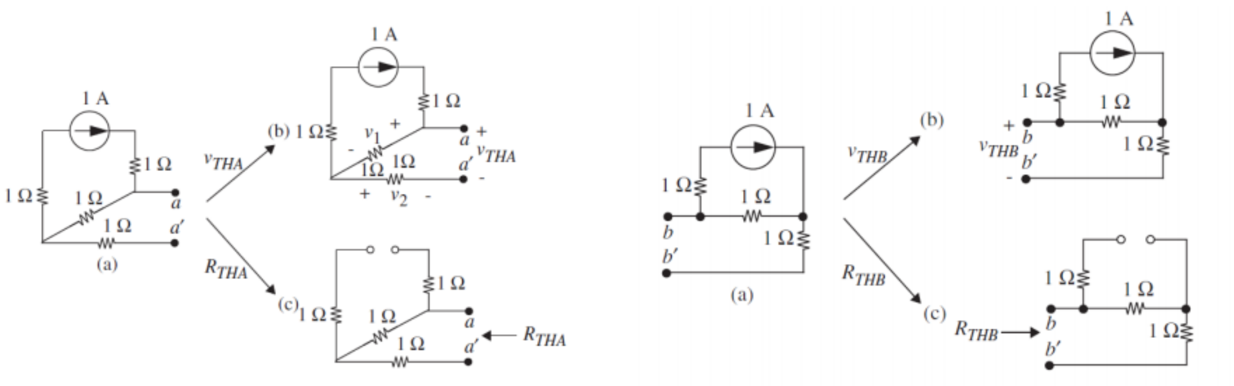
Department of Electrical Engineering
School of Science and Engineering

EE240 Circuits I - Fall 2020

ASSIGNMENT 3 – SOLUTIONS

Q 1. (a) Divide the two networks as shown in the figure below:



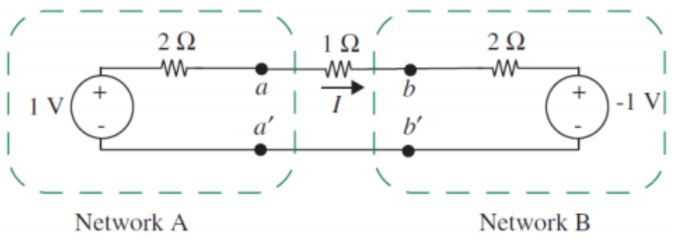
(a) Network A (b) Network B

$$V_{THA} = 1 \cdot 1 = 1V$$

(V_{THA} is simply the voltage across 1Ω resistor)

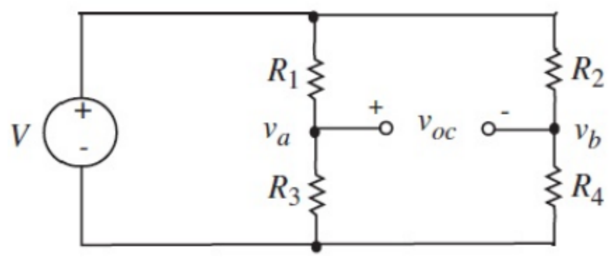
$$R_{THA} = 1 + 1 = 2\Omega$$
$$V_{THB} = -1V$$
$$R_{THB} = 2\Omega$$

Join the Thevenin equivalent of the two networks,



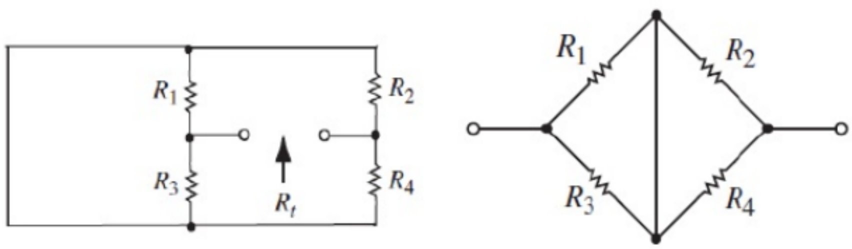
$$I = \frac{1V - (-1V)}{1\Omega + 2\Omega + 2\Omega} = \frac{2}{5}A$$

(b) Let the terminals of R_5 be v_a and v_b,



$$V_{TH} = V_{oc} = V_a - V_b = V(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4})$$

For R_{TH}, switch off the independent sources,

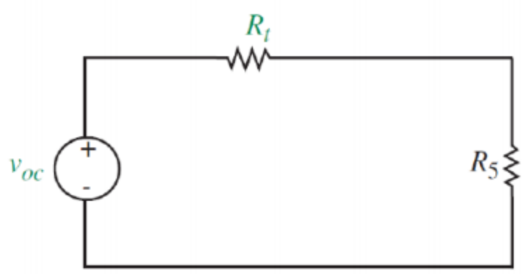


(a) Turn off independent source

(b) Redrawn image

$$R_{TH} = (R_1||R_3) + (R_2||R_4)$$

The simplified thevenin circuit is as follows:



