LUMS
A Not-for-Profit University

Department of Electrical Engineering
School of Science and Engineering

EE240 Circuits I - Fall 2020

ASSIGNMENT 4 – SOLUTIONS

Q 1. (a) $V_C(0^-) = 0V$ (Open circuit, no source to charge the capacitor)
$V_C(0^+) = 0V$ (Capacitor does not allow instantaneous change in voltage)
$V_C(\infty) = 15V$ (In steady state the capacitor is charged to voltage source value since it is in series)
$i(0^-) = 0A$ (Open circuit, no current flows)
$i(0+) = 3A$ (Given)
$i(\infty) = 0A$ (Capacitor is fully charged and behaves as open circuit so no current flows)

(b) Two resistors are in parallel so we will use equivalent resistance.

$$\frac{7R}{7+R}i(t) + \frac{1}{C}\int i(t) = 15 \tag{1}$$

(c) To formulate first order differential equation, take derivative of () to produce the following:

$$\frac{7R}{R+7}\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{i(t)}{C} = 0$$

Convert it into the standard form to produce the following:

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{7+R}{7RC}i(t) = 0$$

(d) Evaluate (1) at $t = 0^+$ and substitute value of $i(0^+)$ which is given,

$$\frac{7R}{7+R} \cdot 3 + 0 = 15$$
$$R = 17.5\Omega$$

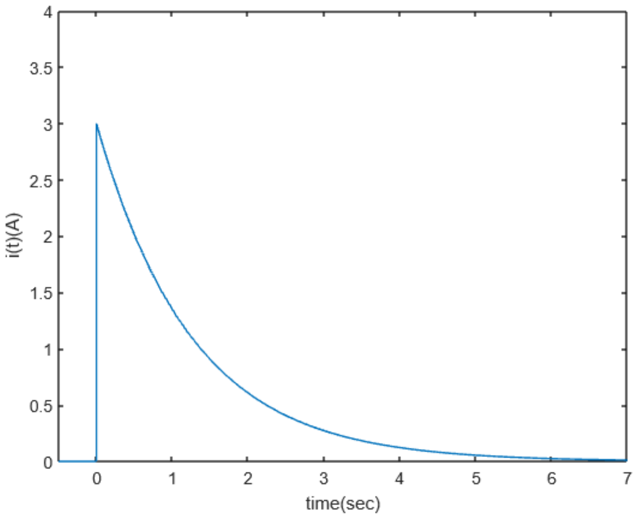
Transient lasts for $6.25s$ means $5\tau = 6.25s$ so $\tau = 1.25s$

$$\tau = R_{eq}C$$
$$R_{eq} = 5\Omega$$
$$C = 0.25F$$

(e)

$$y(t) = K_1 + K_2 e^{-\frac{t}{\tau}}$$
$$K_1 = i(\infty) = 0$$
$$K_2 = i(0^+) - K_1 = 3$$
$$i(t) = 3e^{-\frac{t}{1.25}}A$$

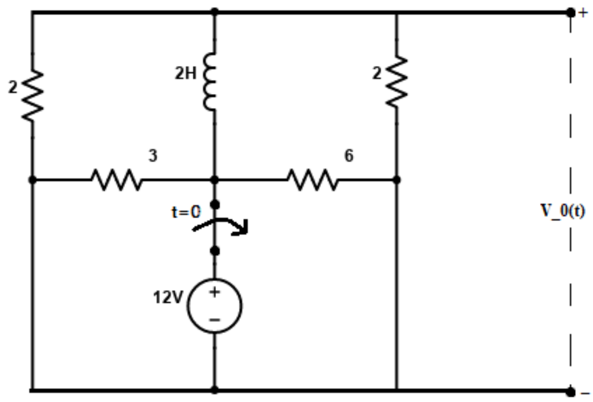
(f) The plot:



