EE240 – Circuits I

Final Examination (Fall 2019)

Solutions

December 18, 2019 11:30 am–02:30 pm

Student ID ____________________________ Name _________________________________

Signature ____________________

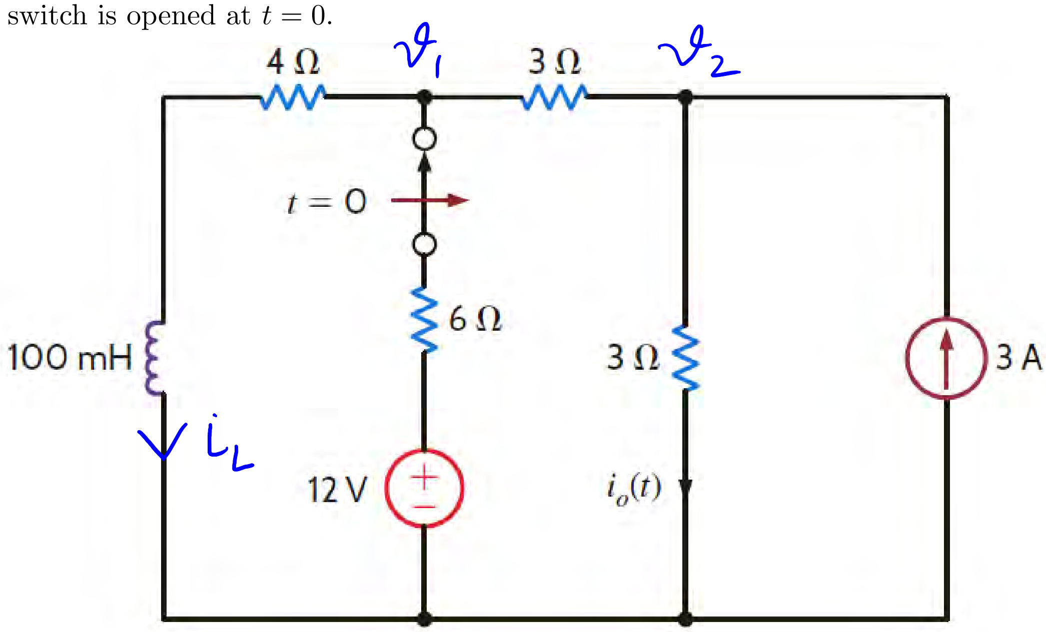
INSTRUCTIONS:

- Do not flip this page over until told to do so.
- The exam consists of 7 problems worth a total of 85 points.
- **The exam needs to be solved on this book and not on blue book.**
- If you need the blue book for rough work, please ask the exam staff.
- The exam is closed book and notes. You are allowed to bring calculator and two A4 sheet with you with *hand-written* notes on both sides.
- Read all the questions before you start working on the exam.
- You cannot keep your mobile phone(s) with you (even on silent mode or switched off).

Problem	**Total Points**	**Obtained Points**
Problem 1	15	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	05	
Problem 6	10	
Problem 7	10	
Problem 8	20	
Total	90	

Part 1: First Order Circuits

Problem 1. (**15 pts**) The circuit given below is in steady state with switch closed. The switch is opened at $t = 0$.



(a) (**4 pts**) Determine the current $i_o(t)$ at $t = 0^-$, that is, just before the switch is operated. Also determine the current through the inductor.

$t = 0^-$

* Inductor sc
* Nodal Eqs:

$$9V_1 - 4V_2 = 24$$

$$2V_2 - V_1 = 9$$

$$\Rightarrow \boxed{V_1 = 6V, \; v_2 = 7.5V}$$

$$i_o(t) = \frac{v_2}{3} = 2.5A, \quad t < 0$$

$$i_L(0^-) = \frac{v_1}{4} = 1.5A$$

(b) (**3 pts**) Determine the current $i_o(t)$ at $t = 0^+$, that is, just after the switch is operated.