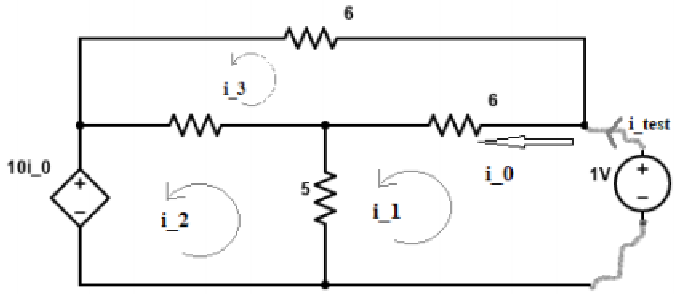
For the voltage across R_5 to be zero, V_{oc} or V_{TH} must be zero. Therefore,

$$\frac{R_3}{R_1 + R_3} = \frac{R_4}{R_2 + R_4}$$

$$\frac{R_3}{R_1} = \frac{R_4}{R_2}$$

Q 2. Since there is no independent source in the circuit,

$$V_{Th} = 0$$



To find R_{Th}, use loop analysis,
Loop 1:

$$6(i_1 - i_3) + 5(i_1 - i_2) = 1$$

Loop 2:

$$5(i_2 - i_3) + 5(i_2 - i_1) = -10i_o$$

Loop 3:

$$6i_3 + 5(i_3 - i_2) + 6(i_3 - i_1) = 0$$

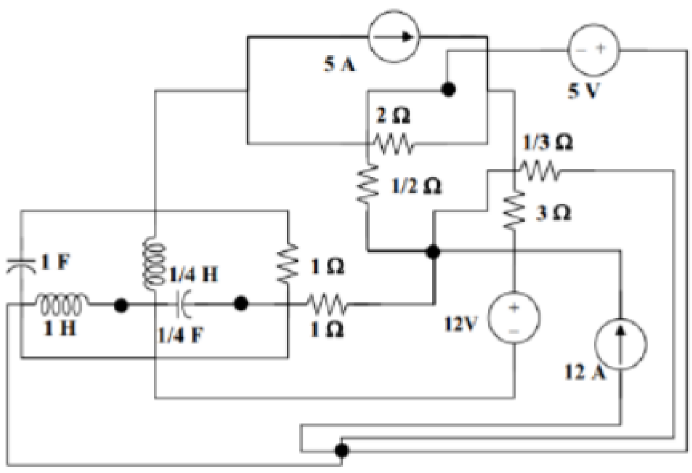
Also,

$$i_o = i_1 - i_3$$

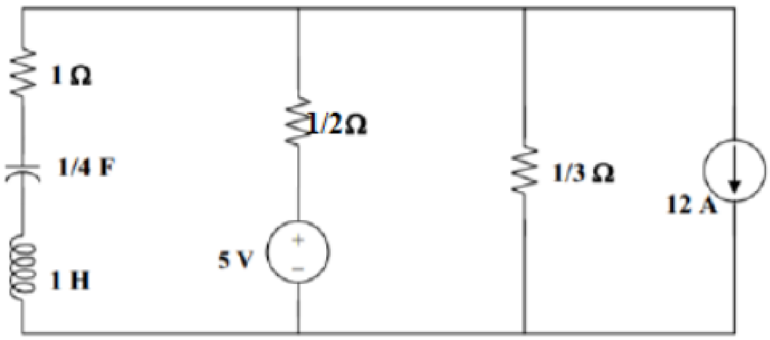
Solving these equations,

$$I_1 = \frac{19}{162},$$
$$I_2 = \frac{1}{162},$$
$$I_3 = \frac{7}{162},$$
$$I_o = \frac{2}{27}$$
$$i_{test} = i_1 = \frac{19}{162}A$$
$$R_{Th} = \frac{V_{Test}}{i_{test}} = \frac{162}{19}\Omega = 8.53\Omega$$