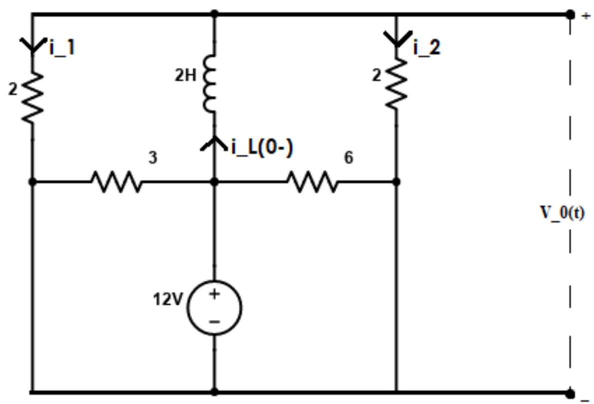
Matlab code:

```
syms t
y = piecewise(t < 0, 0, t ≥ 0, 3.*exp(-t./1.25) )
fplot(y)
axis([-0.5 7 0 4])
xlabel('time(sec)')
ylabel('i(t)(A)')
```

Q 2. Simplify the circuit in the question,

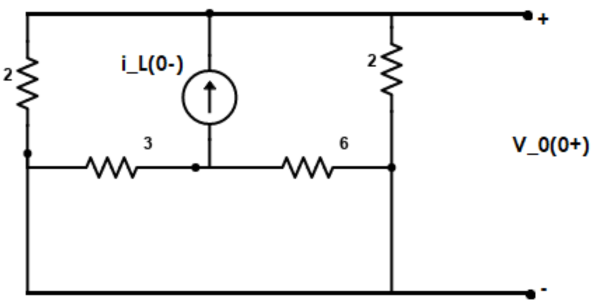


In steady state, inductors behave as short circuit.
At $t = 0^{-}$,



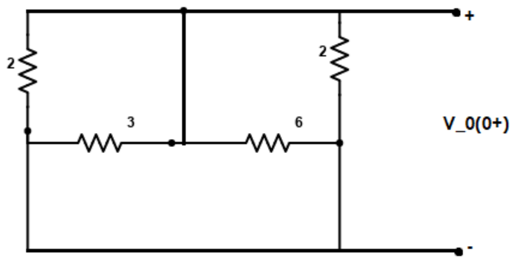
$$i_L(0^-) = i_1 + i_2 = \frac{12}{(2||2)} = 12A$$
$$i_L(0^-) = i_L(0^+) = 12A$$

At $t = 0^+$,

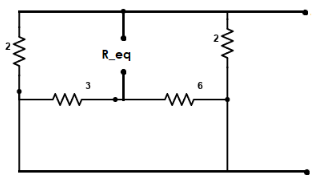


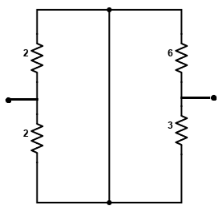
$$V_0(0^+) = 12 \cdot (2||2) = 12V$$

At $t = \infty$,



$$V(\infty) = 0$$



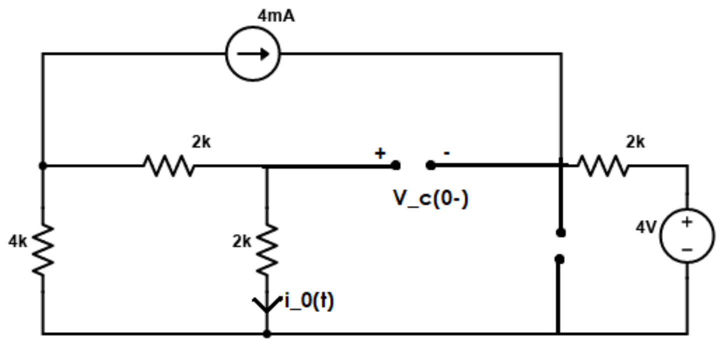


$$R_{eq} = (2||2) + (6||3) = 3\Omega$$
$$\tau = \frac{L}{R_{eq}} = \frac{2}{3} sec$$
$$K_1 = V_0(\infty) = 0V$$
$$K_2 = V_0(0^+) = 12V$$