$t = 0^+$

* Inductor acts as current source of 1.5A

$i_L(0^+)$

$$i_o(0^+) = 3 - i_L(0^+) = 1.5A$$

(c) (**2 pts**) Determine the current $i_o(t)$ at $t = \infty$.

$$i_o(\infty) = \left(\frac{7}{7+3}\right)(3) = 2.1A$$

(d) (**5 pts**) Using the results of the previous parts, or otherwise, determine the current $i_o(t)$ for all times $t > 0$.

$$i_o(t) = K_1 + K_2 e^{-t/\tau}$$

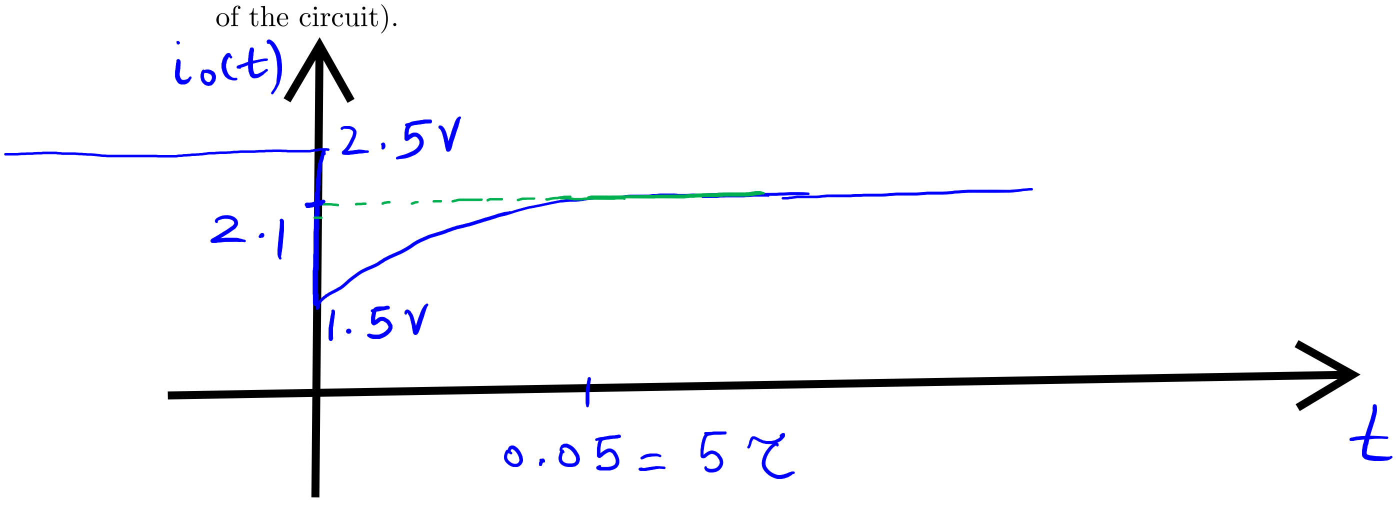
$$K_1 = 2.1$$

$$K_2 = 1.5 - 2.1 = -0.6$$

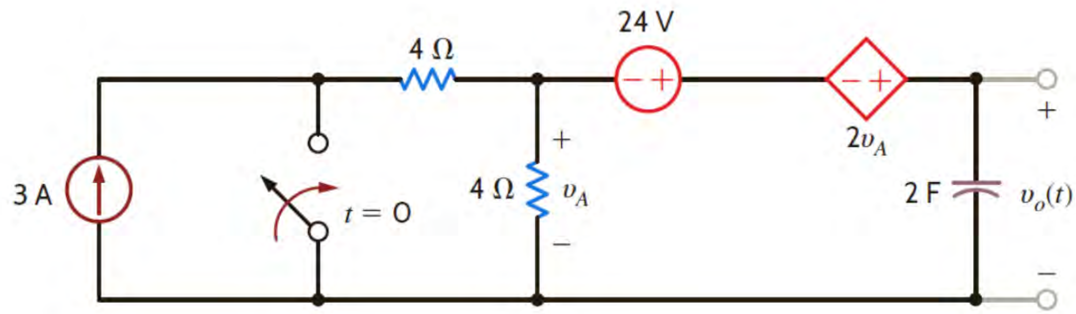
$$\tau = \frac{L}{R_{eq}} = \frac{100m}{10} = 0.01s$$

$$\Rightarrow \quad i_o(t) = 2.1 - 0.6 e^{-100t} \ A$$

(e) (**1 pts**) Plot the current $i_o(t)$ for $-\tau \le t \le 6\tau$ (where τ denotes the time constant of the circuit).



Problem 2. (**10 pts**) The circuit given below is in steady state with switch in open state. The switch is closed at $t = 0$.



(a) (**2 pts**) Determine the voltage $v_o(t)$ at $t = 0^-$.

