EE240 – Circuits I

Mid Examination (Fall 2019)

Solutions

October 31, 2019 06:00 pm–08:45 pm

Student ID ______________________________ Name ___________________________________

Signature _____________________

INSTRUCTIONS:

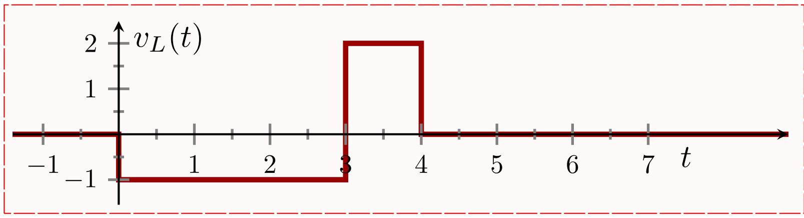
- Reading time: 15 minutes
- Writing time: 2 hours and 30 minutes
- Do not flip this page over until told to do so.
- **The exam needs to be solved on this book and not on blue book.**
- If you need the blue book for rough work, please ask the exam staff.
- The exam is closed book and notes. You are allowed to bring calculator and two A4 sheet with you with *hand-written* notes on both sides.
- Read all the questions before you start working on the exam.
- You cannot keep your mobile phone(s) with you (even on silent mode or switched off).

Mapping between exam parts and course learning outcomes (CLOs)

- Part 1: Sources and I-V Characteristics of R, L, C (CLO1)
- Part 2: Network Topology, Network Equations and Equivalent Circuits (CLO2)
- Part 3: Additional Analysis Techniques (CLO3)

Part 1: Sources and I-V Characteristics of R, L, C

Problem 1. (**8 pts**) The voltage $v_L(t)$ through the inductor of inductance $2H$ is shown in Figure 1 below.



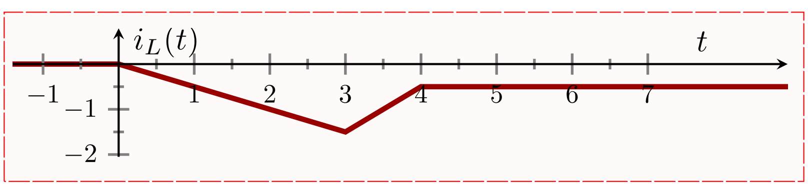
(a) (**1 pts**) Express $v_L(t)$ as piecewise function of time.

$$v_L(t) = \begin{cases} 0 & t < 0 \\ -1 & 0 \le t < 3 \\ 2 & 3 \le t < 4 \\ 0 & 6 \le t \end{cases}$$

(b) (**6 pts**) Assuming that the current is zero for times $t \le -1$ seconds, determine the current through the inductor and **plot** for $0 \le t \le 7$ seconds.
Let $i_L(t)$ be the current through inductor.

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)dt$$

$$i_L(t) = \frac{1}{2}\begin{cases} 0 & t < 0 \\ -t & 0 \le t < 3 \\ 2t - 9 & 3 \le t < 4 \\ -1 & 6 \le t \end{cases}$$

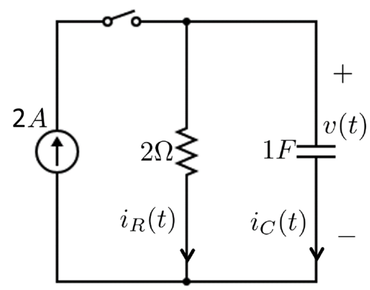


(c) (**1 pts**) Determine the energy stored in an inductor at $t = 3$ seconds.
$w_L(t) = \frac{1}{2}L\big(i_L(t)\big)^2 \Rightarrow w_L(3) = \big(i_L(3)\big)^2 = 2.25\ J.$