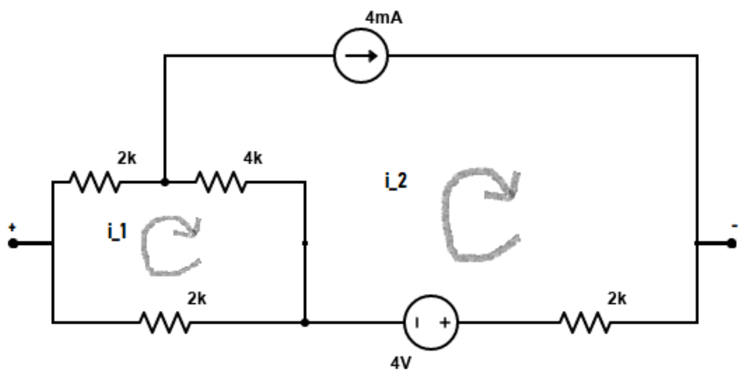
For $t > 0$,

$$V_0(t) = 12e^{-1.5t}.$$

Q 3. At $t = 0^-$, capacitor behaves as an open circuit,



Redraw the circuit and use loop analysis,



$$i_2 = 4mA \tag{2}$$

$$(2+4+2)i_1 - 4i_2 = 0 \tag{3}$$

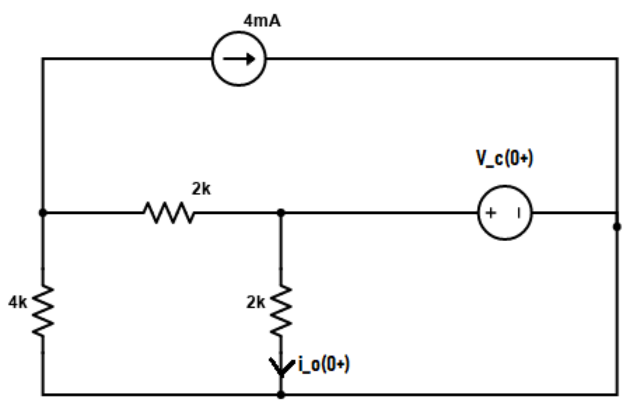
Plugging (2) into (3),

$$i_1 = 2mA$$

For $V_c(0^-)$, take a path along $+$ to $-$ and add up voltages. Take precaution when dealing with polarity,

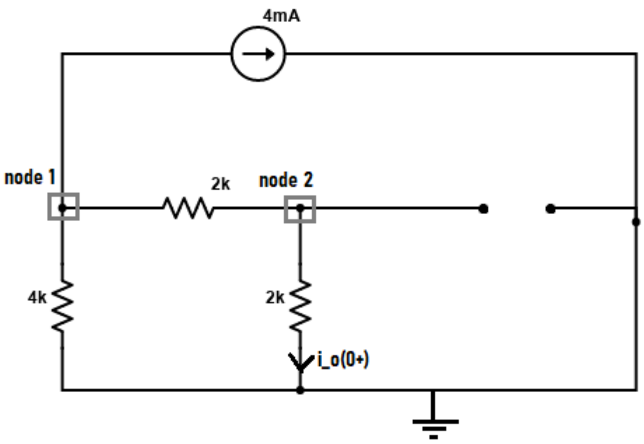
$$V_c(0^-) = -2(2) - 4 - 2(4) = -16V$$

At $t = 0^+$,



$$i_0(0^+) = \frac{-16}{2 \cdot 10^3} = -8mA$$

At $t = \infty$,



Using nodal analysis,
Node 1:

$$\frac{V_1}{4000} + \frac{V_1 - V_2}{2000} + 4 \cdot 10^{-3} = 0$$

Node 2:

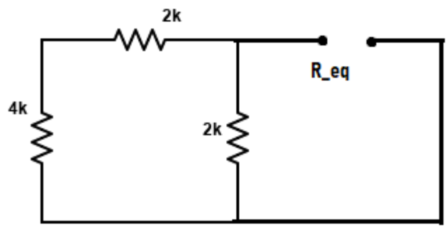
$$\frac{V_2}{2000} + \frac{V_2 - V_1}{2000} = 0$$