* Capacitor OC
* $v_A = 12\,V$
* $v_o(0^-) = 2v_A + 24 + v_A = 60V$

(b) (**1 pts**) Determine the the voltage $v_o(t)$ at $t = 0^+$.

$v_o(0^+) = 60V$

(c) (**1 pts**) the voltage $v_o(t)$ at $t = \infty$

$v_A(\infty) = 0, \quad V_o(\infty) = 24V$

(d) (**6 pts**) Using the results of the previous parts, or otherwise, determine the voltage $v_o(t)$ for all times $t > 0$.

$v_o(t) = K_1 + K_2 e^{-t/\tau} \qquad t \geq 0$

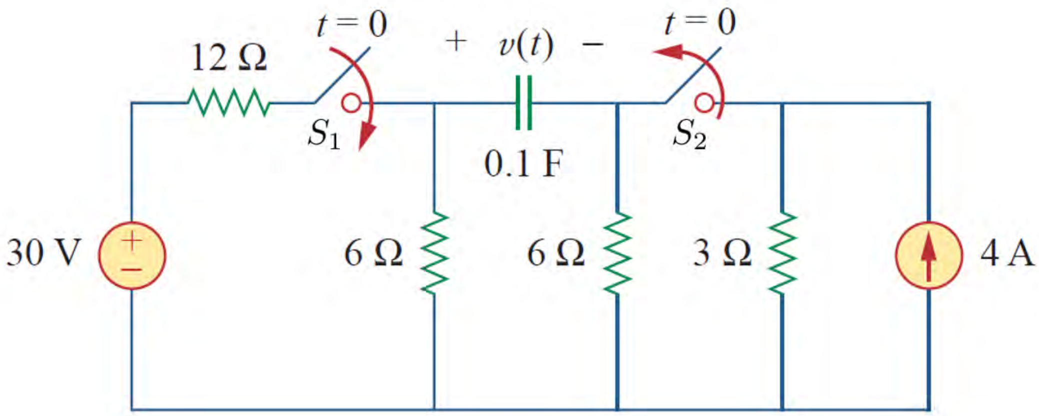
$K_1 = 24, \quad K_2 = 60 - 24 = 36$

$\tau = R_{eq} C = 12\,s$

$R_{eq} = 6\Omega$ (using Thevenin's method)

$\Rightarrow \quad v_o(t) = \begin{cases} 24 + 36 e^{-t/12} \; V & t \geq 0 \\ 60\,V & t < 0 \end{cases}$

Problem 3. (**10 pts**) The circuit given below is operating in the steady state with the switch S_1 in open state and switch S_2 in closed state for $t < 0$. At $t = 0$, the switch S_1 is *closed* and S_2 is *opened*. Determine the voltage $v(t)$ for all times.