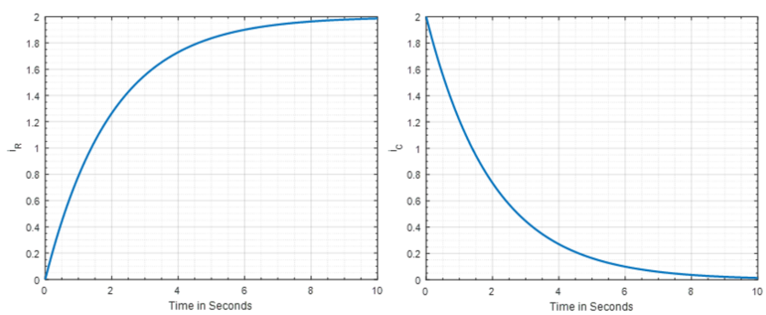
Problem 2. (**5 pts**)

Consider a circuit where the DC current source of $2A$ is connected with a parallel combination of a $2\ \Omega$ resistor and $1F$ capacitor through the switch. Assume that the switch is initially open and is closed at $t = 0$ and the capacitor is not carrying any charge before the switch is closed, that is, the voltage across capacitor $v(t) = 0$ for all $t < 0$.

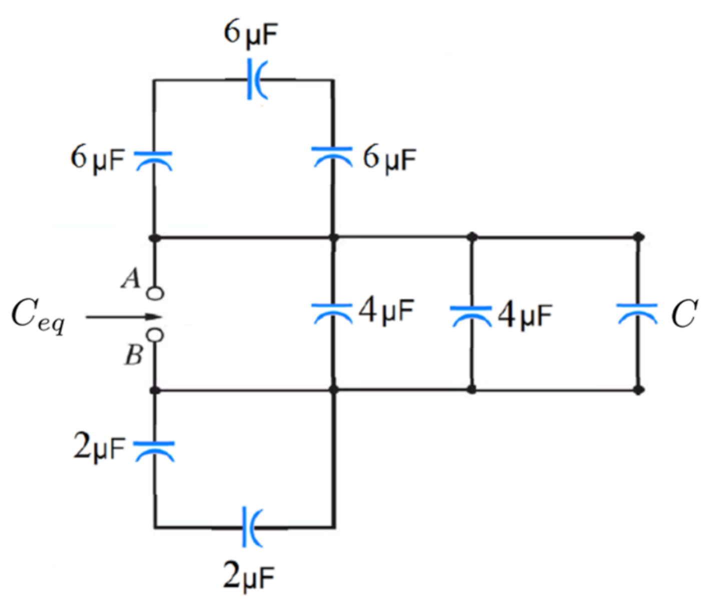
(a) (**1 pts**) Draw the circuit and indicate the voltage $v(t)$ across the capacitor and the currents $i_R(t)$ and $i_C(t)$ through the resistor and capacitor respectively.



(b) (**5 pts**) Plot the waveforms of the currents $i_R(t)$ and $i_C(t)$.



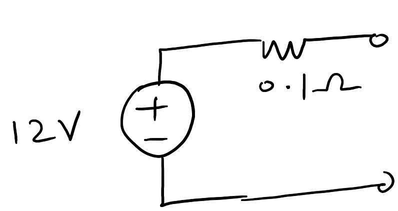
Problem 3. (**3 pts**) For the network of capacitors shown below, determine the capacitance C if the equivalent capacitance C_{eq} across terminals A-B is 16 μF.

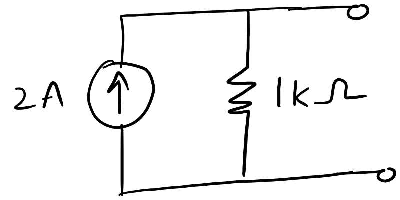


$$C_{eq} = 16 = 4 + 4 + C \quad \Rightarrow \quad C = 8\mu\ F$$

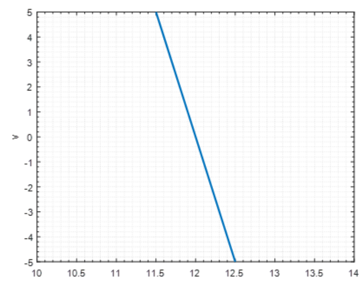
Problem 4. (**4 pts**) We consider a practical $12V$ voltage source of $0.1\ \Omega$ internal resistance and practical $2A$ current source with internal resistance of $1\ k\Omega$.

(a) (**2 pts**) Draw the circuit models of the practical sources.