Solving these equations simultaneously,

$$V_1 = -8V$$
$$V_2 = -4V$$

And,

$$i_0(\infty) = \frac{-4}{2000} = -2mA$$

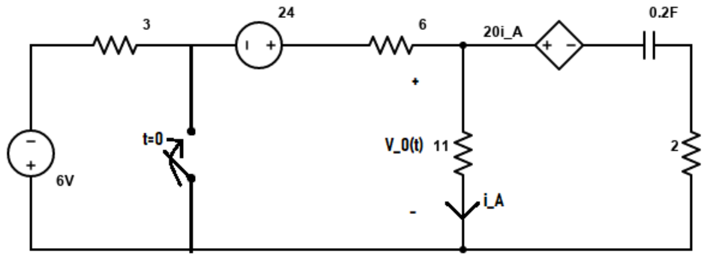


$$R_{eq} = (4+2)||2 = 1.5k\Omega$$
$$\tau = R_{eq} \cdot C = \frac{3}{20} sec$$
$$K_1 = i_0(\infty) = -2mA$$
$$K_2 = i_0(0^+) - i_0(\infty) = -8 - (-2) = -6mA$$

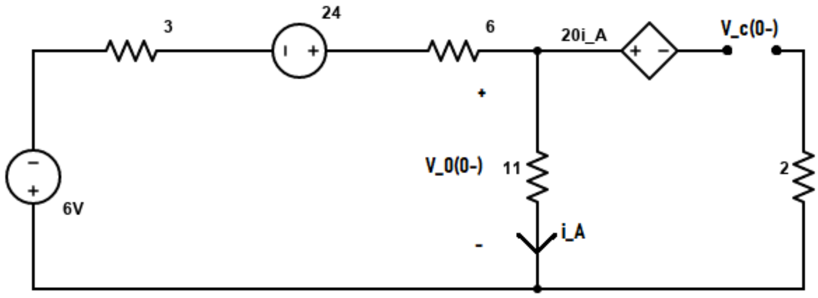
So,

$$i_0(t) = -2 - 6e^{\frac{20t}{3}} mA$$

Q 4. Apply source transformation,

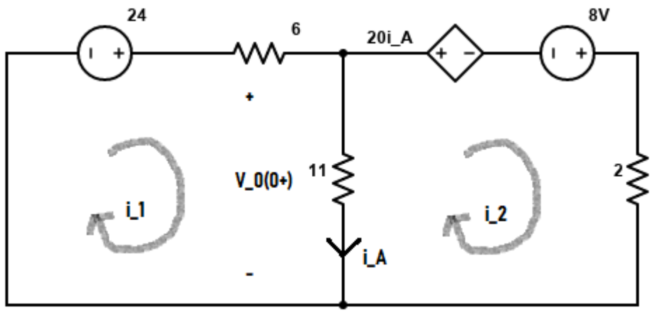


At $t = 0^-$, capacitor acts as an open circuit,



$$
\begin{aligned}
i_A &= \frac{24-6}{11+6+3} = 0.9A \\
V_0(0^-) &= 11(0.9) = 9.9V \\
V_c(0^-) &= (11 \cdot 0.9) - 20(0.9) = -8.1V
\end{aligned}
$$

At $t = 0^+$,



Loop 1:

$$
\begin{aligned}
&-24 + 6i_1 + 11(i_1 - i_2) = 0 \\
&17i_1 - 11i_2 = 24
\end{aligned} \tag{4}
$$

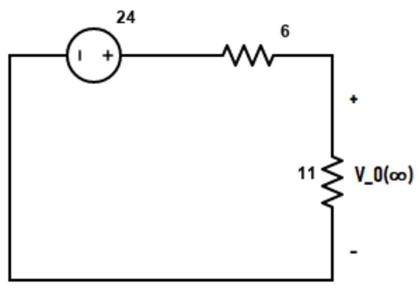
Loop 2:

$$
\begin{aligned}
&20i_A - 8.1 + 2i_2 + 11(i_2 - i_1) = 0 \\
&20(i_1 - i_2) - 8.1 + 2i_2 + 11(i_2 - i_1) = 0 \\
&-7i_2 + 9i_1 = 8.1
\end{aligned} \tag{5}
$$

Solve (7),(8) simultaneously,

$$
\begin{aligned}
i_1 &= 3.945A \\
i_2 &= 3.915A \\
i_A &= i_1 - i_2 = 0.03A \\
V_0(0^+) &= 11(0.03) = 0.33V
\end{aligned}
$$