At $t=0^-$; $v(0^-) = -(4)(2) = -8V$

For $t \geq 0$: $v(\infty) = \left(\frac{6}{18}\right) 30 = 10V$

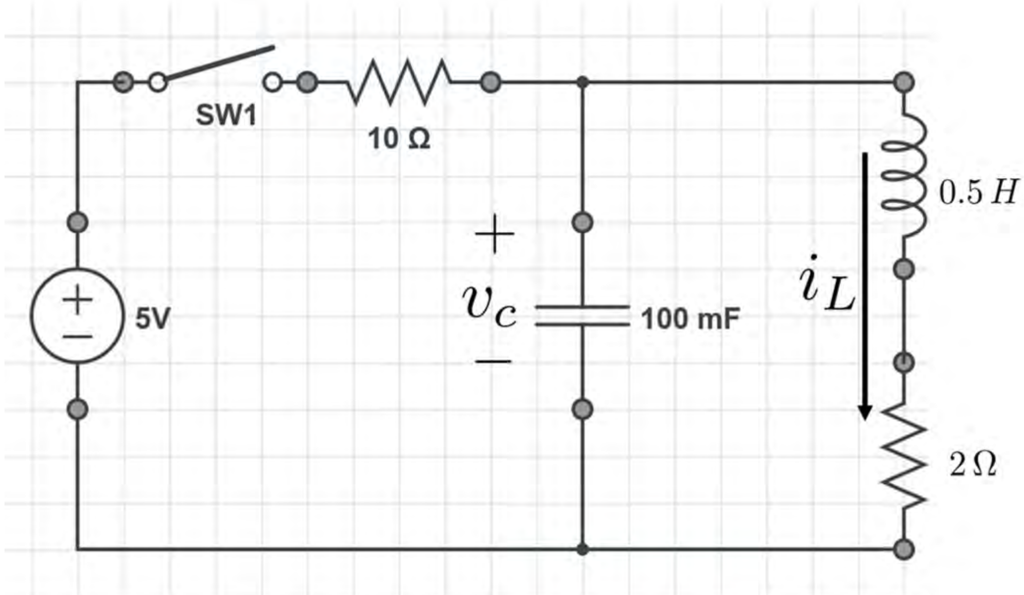
$R_{eq} = 12 || 6 + 6 = 10\Omega$

$\tau = R_{eq} C = 1$ sec

$$\boxed{v(t) = 10 - 18e^{-t} \text{ V}}$$

Part 2: Evaluation of Initial Conditions

Problem 4. (**10 pts**) In the circuit given below, assume that the switch SW1 is initially open and is closed at $t = 0$. Determine $v_c(t)$, $i_L(t)$, $dv_c(t)/dt$, $di_L(t)/dt$ and $dv_C^2(t)/dt^2$ at $t = 0^+$.



$$v_c(0^-) = v_c(0^+) = 0$$

$$i_L(0^-) = i_L(0^+) = 0$$

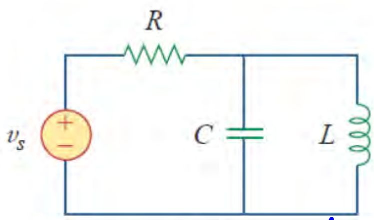
Eq. 01 ; $\quad v_c = 0.5 \dfrac{di_L}{dt} + 2 i_L$

Eq. 02 ; $\quad i_L + (100m)\dfrac{dV_c}{dt} + \dfrac{v_c - 5}{10} = 0$

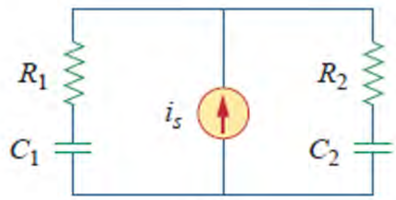
$$\Rightarrow \quad \frac{di_L}{dt} = 0$$

$$\frac{dV_c}{dt} = 5 V/s \quad , \quad \frac{d^2 V_c}{dt^2} = 5 V/s^2$$

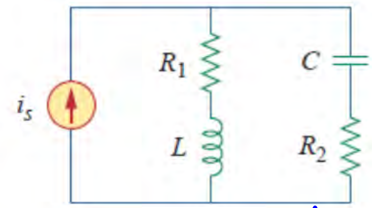
Problem 5. (**5 pts**) Determine the circuit-order for each of the following circuits.