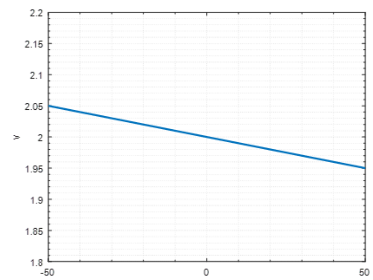




(b) (**2 pts**) Plot the $i - v$ characteristics of the practical sources.



Voltage source



Current source

Problem 5. (**4 pts**) *Consider a practical voltage source with rated voltage of* V_o *and internal resistance* r_s. *When no current is drawn from the circuit, the voltage across terminals of the voltage source is* V_o. When $2W$ load is connected to the circuit, the voltage across the load is $12\ V$. When the terminals of the source are short circuited, the source supplies the current of $8\ A$. Determine the rated voltage V_o and internal resistance r_s.

When terminals are short-circuited, the current is $8A = \frac{V_o}{r_s} \Rightarrow V_o = 8r_s$.

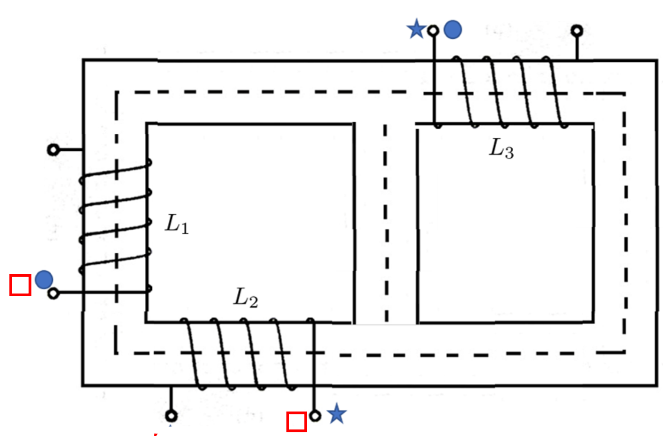
When $2W$ resistor is connected, the voltage is $12\ V$, the current is

$$\frac{1}{12} = \frac{1}{6} = \frac{V_o - 12}{r_s} \Rightarrow 6V_o - 72 = r_s$$

. Solving these two equation yields

$$r_s = \frac{72}{47} = 1.532\ \Omega, \qquad V_o = 12.25\ V.$$

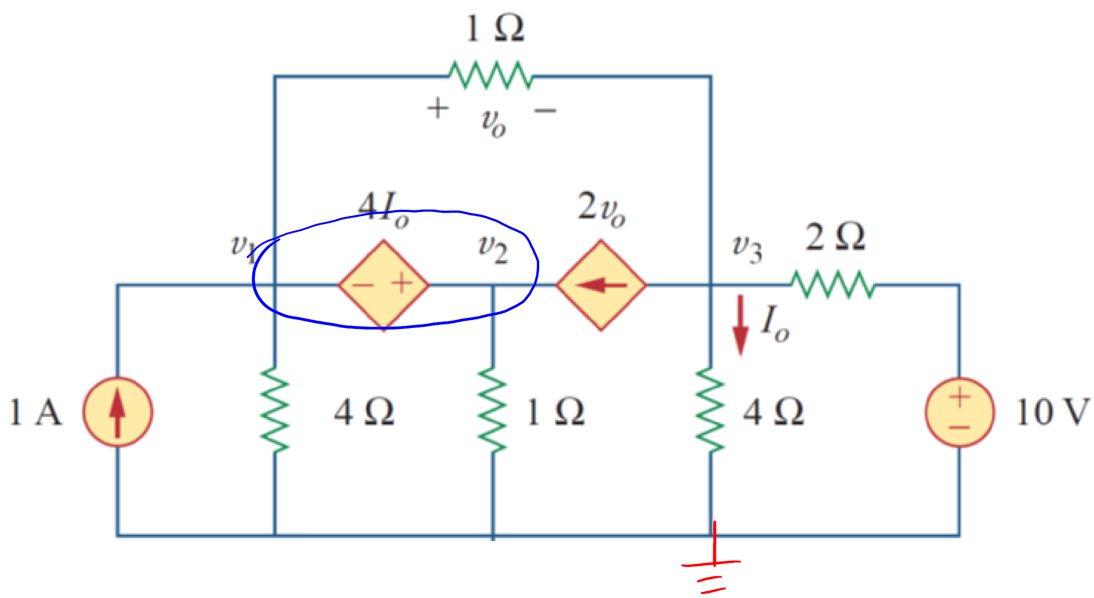
Problem 6. (**6 pts**) The figure (right) shows windings marked on a magnetic flux-conducting core. Mark the dots on the windings to establish the mutual coupling using the dot convention.