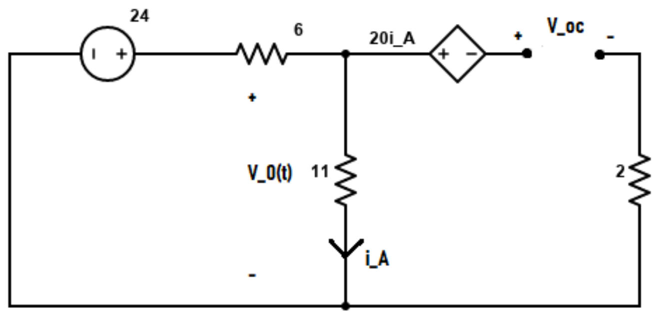
At $t = \infty$, capacitor acts as open circuit,



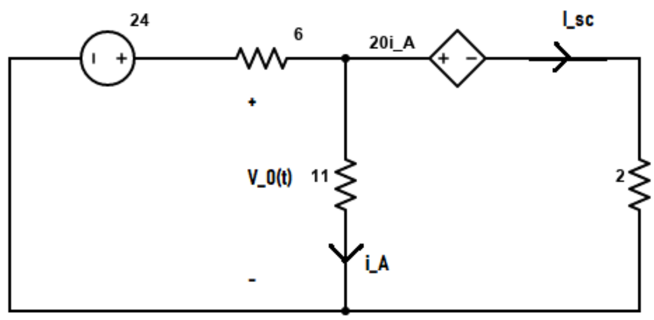
$$V_0(\infty) = 15.5V$$

Next find the resistance of the circuit R_{eq},

$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$



$$i_A = \frac{24}{17}$$
$$V_{oc} = 11 \cdot \frac{24}{17} - 20 \cdot \frac{24}{17}$$
$$V_{oc} = -12.7V$$



Loop 1:

$$\begin{aligned} & -24 + 6i_1 + 11(i_2 - i_2) = 0 \\ & 17i_1 - 11i_2 = 24 \end{aligned} \tag{6}$$

Loop 2:

$$\begin{aligned} & 20i_A + 2i_2 + 11(i_1 - i_2) = 0 \\ & 20(i_1 - i_2) + 2i_2 + 11(i_2 - i_1) = 0 \\ & 9i_1 - 7i_2 = 0 \end{aligned} \tag{7}$$

Solve (6),(7) simultaneously,

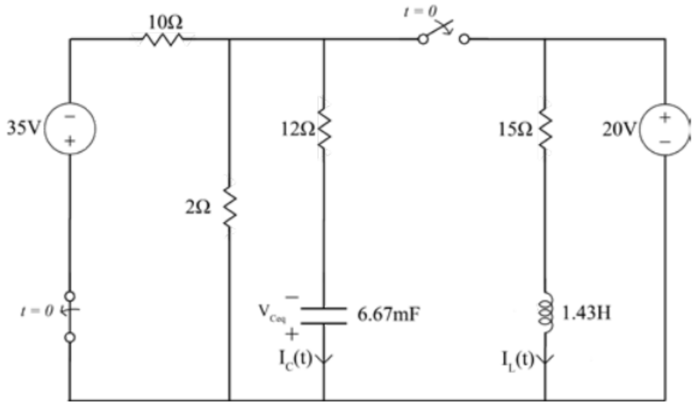
$$i_1 = 8.4$$
$$i_2 = I_{sc} = 10.8$$
$$R_{eq} = \frac{-12.7}{10.8} = -1.17\Omega$$
$$\tau = RC = -0.234$$
$$K_1 = 15.5V$$
$$K_2 = -15.2V$$

So,

$$V_0(t) = \begin{cases} 9.9V & t < 0 \\ 15.5 - 15.2e^{4.27t}V & t \geq 0 \end{cases}$$

Q 5. (a) Order $= 2$

(b) Apply source transformation, simplify resistors, capacitors and inductors.