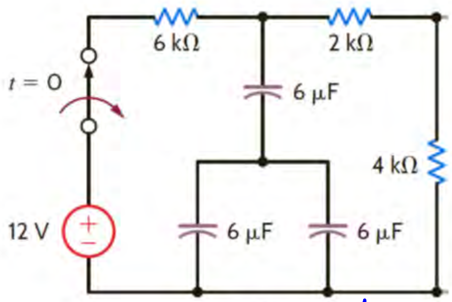
(a) 2nd



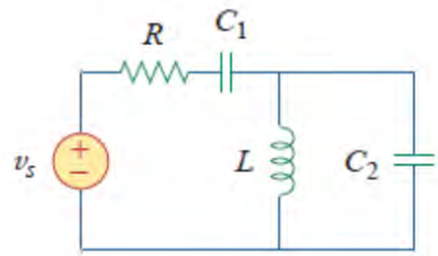
(b) 2nd



(c) 2nd



(d) 1st

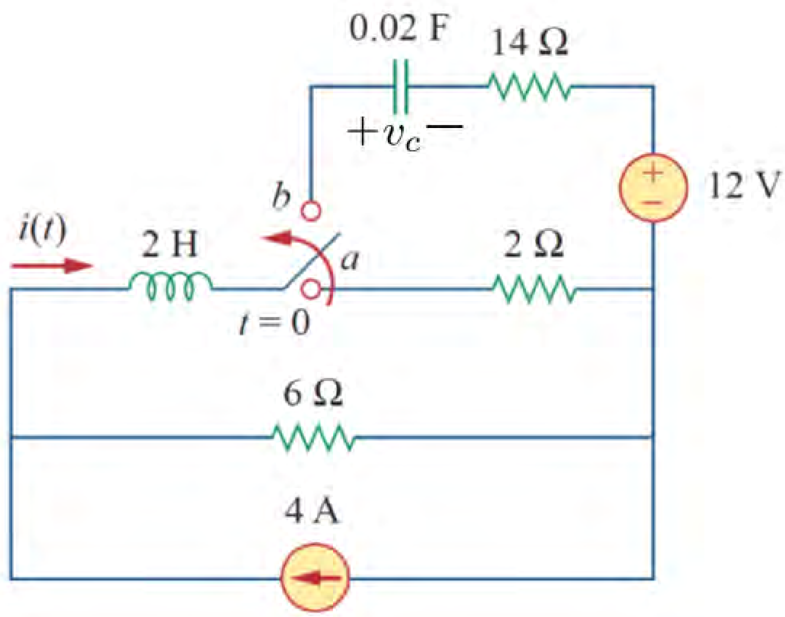


(e) 3rd

Part 3: Second-Order Circuits

Problem 6. (**10 pts**)

Consider the circuit shown below. The circuit is in steady state with switch at position a. At $t = 0$, the switch is moved from position a to position b.



(a) (**1 pts**) Determine $i(t)$ at $t = 0^-$.

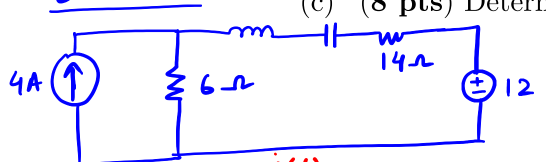
$$i(0^-) = \frac{6}{8} \times 4 = 3A$$

(b) (**1 pts**) Determine $v_c(t)$ at $t = 0^-$.

$$v_c(0^-) = 0$$

t>0 circuit

(c) (**8 pts**) Determine $i(t)$ for all times $t \geq 0$.



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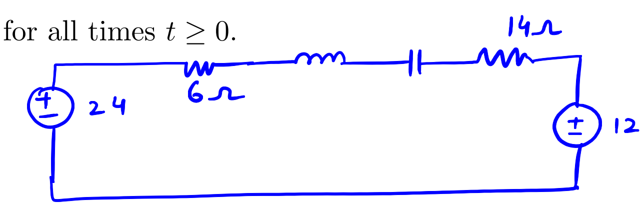
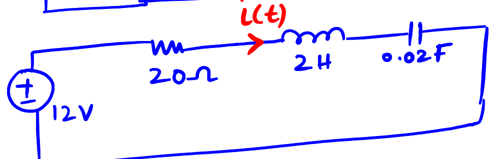




Series RLC

$$L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt = 12 \Rightarrow L\frac{di}{dt}(0^+) = 12 - (3)(20) \Rightarrow \frac{di}{dt}(0^+) = -24\ A/s$$

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25 = 0 \Rightarrow s^2 + 5s + 25 = 0 \Rightarrow s_1, s_2 = -5$$