Part 2: Network Topology, Network Equations and Equivalent Circuits

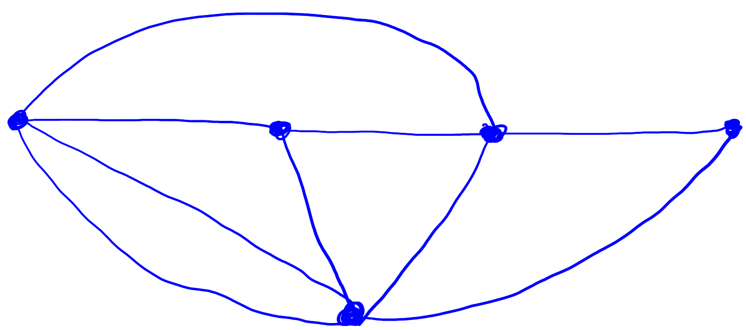
Problem 7. (**18 pts**)

Consider the circuit given below.



(a) (**4 pts**) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in a circuit.

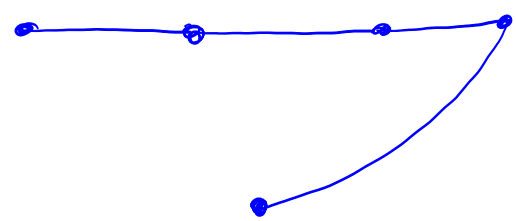
Graph



Tree

$n = 5$

$b = 9$



(b) (**2 pts**) Determine the number of network equations required for carrying out i) nodal analysis and ii) loop analysis.

Nodal Analysis; $n - 1 = 4$

Loop Analysis; $b - n + 1 = 5$

(c) (**8 pts**) Use Kirchhoff current law to determine the nodal voltages.

Nodes 1 and 2 form a supernode

$$\frac{v_1}{4} + \frac{v_2}{1} + \frac{v_1 - v_3}{1} - 1 - 2V_o = 0$$

$$\Rightarrow \frac{v_1}{4} + v_2 + v_1 - v_3 - 2v_1 - 2v_3 = 1$$

$$\boxed{-3v_1 + 4v_2 - 12v_3 = 4}$$

Controlled Sources

$v_1 - v_3 = v_o$

$I_o = \frac{v_3}{4}$

$v_2 - v_1 = 4I_o = v_3$

$\Rightarrow \boxed{-v_1 + v_2 - v_3 = 0}$

Node 3 : $\frac{v_3}{4} + \frac{v_3 - 10}{2} + v_3 - v_1 + 2V_o = 0$

$$\Rightarrow \frac{v_3}{4} + \frac{v_3}{2} + v_3 - v_1 + 2v_1 - 2v_3 = 5$$

$$\boxed{-v_3 + 4v_1 = 20}$$

$$\begin{bmatrix} -3 & 4 & -12 \\ 4 & 0 & -1 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} v_1 &= \frac{164}{33} = 4.97V \\ v_2 &= \frac{160}{33} = 4.85V \\ v_3 &= -\frac{4}{33} = -0.12V \end{aligned}$$