(c) Use potential divider formula to find voltage across 2Ω resistor which is what the capacitors are charged to,

$$V_C = \frac{2}{12} \cdot 35 = 5.83V$$

(d)

$$12I_C(t) + \frac{1}{6.67mH}\int I_C(t)dt = 20 \tag{8}$$

$$15I_L(t) + 1.43\frac{\mathrm{d}I_L(t)}{\mathrm{d}t} = 20 \tag{9}$$

(e) Find $I_C(0^+)$ by evaluating first loop equation (11) at $t = 0^+$ and substituting $V_C(0^+)$ from part (c) to get,

$$I_C(0^+) = 1.18A$$

Derivate first loop equation (11) and evaluate it at $t = 0^+$ and substituting $I_C(0^+)$ to get,

$$\frac{\mathrm{d}I_C(0^+)}{\mathrm{d}t} = -14.75As^{-1}$$

Find $I_C(\infty)$ by evaluating first loop equation (11) at $t = \infty$ and substituting $V_C(\infty) = 20V$ to get,

$$I_C(\infty) = 0$$

Derivate first loop equation (11) and evaluate it at $t = \infty$ and substituting $I_C(\infty)$ to get,

$$\frac{\mathrm{d}I_C(\infty)}{\mathrm{d}t} = 0As^{-1}$$

Find $I_L(0^-)$ by ohm law,

$$I_L(0^-) = 1.3A$$

$I_L(0^-) = I_L(0^+)$ since inductors do not allow instantaneous change in current. Evaluate second loop equation (12) at $t = 0^+$ and substitute $I_L(0^+)$ to get,

$$\frac{\mathrm{d}I_L(0^+)}{\mathrm{d}t} = 0.35As^{-1}$$

Find $I_L(\infty)$ by ohm law,

$$I_L(\infty) = 1.3A$$

Evaluate second loop equation (12) at $t = \infty$ and substitute $I_L(\infty)$ to get,

$$\frac{\mathrm{d}I_L(\infty)}{\mathrm{d}t} = 0.35As^{-1}$$

Q 6. (a) Order = 5

(b) Inductor is shorted so both nodes at same potential, using potential divider find voltage across 10Ω resistor which is the value the capacitor is charged at. Capacitor does not allow instantaneous change in voltage so values at $t = 0^-$and $t = 0^+$are same,

$$V_3(0^-) = V_3(0^+) = V_1(0^-) = V_1(0^+) = 6.67V$$

Inductor shorts the capacitor to ground. Capacitor does not allow instantaneous change in voltage so values at $t = 0^-$and $t = 0^+$ are same,

$V_2(0^-) = V_2(0^+) = 0V$
$I_2(0-) = I_2(0+) = 0A$ (Open circuit, capacitor is fully charged)
$I_1(0-) = I_1(0+) = 0.67A$ (Ohm's law)

(c) The three nodal equations that describe the circuit at $t > 0$,

$$2\frac{\mathrm{d}V_1(t)}{\mathrm{d}t} + \frac{V_1(t) - V_2(t)}{10} + \frac{1}{5}\int (V_1(t) - V_3(t))dt = 2 \tag{10}$$

$$2\frac{\mathrm{d}V_2(t)}{\mathrm{d}t} + \frac{V_2(t) - V_1(t)}{10} + \frac{1}{5}\int V_2(t)dt = 0 \tag{11}$$

$$2\frac{\mathrm{d}V_3(t)}{\mathrm{d}t} + \frac{1}{5}\int (V_3(t) - V_1(t))dt = 0 \tag{12}$$

(d) Evaluate first node equation at $t = 0^+$ and substitute $V_1(0^+)$ and $V_2(0^+)$ from part (b) to get,

$$\frac{\mathrm{d}V_1(0^+)}{\mathrm{d}t} = 0.67Vs^{-1}$$

Evaluate second node equation at t = 0+ and substitute $V_1(0^+)$ and $V_2(0^+)$ from part (b) to get,

$$\frac{\mathrm{d}V_2(0^+)}{\mathrm{d}t} = 0Vs^{-1}$$

Derivate second node equation and evaluate it at $t = 0^+$ and substitute $\frac{\mathrm{d}V_1(0^+)}{\mathrm{d}t}$ and $\frac{\mathrm{d}V_2(0^+)}{\mathrm{d}t}$,

$$\frac{d^2V_2(0^+)}{dt^2} = 0.335Vs^{-1}$$

Derivate third node equation and evaluate it at $t = \infty$ and substitute $V_1(\infty)$ and $V_3(\infty)$ (both will be 20V) to get,

$$\frac{d^2V_3(\infty)}{dt^2} = 0Vs^{-1}$$