Q 3. The dual is constructed in Figure (a) and redrawn in Figure (b),

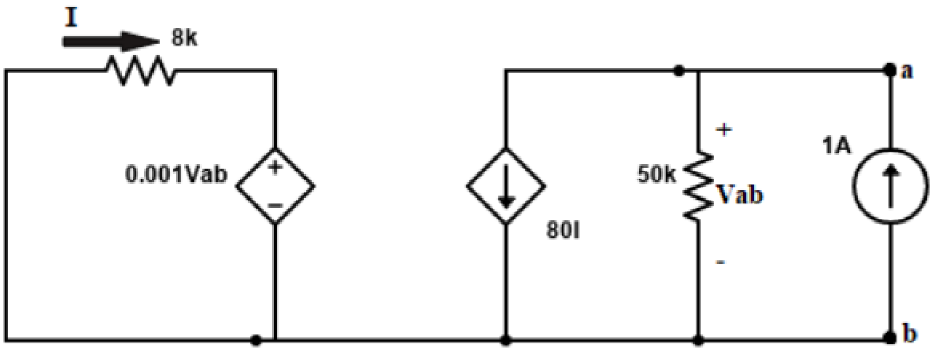


(a) construction



(b) Dual Circuit

Q 4. To find R_N (equivalent of R_{TH}), use a test source (notice the dependent source) and remove the independent sources (short circuit for a voltage source),



The equation for node a,

$$\frac{V_{ab}}{50000} + 80I = 1 \tag{1}$$

The loop equation for the left loop,

$$-8000I = 0.001V_{ab} \tag{2}$$

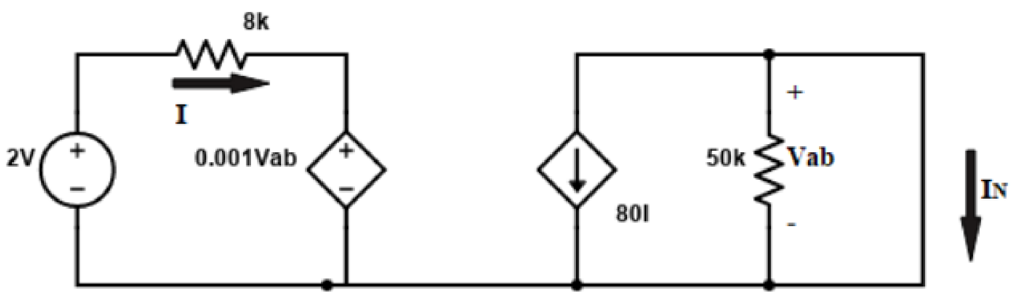
From (1) and (2),

$$V_{ab} = 100V$$

So,

$$R_N = \frac{V_{ab}}{1} = 100\Omega$$

To find I_N, consider the circuit below,



$$V_{ab} = 0$$
$$I_N = -80I$$

Hence,

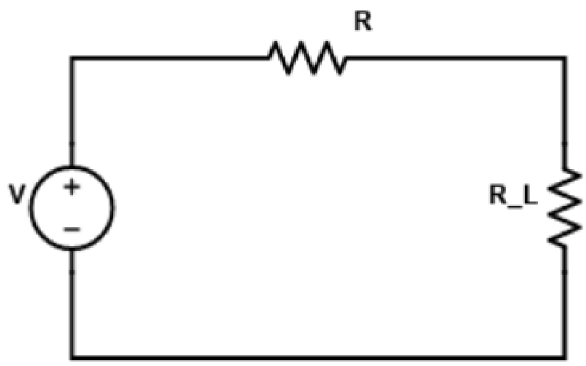
$$8000I = 2$$
$$I = 0.25mA$$

Finally,

$$I_N = -20mA$$

Q 5. (a)