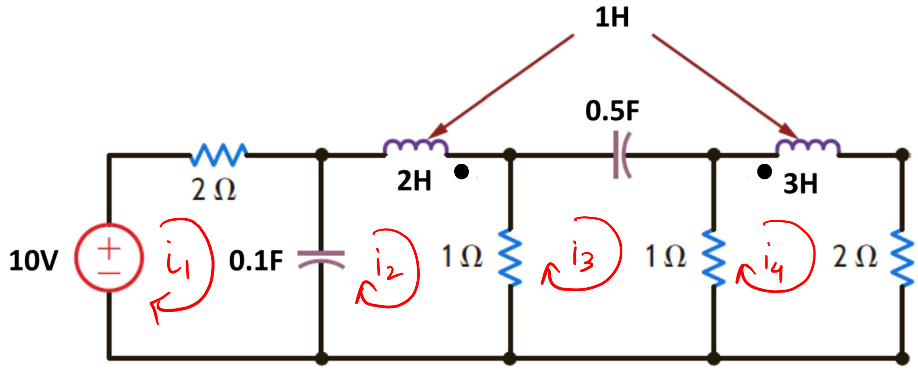
(d) (**4 pts**) Determine the power supplied by (i) voltage source and (ii) current source.

Power supplied by current source $(1A)(v_1) = 4.97\ W$

Power supplied by voltage source

$$(10)\underbrace{\left(\frac{10 - v_3}{2}\right)}_{\text{current supplied by voltage source}} = (10)(5.06) = 50.6W$$

Problem 8. (**7 pts**) For the circuit given below, formulate the network equations using Kirchhoff voltage law.



Loop 1

$$2 i_1 + 10 \int (i_1 - i_2)\, dt = 10$$

Loop 2

$$10 \int (i_2 - i_1)\, dt + (i_2 - i_3) + 2 \frac{d i_2}{dt} - \frac{d i_4}{dt} = 0$$

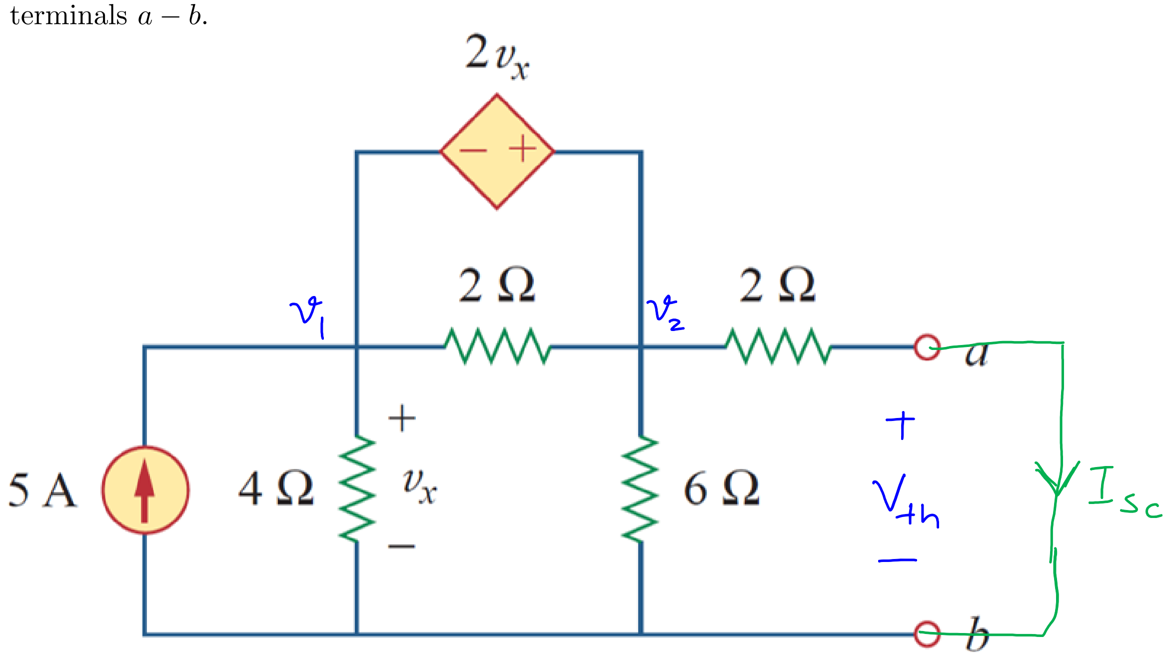
$$(i_3 - i_2) + (i_3 - i_4) + 2 \int i_3\, dt = 0$$

$$(i_4 - i_3) + 2 i_4 + 3 \frac{d i_4}{dt} - \frac{d i_2}{dt} = 0$$

— terms due to mutual inductance.