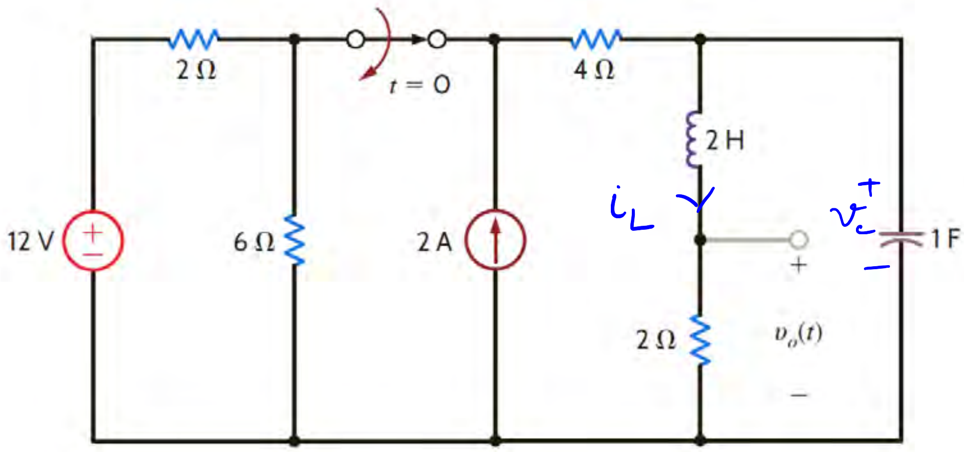
$$i(t) = (K_1 + K_2 t)e^{-5t}$$

$K_1 = 3$, $K_2 = 5K_1 - 24 = -9$

$$\Rightarrow \boxed{i(t) = (3 - 9t)e^{-5t} A}$$

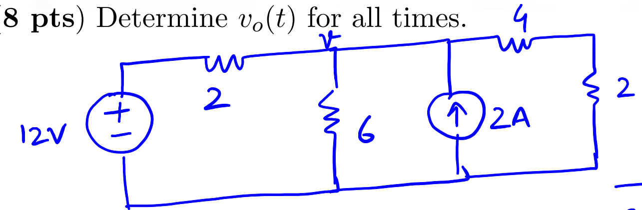
Problem 7. (10 pts)

The switch in the following circuit is opened at $t = 0$.



(a) (**8 pts**) Determine $v_o(t)$ for all times.

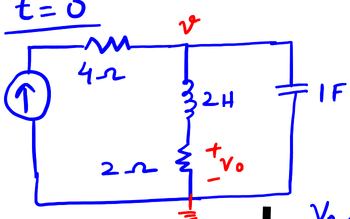
$t = 0^-$;



$$\frac{v}{6} + \frac{v}{6} + \frac{v-12}{2} = 2 \Rightarrow v = \frac{42}{5} = \boxed{8.4\,V = v_c(0^-)}$$

$$i_L(0^-) = \frac{v}{2} = \boxed{4.2\ A}$$

$t = 0^+$



$v = v_o + \frac{dv_o}{dt}$ — ①

Nodal Eq. $\frac{v_o}{2} + C\frac{d}{dt}v = 2 \Rightarrow \frac{v_o}{2} + v_o + \frac{d^2v_o}{dt^2} = 2 \Rightarrow \frac{d^2v_o}{dt^2} + \frac{dv_o}{dt} + \frac{v_o}{2} = 2$

$v_o = v_{o,p} + v_{o,c}$, $\boxed{v_{o,p} = 4V}$

$v_{o,c}$; $s^2 + s + 0.5 = 0 \Rightarrow s = -\frac{1}{2} \pm j\frac{1}{2}$, $v_{o,c} = e^{-t/2}\left(K_1\cos\left(\frac{t}{2}\right) + K_2\sin\left(\frac{t}{2}\right)\right)$

Initial Conditions

$v_o(0^+) = i_L(0^+)(2) = 8.4V$

Using (1)

$\frac{dv_o}{dt}(0^+) = 0$

$\because v(0^+) = 8.4V$

voltage across capacitor

$v_o(t) = 4 + e^{-t/2}\left(K_1\cos\left(\frac{t}{2}\right) + K_2\sin\left(\frac{t}{2}\right)\right)$

$4 + K_1 = 8.4 \Rightarrow \boxed{K_1 = 4.4}$

$v_o'(0) = -\frac{K_1}{2} + \frac{K_2}{2} = 0 \Rightarrow \boxed{K_2 = K_1}$

$$v_o(t) = \begin{cases} 4 + 4.4\,e^{-t/2}\left(\cos\frac{t}{2} + \sin\frac{t}{2}\right)\ V & t \geq 0 \\ 8.4\,V & t < 0 \end{cases}$$

(b) (**2 pts**) Determine the damping ratio ζ and undamped natural frequency ω_n for the circuit after the switch is operated.

