roots $s_1, s_2 = -2, -5$. Consequently, $i_c(t)$ is given by

$$i_c(t) = K_1 e^{-2t} + K_2 e^{-5t}, \quad t \geq 0.$$

To determine $i_p(t)$, we assume

$$i_p(t) = A\cos(2t) + B\sin(2t) = -1.3793\cos(2t) + 3.4483\sin(2t),$$

where the constants A and B are obtained by substituting the assumed form of $i_p(t)$ in the differential equation such that

$$6A + 14B = 40, \quad -14A + 6B = 40.$$

The overall solution is

$$i(t) = K_1 e^{-2t} + K_2 e^{-5t} - 1.3793\cos(2t) + 3.4483\sin(2t).$$

Initial Conditions:
- $i_L(0^-) = i_L(0^+) = 0\,A, \quad v_c(0^-) = v_c(0^+) = 0\,V.$
- $\frac{di_L(0+)}{dt} = 0$ (Using (1))

Constants of Integration:
We solve

$$K_1 + K_2 = 1.3793, \quad 2K_1 + 5K_2 = 6.8966$$

to obtain $K_1 = 0$ and $K_2 = \frac{40}{29}$.