Part 3: Additional Analysis Techniques

Problem 9. (**12 pts**) For the circuit given below, determine the Thevenin equivalent circuit at terminals $a - b$.



V_{th} :

Super Node :

$$\frac{v_1 - v_2}{2} + \frac{v_1}{4} - 5 + \frac{v_2}{6} + \frac{v_2 - v_1}{2} = 0$$

$$\Rightarrow \frac{v_1}{4} + \frac{v_2}{6} = 5$$

Controlled Source :

$$v_2 - v_1 = 2V_x$$
$$v_1 = v_x$$
$$\boxed{v_2 = 3v_1}$$

$$\Rightarrow \frac{v_1}{1} + \frac{v_1}{2} = 5 \Rightarrow \boxed{v_1 = \frac{20}{3} V}$$

$$\boxed{v_2 = 20V = V_{th}}$$

I_{sc} : Again : $\boxed{v_2 = 3v_1}$

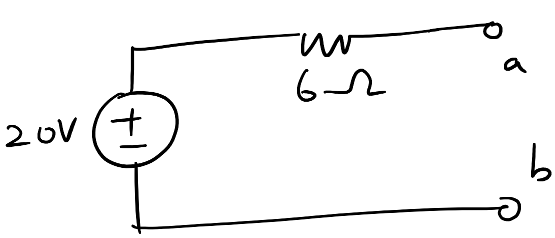
Super-node :

$$\frac{v_1}{4} + \frac{v_2}{6} + \underbrace{\frac{v_2}{2}}_{} = 5$$

This term added when a-b are short-circuited

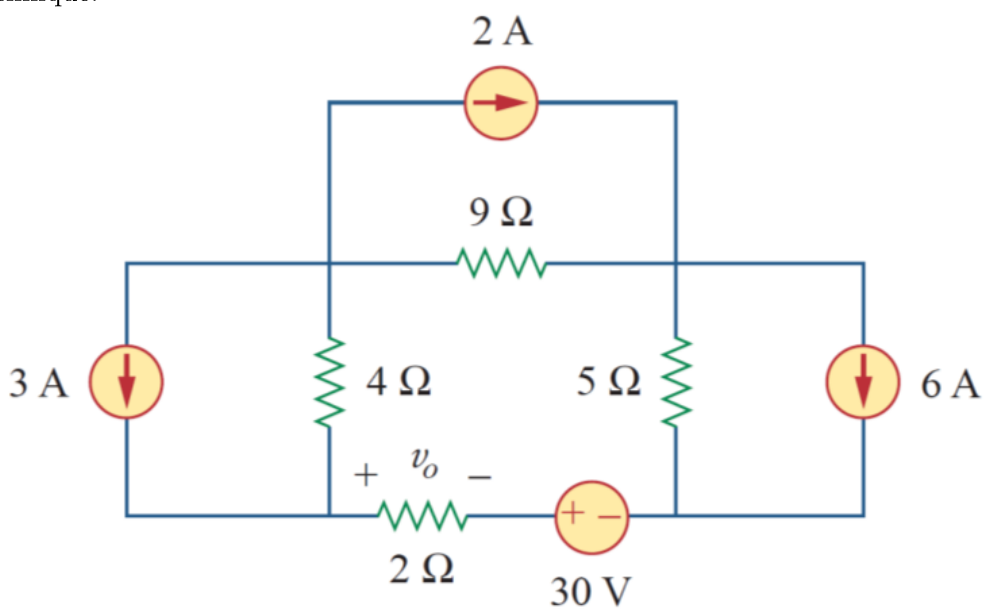
$$v_1 = \frac{20}{9} V$$
$$v_2 = \frac{10}{9} V$$
$$I_{sc} = \frac{v_2}{2}$$
$$\Rightarrow \boxed{I_{sc} = \frac{10}{3} A}$$