$$P_{Load} = I^2 \cdot R_L$$



$$P_{Load} = (\frac{V}{R + R_L})^2 \cdot R_L$$

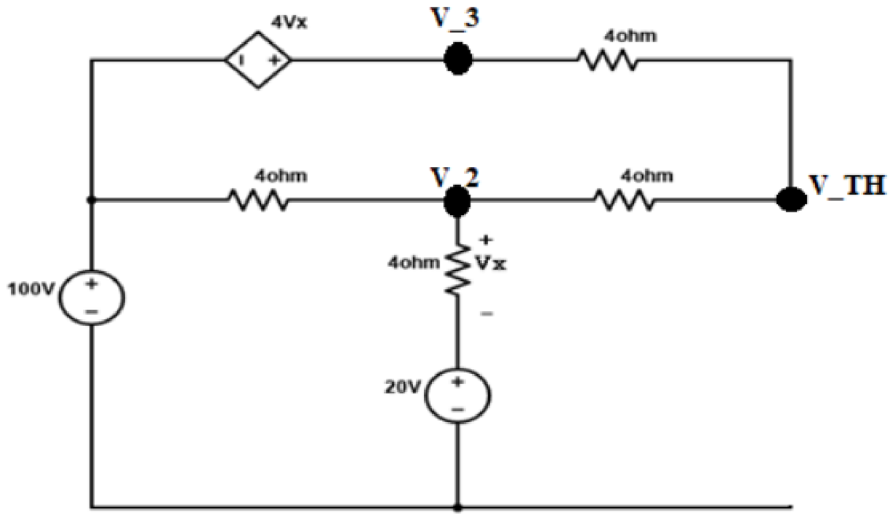
To find the maximum power,

$$\frac{\mathrm{d}P_{Load}}{\mathrm{d}R_L} = 0$$

Apply chain rule,

$$(\frac{V}{R + R_L})^2 - 2\frac{V^2}{R + R_L} \cdot R_L = 0$$
$$(\frac{V}{R + R_L})^2 = \frac{2V^2}{(R + R_L)^3}$$
$$1 = \frac{2R_L}{R + R_L}$$
$$R = R_L$$

(b) Background: To find R_L start with creating the thevenin/nortan equivalent of the given circuit. As shown in the previous part, maximum power transfer will occur when R_I equals $R_{thevenin}(R_{TH})$. Then, use $V_{Thevenin}(V_{TH})$ to find the maximum power transferred.
To find the V_{TH} (open circuit voltage), label the nodes, and write the nodal equations,



Node 1 (V_{TH}):

$$\frac{V_{TH}-V_3}{4}-\frac{V_{TH}-V_2}{4}=0 \tag{3}$$

Node 2 (V_2):

$$\frac{V_2-V_{TH}}{4}+\frac{V_2-20}{4}+\frac{V_2-100}{4}=0 \tag{4}$$

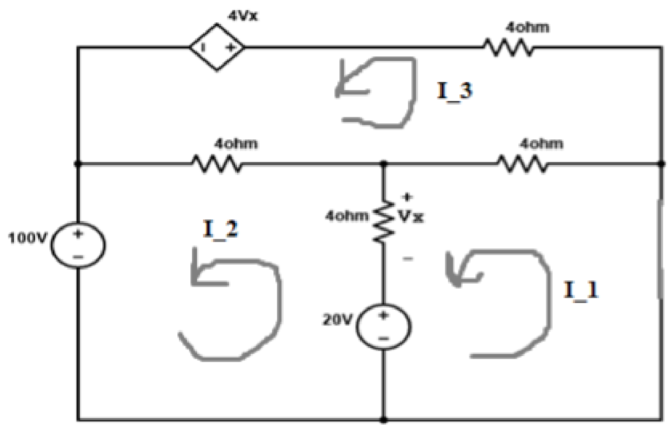
Node 3 (V_3):

$$V_3=100+4V_x \tag{5}$$

Solve (3),(4),(5) simultaneously,

$$V_{TH}=660V$$

To find R_{TH}, first use loop analysis to find short circuit current,



Loop 1 (I_1):

$$4(I_1-I_3)-20+4(I_1-I_2)=0 \tag{6}$$

Loop 2 (I_2):

$$4(I_2-I_3)+4(I_2-I_1)=80 \tag{7}$$

Loop 3 (I_3):

$$4I_3+4(I_3-I_1)+4(I_3-I_2)=4V_x \tag{8}$$

Where,

$$V_x=4(I_2-I_1)$$

So (8) reduces into,

$$12I_3+12I_1-20I_2=0 \tag{9}$$

Solve (6), (7),(9) simultaneously,

$$I_1 = I_{shortcircuit} = 55A$$

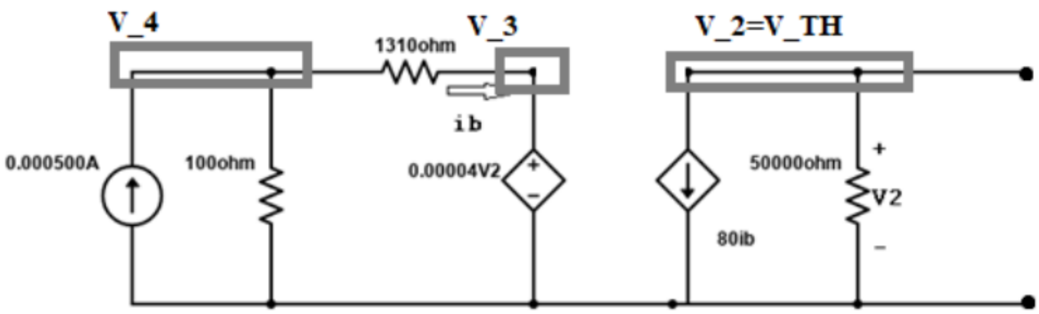
Using V_{TH} and $I_{shortcircuit}$, find R_{TH} to determine R_I,

$$R_{TH} = R_I = \frac{V_{TH}}{I_{shortcircuit}} = \frac{660}{55} = 12\Omega$$

Finally,

$$P_{Load} = \frac{660}{12+12} \cdot 12 = 9075W$$

(c) The approach to this part will be same as part(b).
To find V_{TH}, label the nodes and form nodal equations,



Node 1(V_2):

$$\frac{V_2}{50000} + 80i_b = 0 \tag{10}$$

Where,

$$i_b = \frac{V_4 - V3}{1310}$$

So,

$$\frac{V_2}{50000} - \frac{80V_3}{1310} + \frac{80V_4}{1310} = 0 \tag{11}$$

Node 2(V_3):

$$V_3 = 4 \cdot 10^{-5} \cdot V_2 \tag{12}$$

Node 3(V_4):

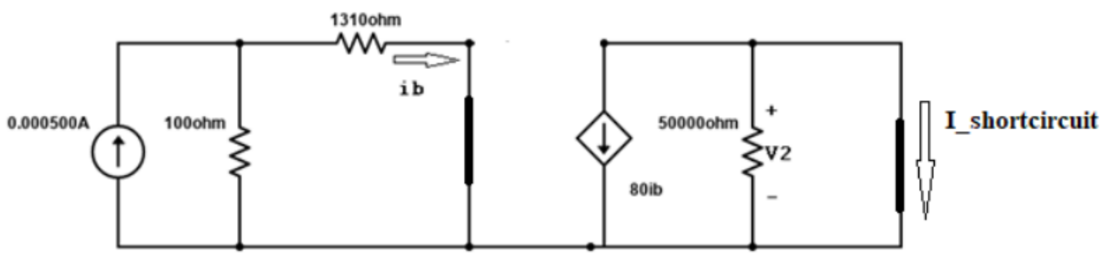
$$\frac{V_4}{100} + \frac{V_4 - V_3}{1310} = 0.0005 \tag{13}$$

Solve (11),(12),(13) simultaneously,

$$V_{TH} = V_2 = -160V$$

To find R_{TH}, short circuit the terminals,

$$V_2 = 0$$



Now,

$$I_{shortcircuit} = -80 i_b$$

Where, using the current divider method,

$$i_b = \frac{100}{1310 + 100} \cdot 0.0005 = 3.546 \cdot 10^{-5}$$

So,

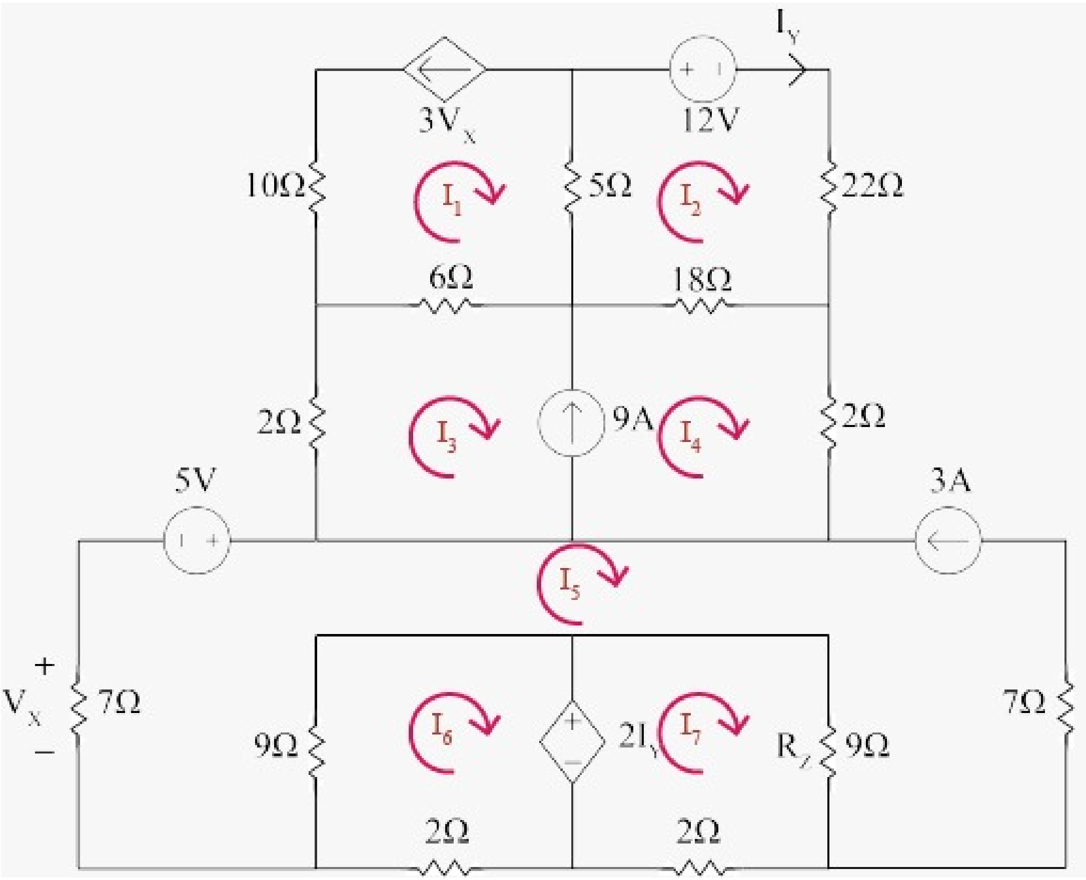
$$I_{shortcircuit} = -2.8mA$$

With $I_{shortcircuit}$ and V_{TH} known, find R_{TH},

$$R_{TH} = R_b = \frac{-160}{-2.8 \cdot 10^{-3}} = 57k\Omega$$

$$P_{Load} = 0.1122W$$

Q 6. (a) (i) Label all loops in clockwise direction.



(ii) Produce the complete set of loop equations by using mesh analysis,

Loop 1:

$$10I_1 + 5(I_1 - I_2) + 6(I_1 - I_3) = 0$$

Loop 2:

$$22I_2 + 18(I_2 - I_4) + 5(I_2 - I_1) = -12$$

Super loop 3+4:

$$2I_3 + 6(I_3 - I_1) + 18(I_4 - I_2) + 2I_4 = 0$$

Super loop relation:

$$I_4 - I_3 = 9$$

Loop 5:

$$7I_5 + 7I_5 + 9(I_5 - I_7) + 9(I_5 - I_6) = 5$$

Loop 6:

$$2I_2 + 11I_6 = -27$$

Loop 7:

$$-2I_2 + 11I_7 = -27$$

(b) (i) By simplifying your loop equations, express the circuit in a matrix equation of the form $AI = V$. Remember to exclude those loops for which you already know the currents for.

$$I_1 = -3V_x$$
$$V_x = 7I_5$$
$$I_1 = -63A$$
$$I_5 = -3A$$

Since currents in loops 1 and 5 are known, substitute their values in the loop equations to formulate the following matrix.
Note: Since we only have 5 unknowns now, we only use the respective 5 equations to formulate the matrix

$$\begin{bmatrix} 45 & 0 & -18 & 0 & 0 \\ -18 & 8 & 20 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 11 & 0 \\ -2 & 0 & 0 & 0 & 11 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} -327 \\ -378 \\ 9 \\ -27 \\ -27 \end{bmatrix}$$

(ii) No, the above matrix is in reduced form, hence it can not be symmetric. Even if you formulate the matrix with all loop equations, the matrix will not be symmetric due to presence of controlled sources.

(c) Solve the matrix equation in part (b) to find all loop currents using Gauss Jordan Elimination or Cramer's Rule.

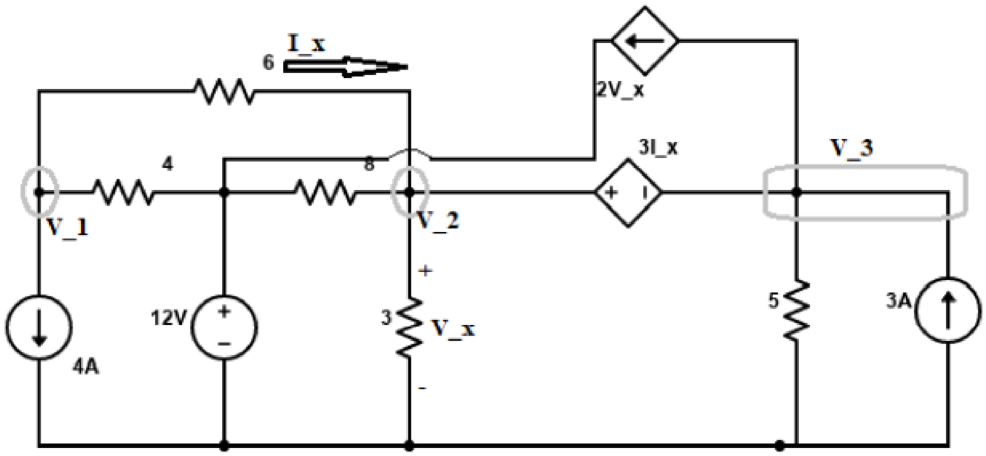
$$I_2 = -15.7A$$
$$I_3 = -30A$$
$$I_4 = -21A$$
$$I_6 = 0.4A$$
$$I_7 = -5.3A$$

(d) Now that all the loop currents are known, explain if you need to change the resistor 'Z' to a different power rating. Current flowing through R_Z is 2.3A.

$$P_{R_Z} = I^2R = (2.3)^2(9) = 47.6W$$

As the power dissipated is less than the 50W threshold of the resistor so no change is required.

Q 7. (a) Label the unknown voltages and form the nodal equations,



Node 1 (V_1):

$$4 + \frac{V_1 - V_2}{6} + \frac{V_1 - 12}{4} = 0$$

Node 2 (Super node):

$$\frac{V_2 - V_1}{6} + \frac{V_2 - 12}{8} + \frac{V_3}{5} + \frac{V_2}{3} + 2V_x = 3$$
$$V_2 - V_3 = 3I_x$$

Where,

$$V_x = V_2$$
$$I_x = \frac{V_1 - V_2}{6}$$

The equations reduce to the following three equations,

$$\frac{5V_1}{12} - \frac{1V_2}{6} = -1 \tag{14}$$

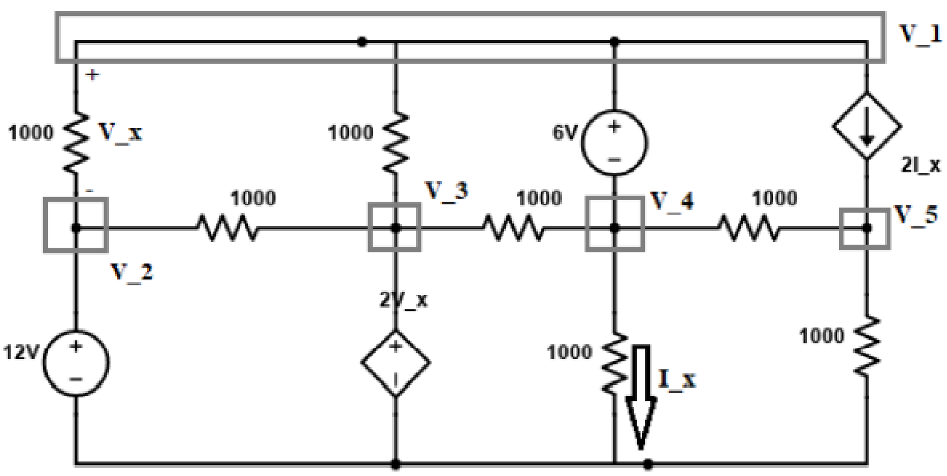
$$\frac{-1V_1}{6} + \frac{21V_2}{8} + \frac{V_3}{5} = 4.5 \tag{15}$$

$$-V_1 + 3V_2 - 2V_3 = 0 \tag{16}$$

Solving(14),(15),(16) simultaneously,

$$V_1 = -2.5V$$
$$V_2 = -0.21V$$
$$V_3 = 0.93V$$

(b) Simplifying the resistor combinations in the network yields(all nodes are labelled, known and unknown),



Notice that,

$$V_2 = 12V$$

Node 1 (Super node):

$$\frac{V_1 - 12}{1000} + \frac{V_1 - V_3}{1000} + 2I_x + \frac{V_4}{1000} + \frac{V_4 - V_3}{1000} + \frac{V_4 - V_5}{1000} = 0$$

Where,

$$I_x = \frac{V_4}{1000}$$

So the equation reduces to,

$$2V_1 - 2V_3 + 5V_4 - V_5 = 12 \tag{17}$$

$$V_1 - V_4 = -6 \tag{18}$$

Node 3 (V_3):

$$-2V_1 + V_3 = -24 \tag{19}$$

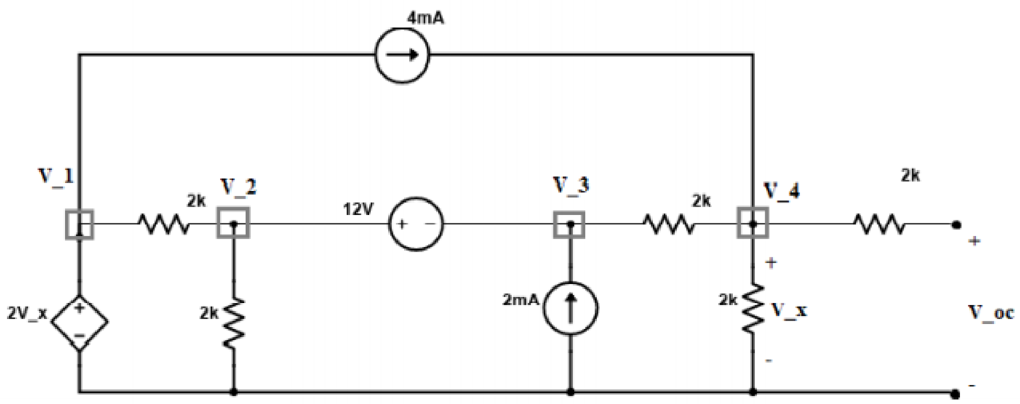
Node 5 (V_5):

$$2V_5 - 3V_4 = 0 \tag{20}$$

Solve (17),(18),(19),(20) simultaneously,

$$V_1 = -38V$$
$$V_3 = -100V$$
$$V_4 = -32V$$
$$V_5 = -48V$$

Q 8. Find V_{TH} using an open circuit,



Using nodal analysis,
Node 1 (V_1):

$$V_1 = 2V_X \tag{21}$$

Node 2 (Super node):

$$\frac{V_2 - V_1}{2000} + \frac{V_2}{2000} + \frac{V_3 - V_4}{2000} = 0.002 \tag{22}$$

And,

$$V_3 - V_2 = 12 \tag{23}$$

Node 4 (V_4):

$$\frac{V_4 - V_3}{2000} + \frac{V_4}{2000} = 0.004 \tag{24}$$

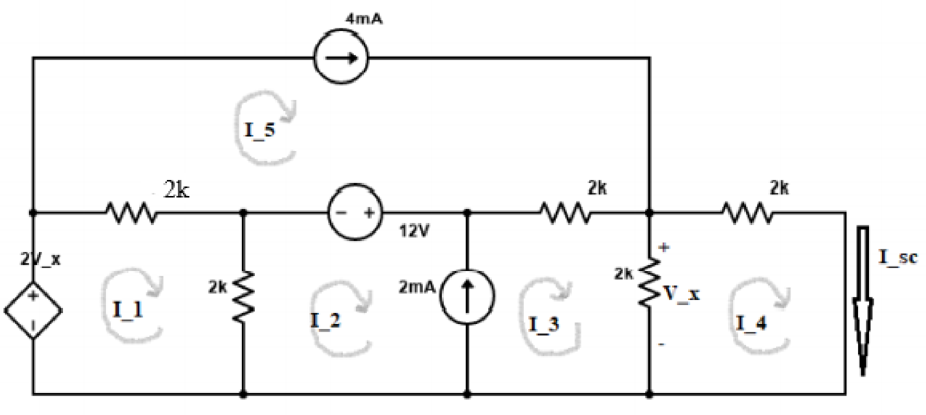
Note that,

$$V_{oc} = V_x = V_4$$

Solve (29),(30),(31),(32) simultaneously,

$$\begin{aligned} V_1 &= \frac{104}{3}V \\ V_2 &= \frac{44}{3}V \\ V_3 &= \frac{80}{3}V \\ V_4 &= V_{oc} = \frac{52}{3}V \end{aligned} \tag{25}$$

Use loop analysis to determine $I_{shortcircuit}(I_{sc})$,



Loop 1 (I_1):

$$2000(I_1 - I_5) + 2000(I_1 - I_2) = 2V_x \tag{26}$$

Loop 2 (Super loop):

$$2000(I_2 - I_1) + 2000(I_3 - I_5) + 2000(I_3 - I_4) = 12 \tag{27}$$

And,

$$I_3 - I_2 = 0.002 \tag{28}$$

Loop 4 (I_4):

$$2000(I_4 - I_3) + 2000I_4 = 0 \tag{29}$$

Loop 5 (I_5)

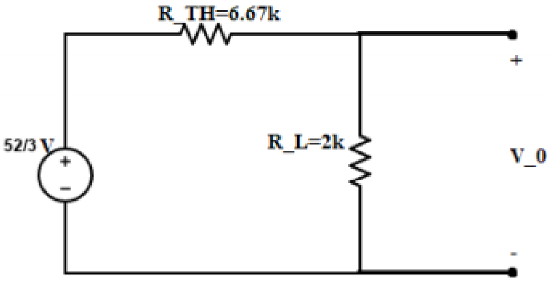
$$I_5 = 0.004A \tag{30}$$

Solve (34),(35),(36),(37),(38) simultaneously,

$$\begin{aligned} I_1 &= 0.001A \\ I_2 &= 0.0032A \\ I_3 &= 0.0052A \\ I_4 &= 0.0026A \\ V_x &= -5.2V \\ I_{sc} &= 0.0026A \end{aligned}$$

R_{TH} is the obtained,

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = 6.67k\Omega$$



$$V_0 = V_{oc} \cdot \frac{R_L}{R_L + R_{TH}} = 3.998V$$