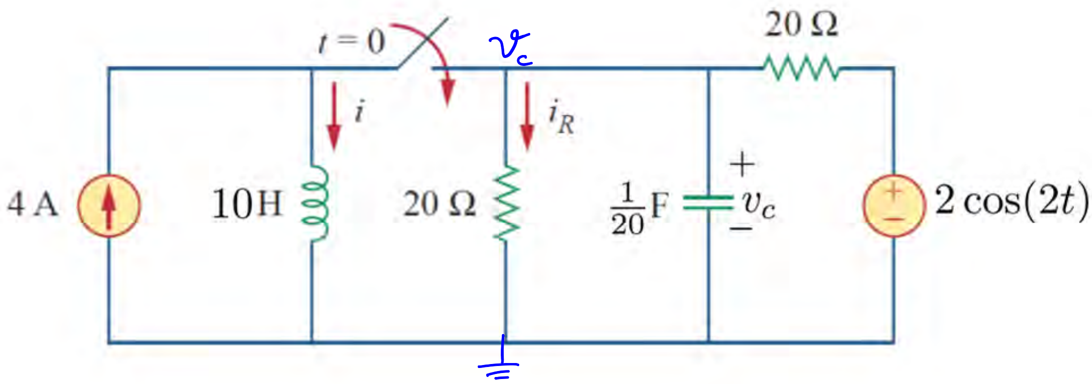
$\omega_n^2 = \frac{1}{2}$

$2\zeta\omega_n = 1$

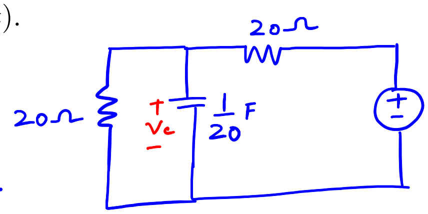
$\omega_n = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ rad/sec

$\zeta = \frac{1}{\sqrt{2}}$

Problem 8. (**20 pts**) In the circuit given below, assume that the switch is initially open and is closed at $t = 0$.



(a) (**5 pts**) For $\mathbf{t < 0}$, determine the voltage across capacitor, $v_c(t)$ and the current through inductor, $i(t)$.



* $i(0^-) = 4A = i(0^+)$
* $\underline{v_c(0^-)}$

Partial Circuit for $t<0$ is drawn.

$\underline{\text{Nodal Eq.}}$ $\quad \frac{v_c}{20} + \frac{1}{20}\frac{dv_c}{dt} + \frac{v_c - 2\cos 2t}{20} = 0$

$$2v_c + \frac{dv_c}{dt} = 2\cos 2t$$

*Switch is connected for long time; we determine particular solution

$$v_c(t) = A\cos 2t + B\sin 2t, \quad t<0$$

$2A + 2B = 2$ $\Rightarrow$ $A = 0.5$, $B = 0.5$

$2B - 2A = 0$

$v_c(t) = 0.5(\cos 2t + \sin 2t), \quad t<0$

$$\boxed{v_c(0^-) = 0.5\text{ V}}$$

$$\boxed{v_c(0^+) = 0.5\text{ V}}$$

(b) (**4 pts**) Formulate a second-order differential equation in $i(t)$.

$$v_c = L\frac{di}{dt}$$

$$i + \frac{v_c}{20} + C\frac{dv_c}{dt} + \frac{v_c - 2\cos 2t}{20} = 4 \Rightarrow i + \frac{di}{dt} + \frac{1}{2}\frac{d^2i}{dt^2} = 4 + \frac{1}{10}\cos 2t$$

$$\boxed{\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 2i = 8 + \frac{1}{5}\cos 2t}$$

(c) (8 **pts**) Determine $i(t)$ for all times $t \geq 0$.

$i = i_p + i_c$

i_c; $s^2 + 2s + 2 = 0 \Rightarrow \boxed{s_1, s_2 = -1 \pm j}$ $\qquad i_c(t) = e^{-t}(K_1 \cos t + K_2 \sin t)$

i_p; $i_p(t) = A + B\cos 2t + C \sin 2t$

$2A = 8 \Rightarrow \boxed{A = 4}$, $\quad \begin{matrix} -2B + 2C = \frac{1}{5} \\ -2B - 2C = 0 \end{matrix} \Rightarrow \boxed{\begin{matrix} C = \frac{1}{20} \\ B = -\frac{1}{20} \end{matrix}}$

$i(t) = 4 - \frac{1}{20}\cos 2t + \frac{1}{20}\sin 2t + e^{-t}(K_1 \cos t + K_2 \sin t)$, $\quad t \geq 0$

Initial Conditions : $i(0^+) = 4A$, $\quad L\frac{di}{dt} = v_c \Rightarrow \frac{di}{dt}(0^+) = 0.05\ A/s$

$K_1 = \frac{1}{20}$, $\quad \frac{1}{10} + 2K_2 - K_1 = 0.05 \Rightarrow \boxed{K_2 = 0}$

$$i(t) = 4 - \frac{1}{20}\cos 2t + \frac{1}{20}\sin 2t + \frac{e^{-t}}{20}\cos t \quad A$$

(d) (3 **pts**) Determine $i_R(t)$ for all times $t \geq 0$.

$$i_R(t) = \frac{10}{20}\frac{di}{dt} = \boxed{\frac{1}{2}\frac{di}{dt}} A \qquad t \geq 0$$