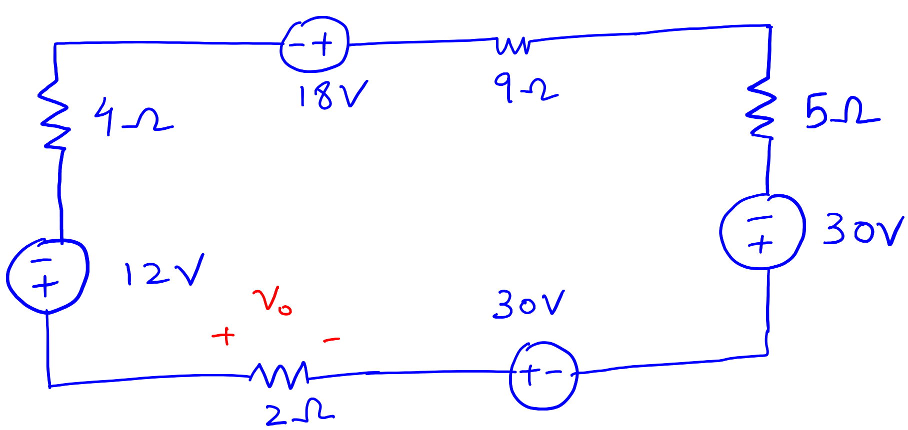
Equivalent circuit

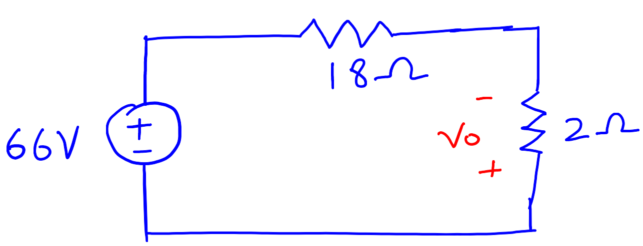


$$R_{th} = \frac{V_{th}}{I_{sc}}$$
$$\boxed{R_{th} = 6\,\Omega}$$

Problem 10. (**7 pts**) Determine v_o in the following circuit using the source transformation technique.



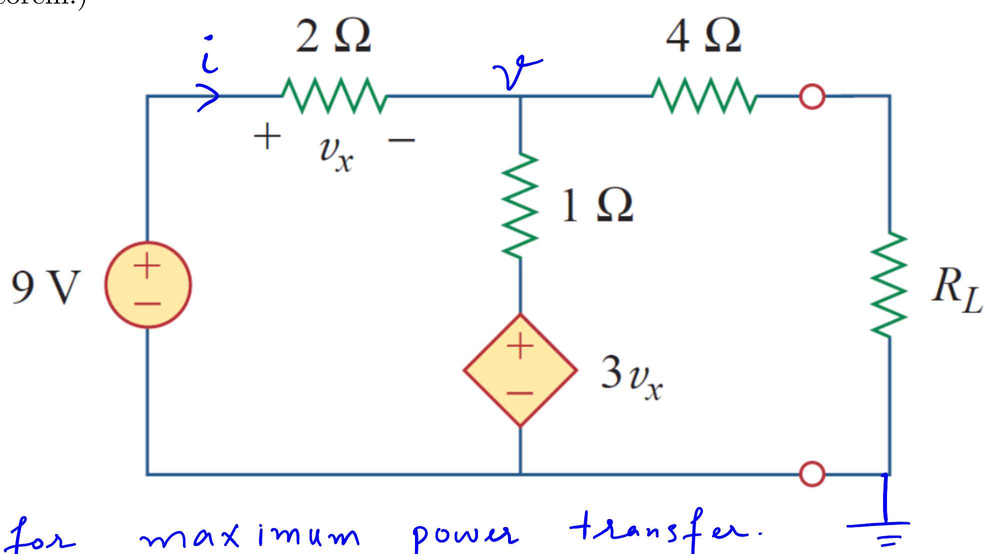




$$V_o = -\frac{2}{20} \times 66$$

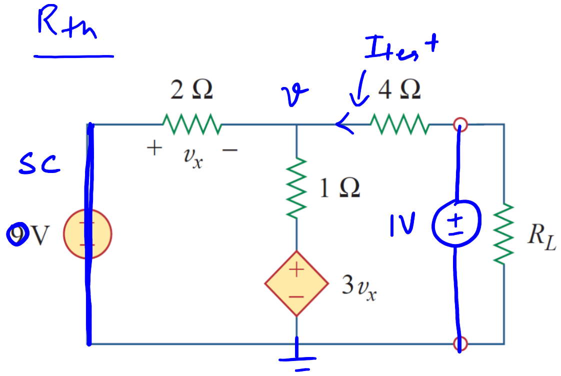
$$\boxed{V_o = -6.6V}$$

Problem 11. (**8 pts**) Consider the circuit given below. For the value of the load resistance R_L that draws maximum power from the circuit, determine the power supplied by the voltage source.
(Hint: Determine the value of load resistance R_L first using the maximum power transfer theorem.)



* $R_L = R_{th}$ for maximum power transfer.

* R_{th}



* $v = -v_x$
* $\frac{v}{2} + \frac{v+3v}{1} + \frac{v-1}{4} = 0 \Rightarrow \boxed{v = \frac{1}{19} V}$

$I_{test} = \frac{1-\frac{1}{19}}{4} = \frac{18}{76} \Rightarrow R_{th} = \frac{1}{I_{test}} = \boxed{\frac{76}{18} = 4.22\Omega}$

When $R_L = R_{th}$ connected;

$V_x = 9 - v$

Node Equation: $\frac{v-9}{2} + \frac{v}{8.22} + v - 3(9-v) = 0$

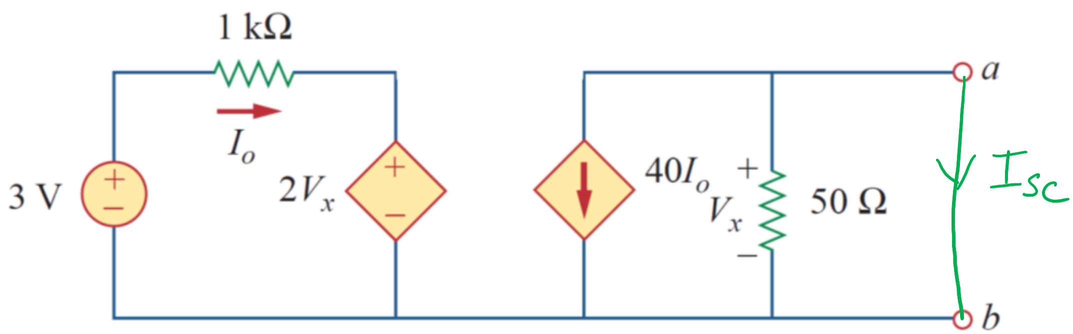
$\frac{v}{2} + \frac{v}{8.22} + 4v = 31.5 \Rightarrow v = \frac{31.5}{4.62} = \boxed{6.818V}$

i – indicated in the circuit:

$i = \frac{9 - 6.818}{2} = \boxed{1.091\ A}$

Power, $P = i \times 9 = \boxed{9.818\ W}$ qed!
↓
supplied by source

Problem 12. (**8 pts**) Find the Norton equivalent of the following circuit at terminals $a - b$.



Determine V_{oc}: $V_{oc} = V_x = -(50)(40I_o) = -2000 I_o$

$I_o = \dfrac{3 - 2V_x}{1k} \Rightarrow 3 - 2V_x = 1000 I_o$

$\Rightarrow 3 + 4000 I_o = 1000 I_o \Rightarrow \boxed{I_o = -1\,mA} \Rightarrow \boxed{V_x = V_{oc} = 2V}$

I_{sc}: * When terminals are short-circuited, $V_x = 0$

and $I_{sc} = -40 I_o$

* $2V_x = 0$; (controlled source) $\therefore$ $I_o = \dfrac{3}{1k} = \boxed{3\,mA}$

$I_N = I_{sc} = -(40)(3mA) = \boxed{0.12\,A}$

R_{th}; $R_{th} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{50}{3}\,\Omega$

Norton Equivalent: