# EE 240 Circuits I


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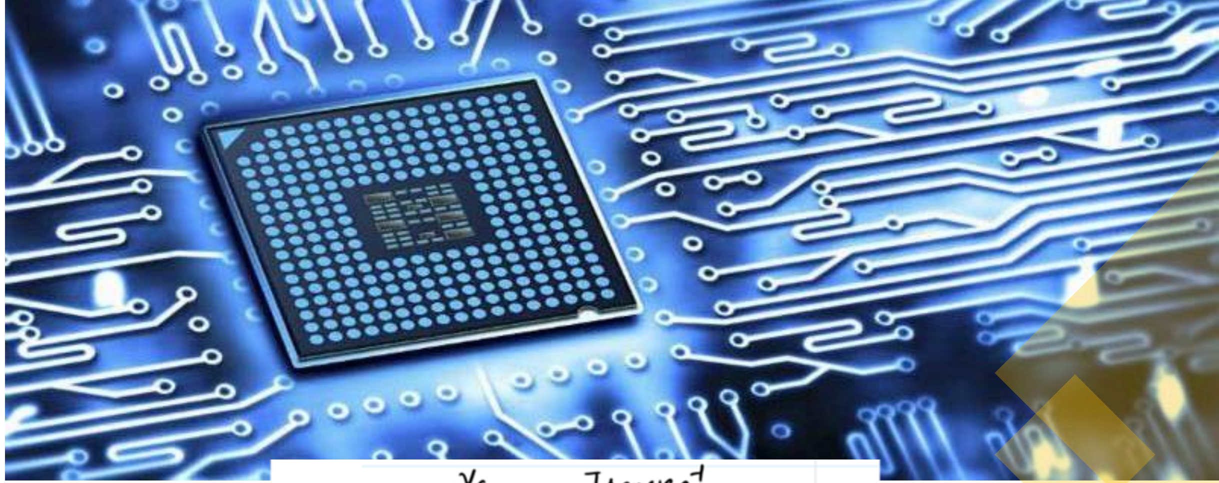
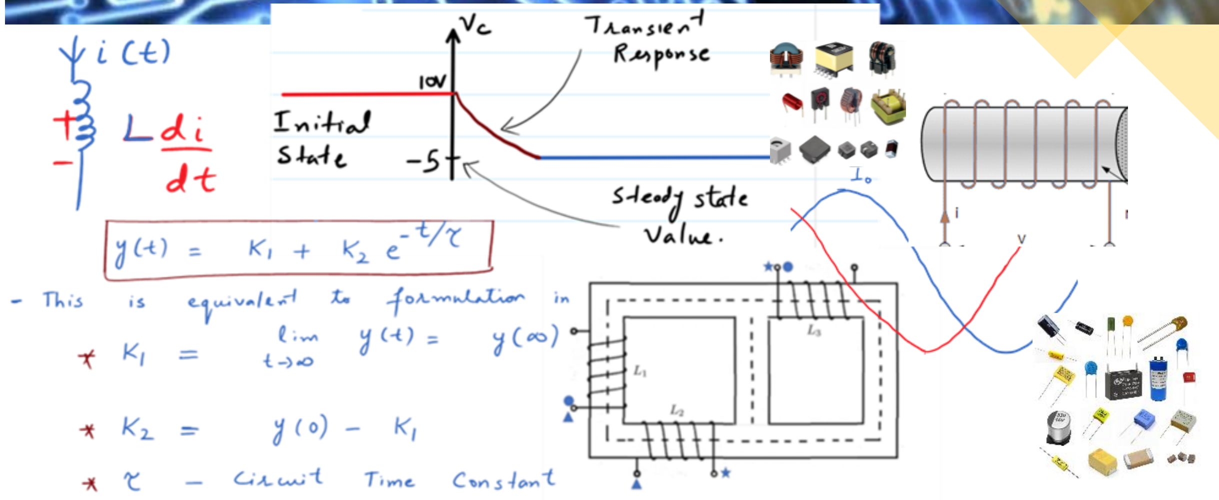
- Series, parallel connection of voltage sources
- Practical models of R,L,C, Voltage source and Current Source
- Controlled Sources
- Mutual Inductance, Dot Convention
- Graphical Representation of Circuits





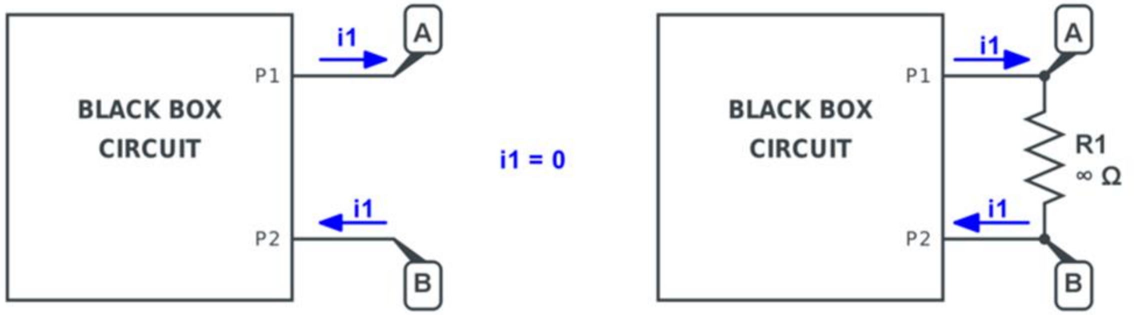

# Concept of Open Circuit and Short Circuit

## Open Circuit:

*Idea: zero current*

Two terminals are disconnected points; zero current can flow between the two terminals irrespective of the potential difference between the terminals.

Equivalent to $R = \infty$ between the terminals

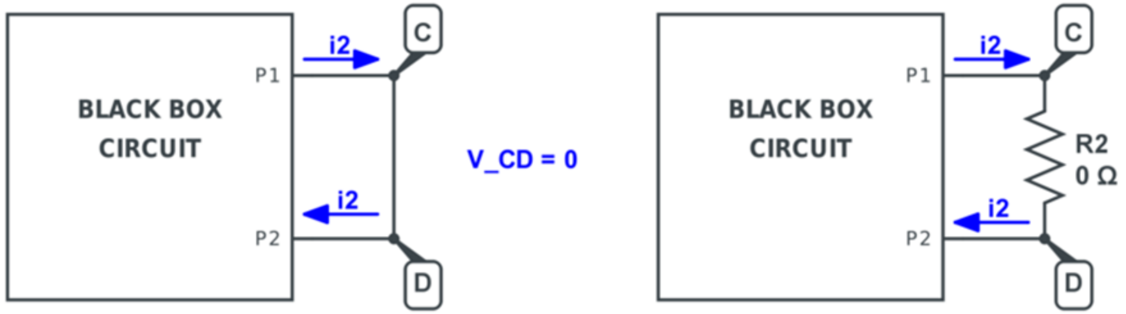



## Short Circuit:

*Idea: zero voltage*

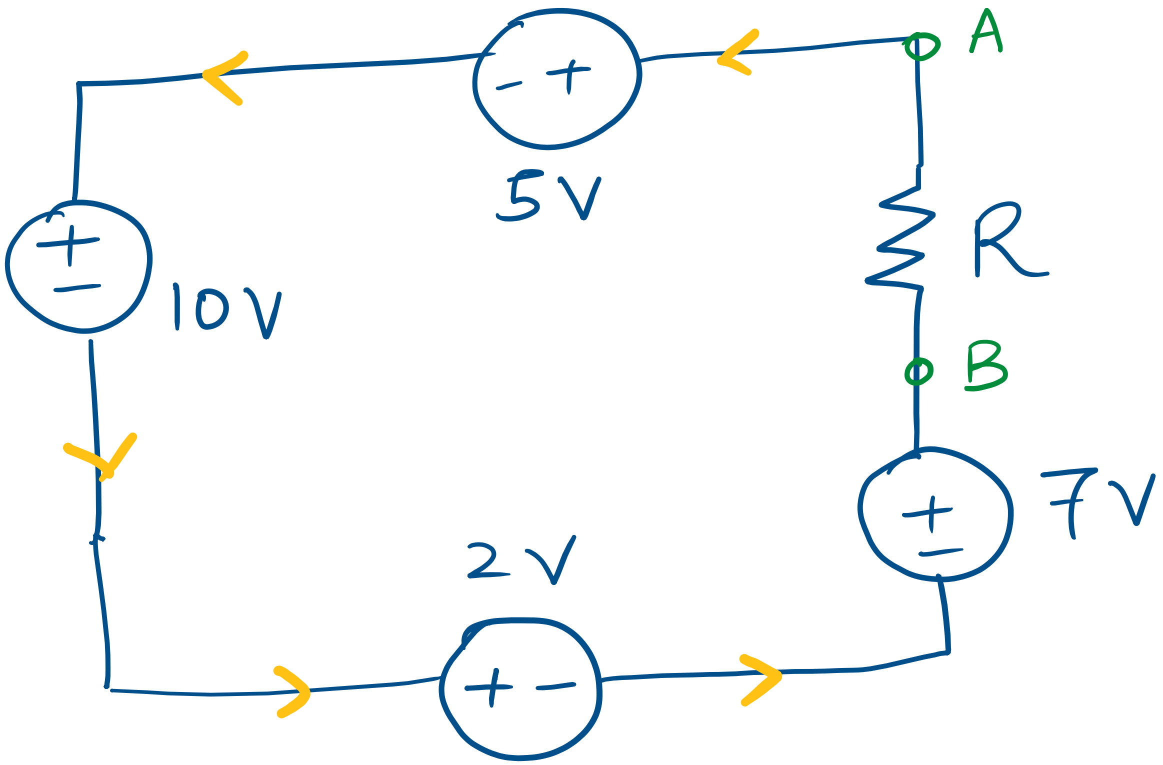
Two terminals connected with zero resistance (wire); zero potetnial difference across the two terminals irrespective of the current flowing between the terminals.

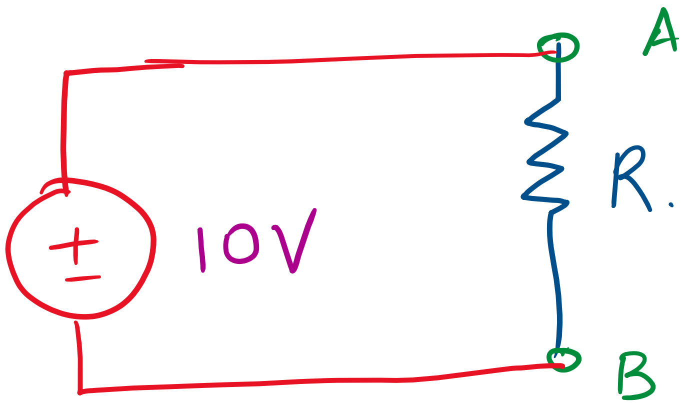
Equivalent to $R = 0$ between the terminals



# Voltage Sources in Series

Voltage sources in series can be represented with an equivalent single voltage source.



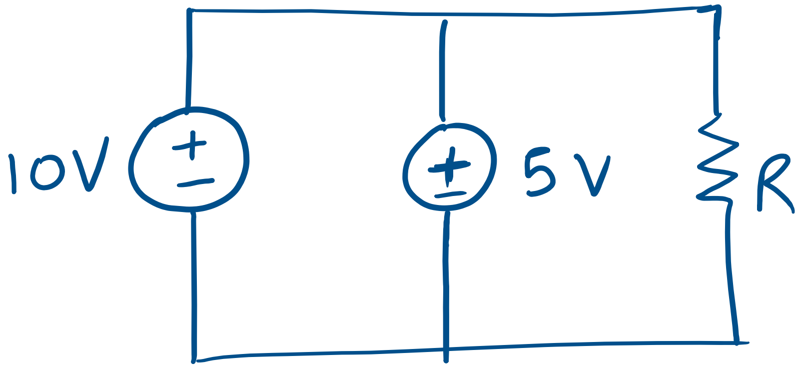



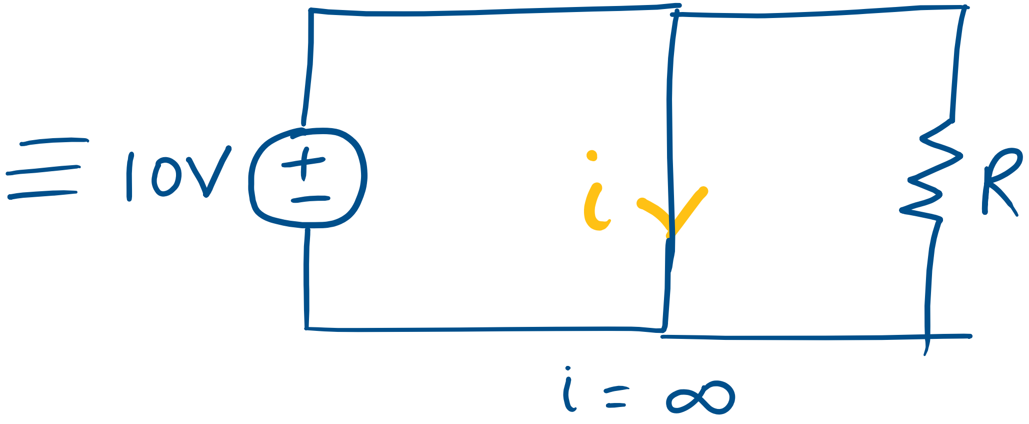

* Let's traverse from A to B.
* Drop of 5 + 10 + 2 = 17V ⇒ Net Drop = 10V
* Gain of 7V = Drop of −7V

# Voltage Sources in Parallel

Q: *Determine the internal resistance of the ideal voltage source?*

A: *Zero resistance (How? Analyze i-v characteristics)*



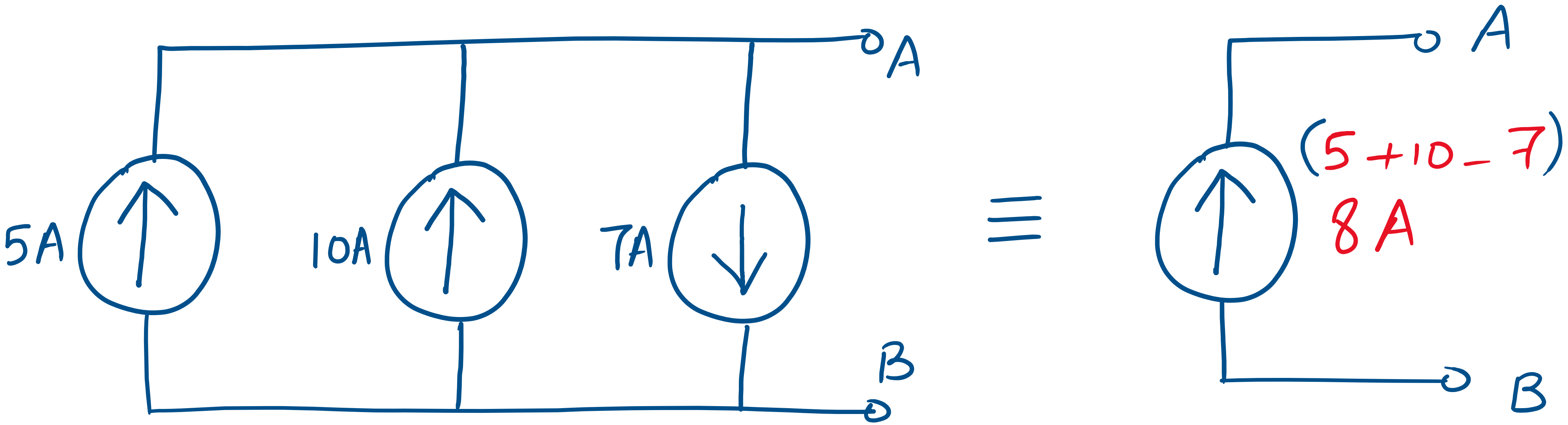




* Infinite current flow due to zero resistance

Voltage sources are **NOT** connected in parallel *except* for the case when both sources have the same voltage level. For circuit analysis, voltage sources of same voltage level, connected in parallel, can be replaced with a single voltage source.

# Current Sources in Parallel

Current sources in parallel can be represented with an equivalent single current source.



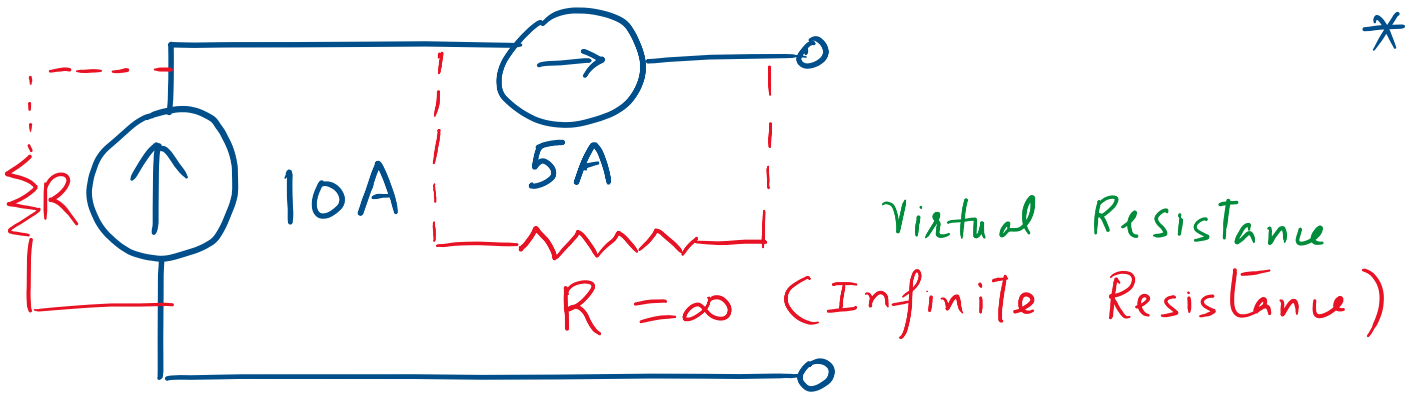

* 5 A and 10 A is being fed to terminal A
* 7A is being drawn from terminal A.

# Current Sources in Series

Q: Determine the internal resistance of the ideal current source?

A: Infinite resistance (How? Analyze i-v characteristics)

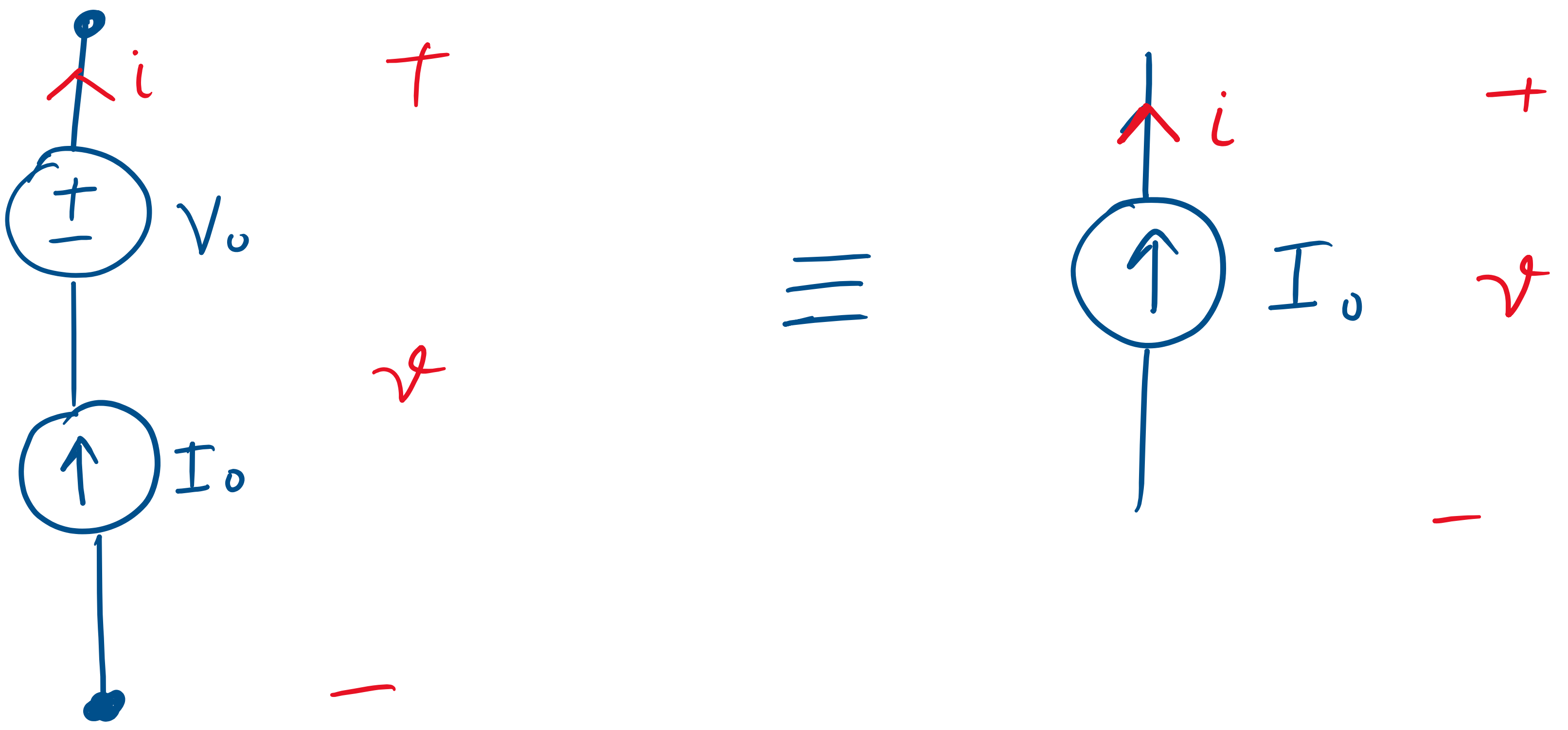
In other words,
- Ideal current source with zero current is equivalent to **open circuit.**
- Voltage across the current source depends on the circuit it is connected to.




* If KCL is to be satisfied; 5A current flow through infinite resistance due to which infinite voltage develops across a source.

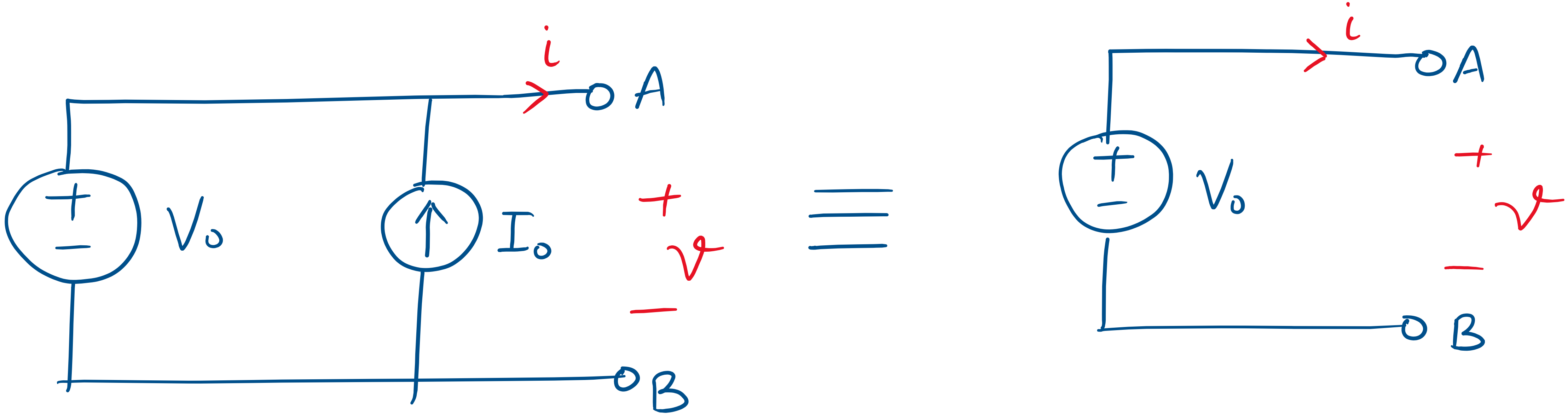
**Current sources are NOT connected in series except for the case when both sources have the same current rating (value + direction).**
**For circuit analysis, current sources of same value, connected in series, can be replaced with a single current source.**

# Current Source and Voltage Source in Series



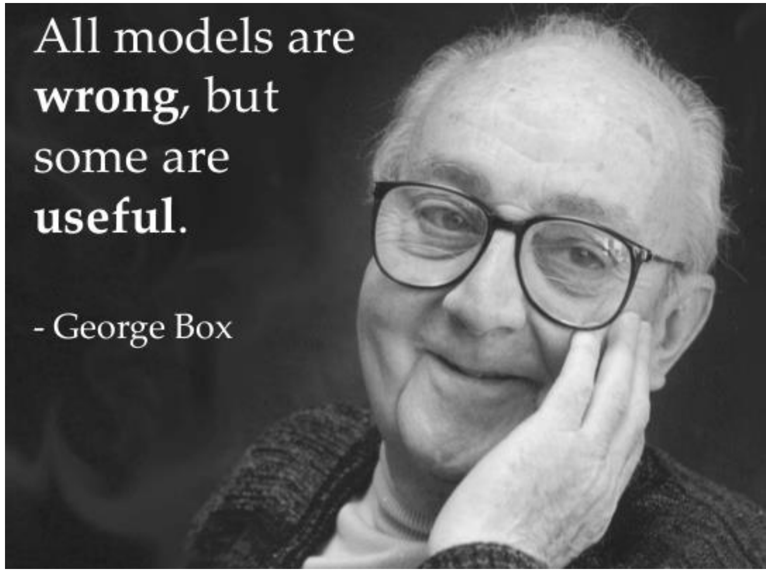

* Note here that $i = I_0$ for any value of $v$
* But these are the characteristics of a current source.

# Current Source and Voltage Source in Parallel




* $v = V_0$ for any value of $i$

# Practical Models – Resistor

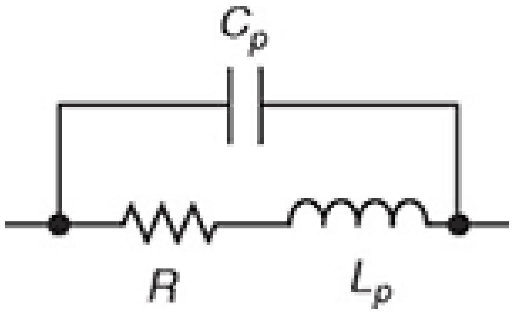



**Practical Resistor:**

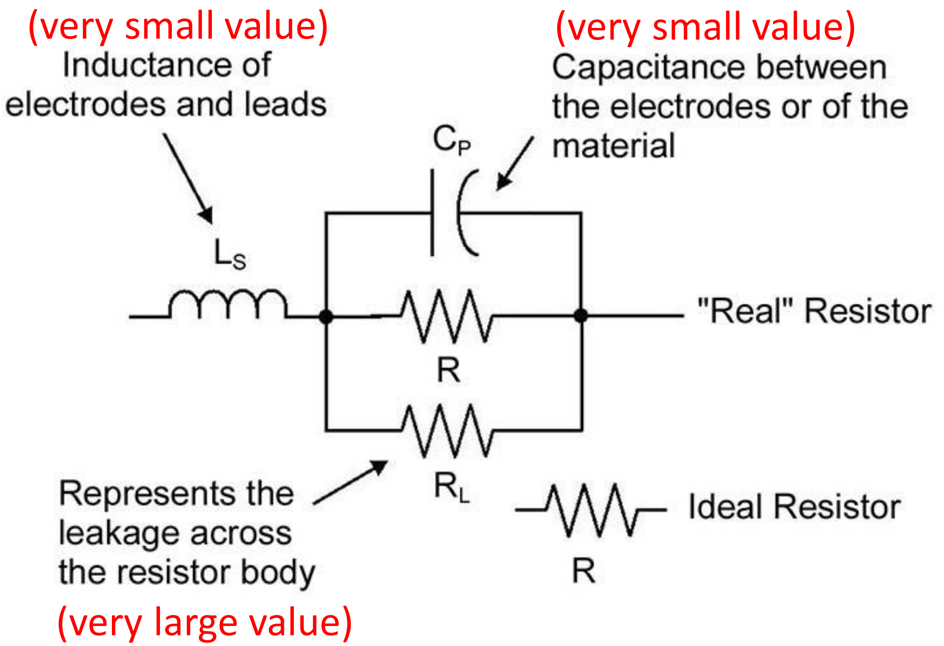
- Stray capacitance – or Parasitic (unwanted) Capacitance
- Parasitic Inductance
- Frequency dependency (prominent effect of stray capacitance and inductance at higher frequencies )
- Non-linear relationship between current and voltage
- Change in resistor due to variations in the temperature and voltage levels.

**Circuit Model 1:**

$L_p$ - Parasitic Inductance
$C_p$ - Parasitic Capacitance

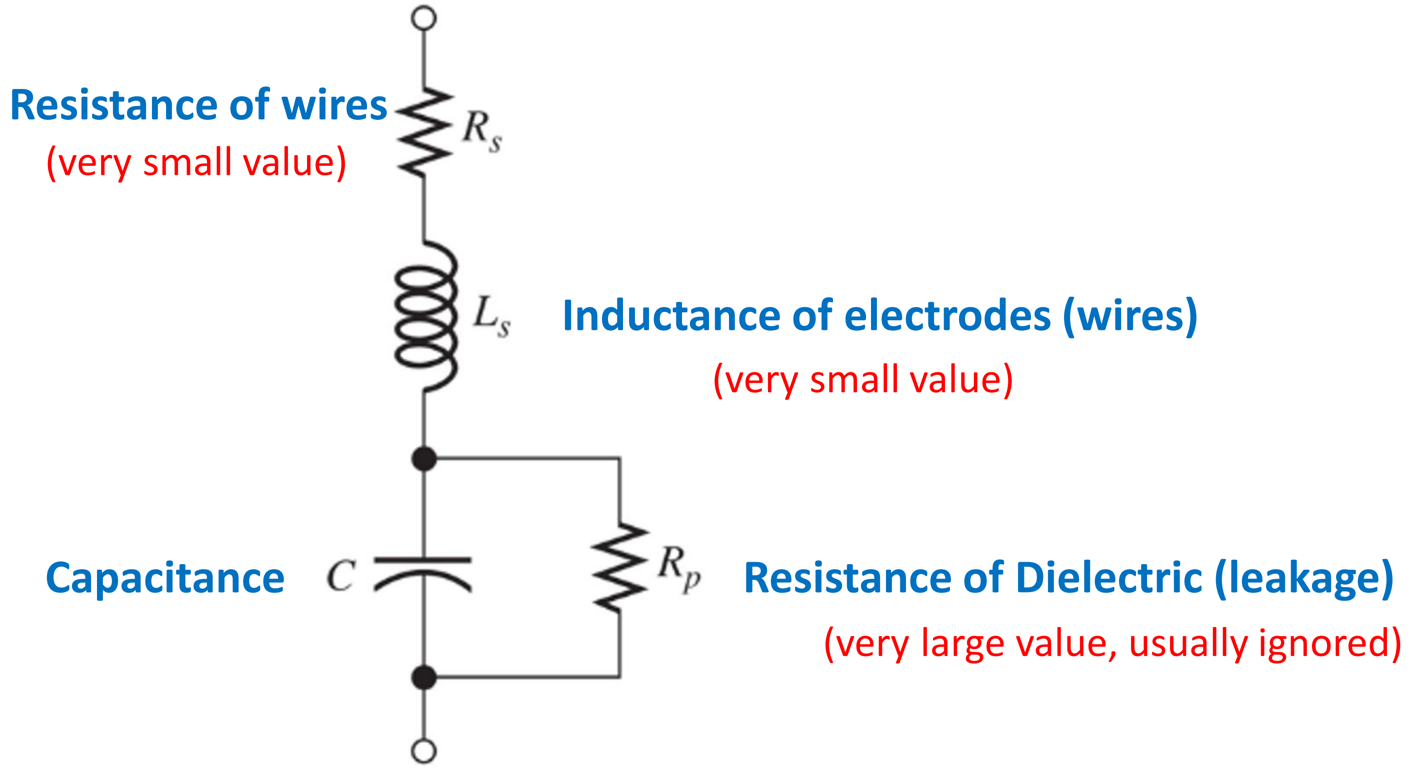



**Circuit Model 2:**



# Practical Models – Capacitor

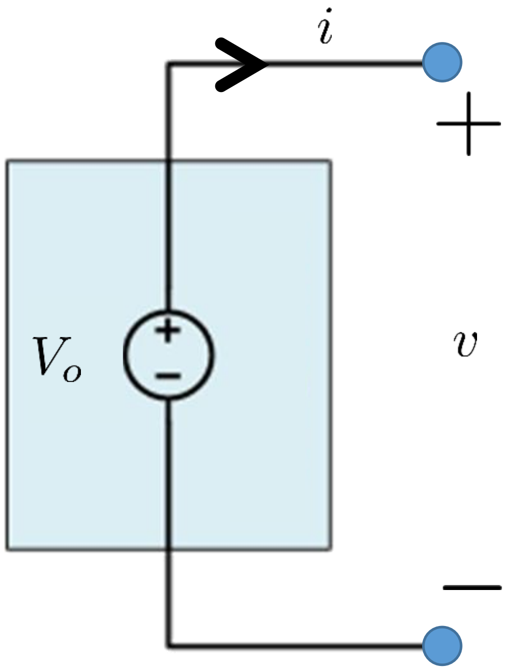
**Circuit Model:**



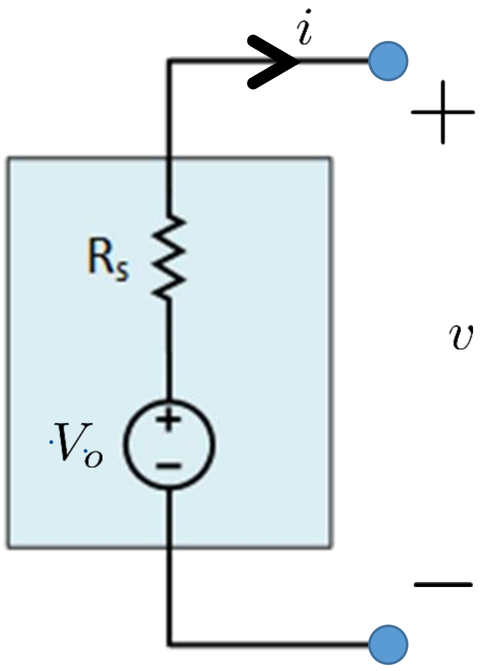

$L_s$ - also referred to as equivalent series inductance (ESL)

$R_s$ - also referred to as equivalent series resistance (ESR)

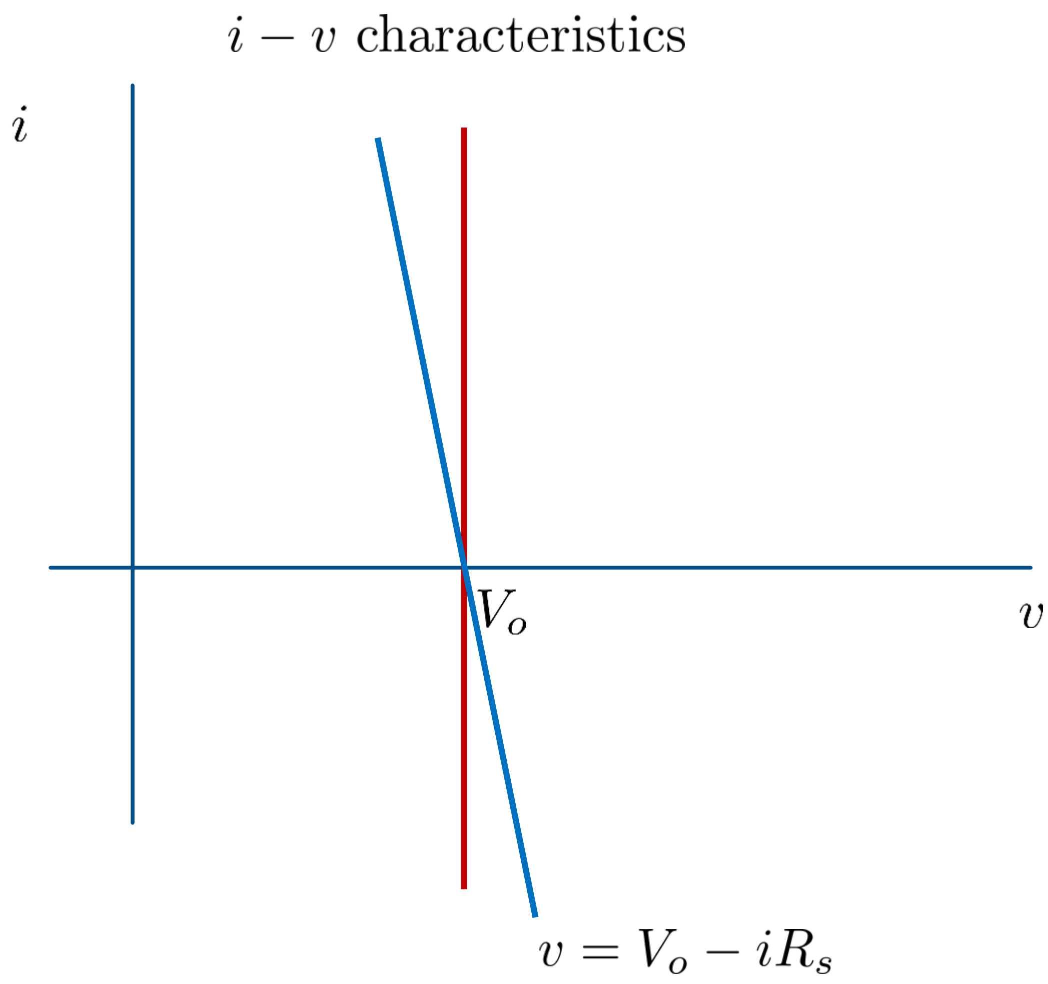
# Practical Models – Voltage Source




**Ideal Voltage Source**




**Practical Voltage Source (Model)**




Practical voltage source: ideal voltage source with reistance $R_s$ in series.

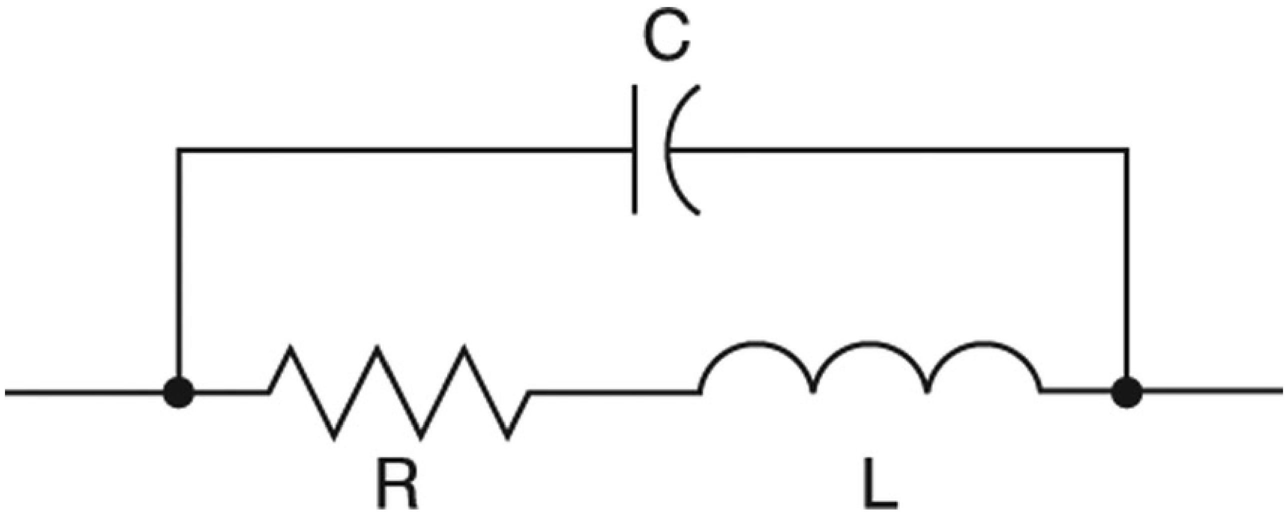
*Consequence:* *Voltage drops with the increase in current across the terminals.*

# Practical Models – Inductor

**Circuit Model:**

(small value)

**Inter-winding Capacitive Effect**

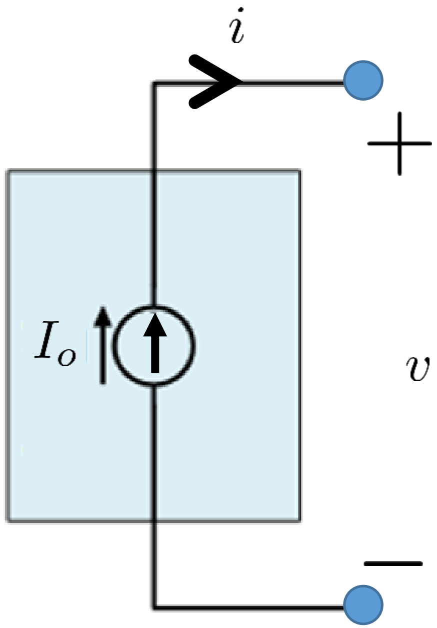



**Resistance of coil**

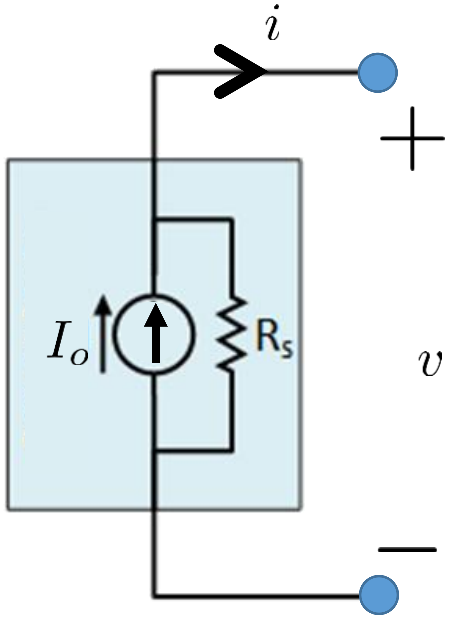
(small value)

**Inductance**

# Practical Models – Current Source

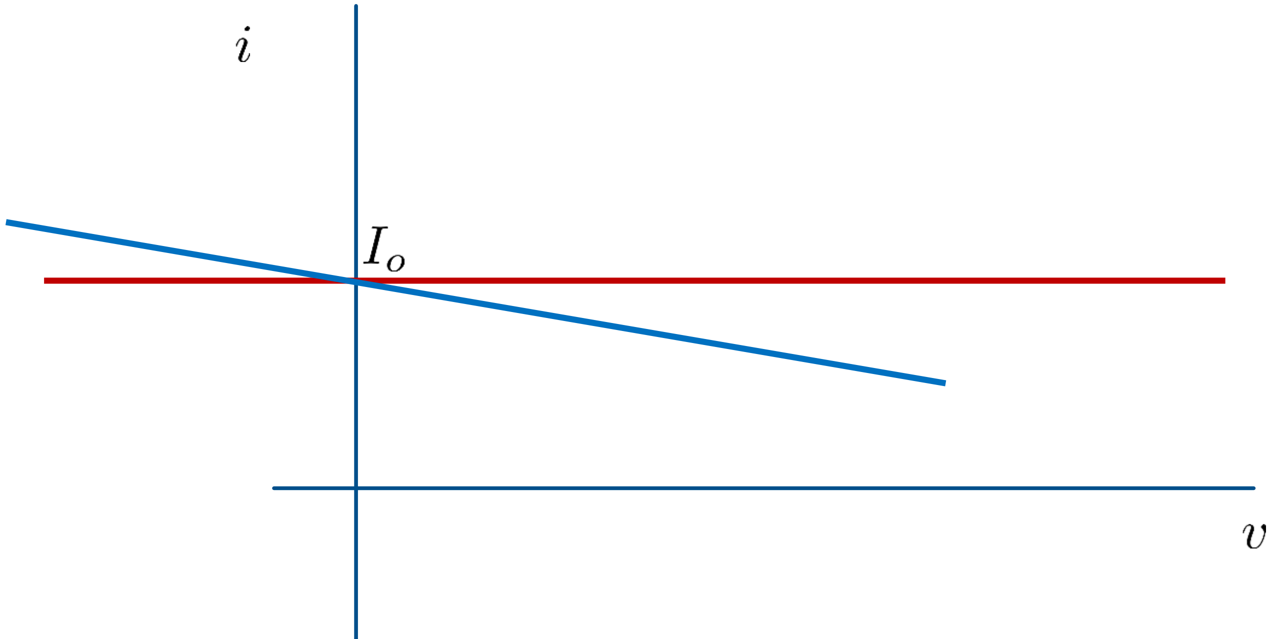



**Ideal Current Source**




**Practical Current Source (Model)**

$i - v$ characteristics




$$i = I_o - \frac{v}{R_s}$$

Practical current source: ideal current source with large reistance $R_s$ in parallel.

*Consequence: current drops with the increase in the voltage across the terminals.*

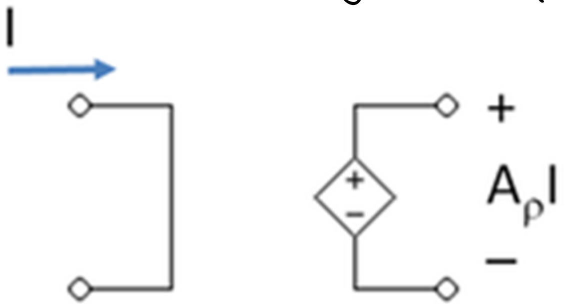
# Dependent or Controlled Sources

So far, we have been dealing with independent sources.

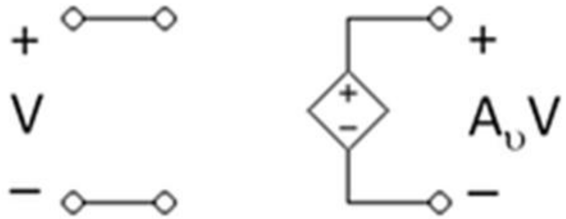
Now we are going to introduce dependent sources, also referred to as controlled sources.

Idea: Current or voltage source depends on the current through some element or voltage across some element of the circuit. We have

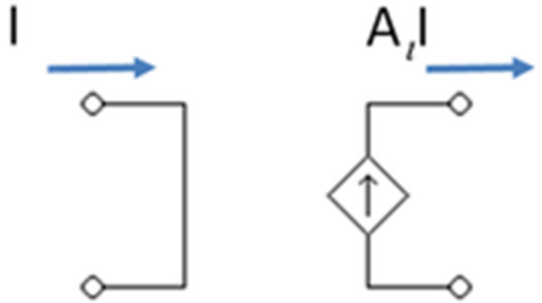
Current-controlled Voltage source (CCVS)



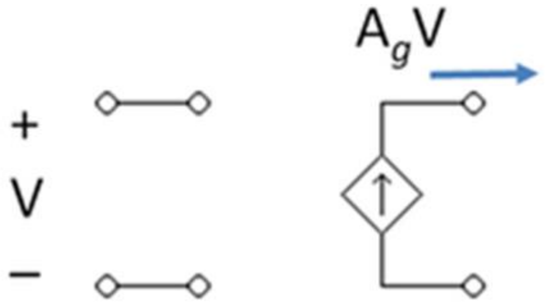

Voltage-controlled Voltage source (VCVS)




Current-controlled Current source (CCCS)




Voltage-controlled Current source (VCVS)



# Coupled Inductors and Mutual Inductance

The two inductors are said to be **coupled** if the flux produced due to the current in one inductor is linked to the other inductor. In other words, the inductors (two or more) are said to be coupled if they are magnetically linked together by a common magnetic flux.

This linking or coupling is quantified by the 'Mutual inductance'.

Let's understand this in more detail. Consider iron core with two coils as shown below

Current $I$ produces magnetic flux density $B$

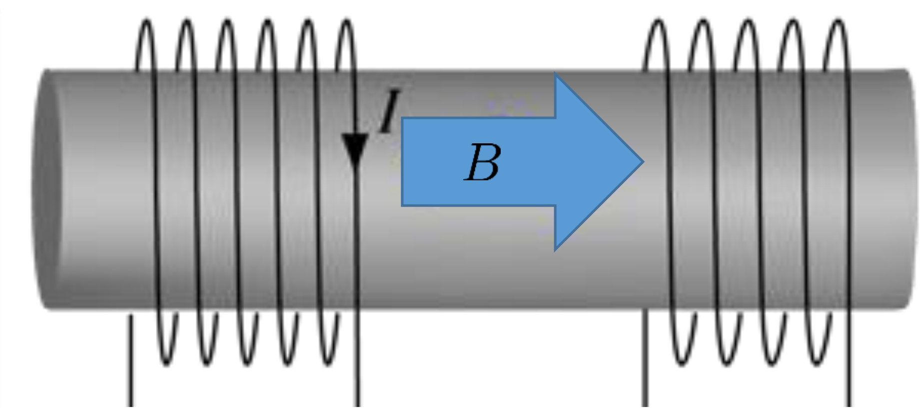
This magnetic field desnity is linked to coil 2 as well.

What we know already that flux in coil 1 is given by

$$\phi_1 = L_1 I$$

Flux in coil 2 is given by

$$\phi_2 = M_{21} I$$




$M_{21}$ Mutual inductance, relates the flux in Coil 2 due to the current in Coil 1

# Coupled Inductors

*Position of the coils on a common core or by increasing the number of turns of either of the coils increases the flux linkage and consequently increases the mutual inductance.*

*For example; Transformer*

## Reciprocity of Mutual Inductance:

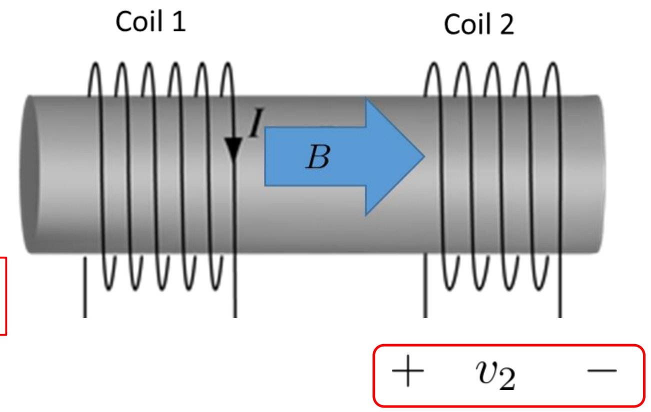
*It follows from the* ***Reciprocity Theorem*** *(proof is beyond the scope here) that Mutual inductance is reciprocate from one side to other equally, that is,*

$$M_{12} = M_{21} = M$$

## Induced voltage due to mutual induction:

Due to flux $\phi_2 = M_{12}I$ in coil 2 due to current in coil 1, the voltage $v_2$ is induced in coil 2 (Faraday's Law), that is

$$v_2 = \frac{d\phi_2}{dt} = M\frac{dI}{dt}$$




*Q: How do we determine the polarity of the induced voltage?*

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# Coupled Inductors

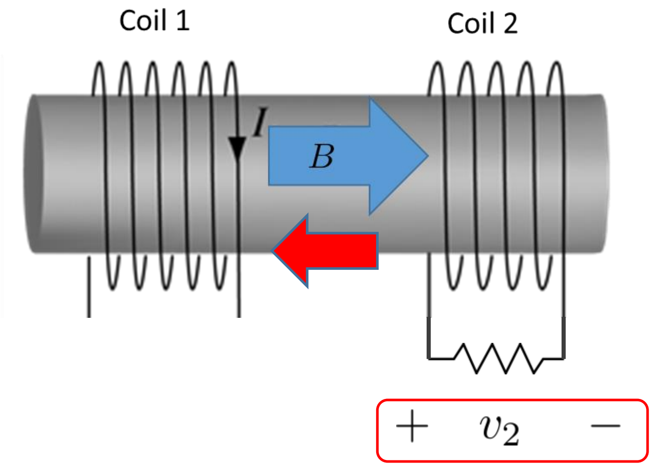
**Determine polarity of the voltage:**

Using Lenz's Law.

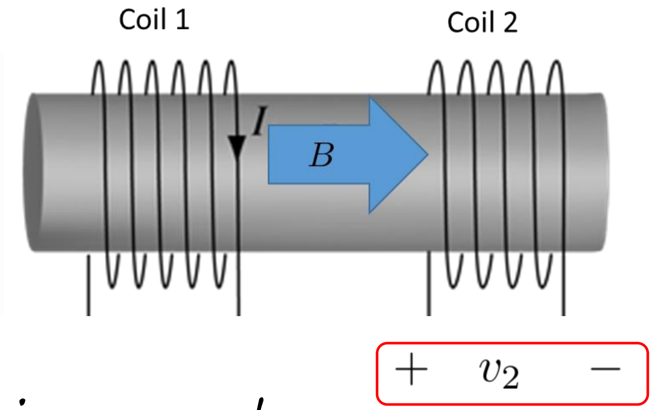
**Idea:** The current produced in Coil 2 due to the induced voltage across coil 2 creates a magnetic field that should oppose the magnetic field due to the current in coil 1 (the current that is causing induced voltage to develop).

Let's understand this further.

- To determine the polarity, connect a resistor across ends of coil 2.
- Once the resistor is connected, current will flow out of the coil from the positive terminal and enter into the coil from the negative terminal.
- Applying right hand-rule, this will produce magnetic field in the direction indicated by the right arrow, that is, opposing the flux indicated in blue (that is due to the current in coil 1).



# Coupled Inductors




## Determine polarity of the voltage:

So the polarity indicated is correct.

If coil 2 winding direction is reversed, the polarity of the induced voltage is reversed.

Therefore, the polarity of the voltage depends on the construction of mutual inductors.

Once the inductors are packaged, the user does not know the direction of the winding.

To facilitate users and indicate the polarity of the voltage, engineers use the dot convention.

## Dot Convention:

- How to use the dots marked on the coupled inductors?
- How to mark the dots given the construction (core, windings directions)?

# Coupled Inductors

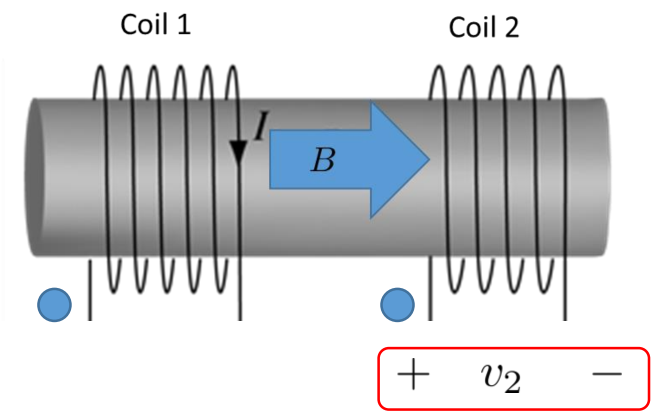
## Dot Convention:

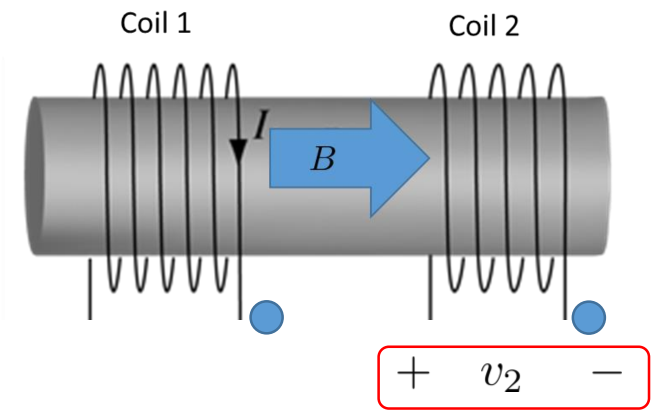
- *How to use the dots marked on the coupled inductors?*

*If the current enters at the dotted terminal of one inductor, it induces a voltage at another (coupled) inductor with positive polarity at the dotted terminal.*

*OR*

*If the current leaves at the dotted terminal of one inductor, it induces a voltage at another (coupled) inductor with negative polarity at the dotted terminal.*



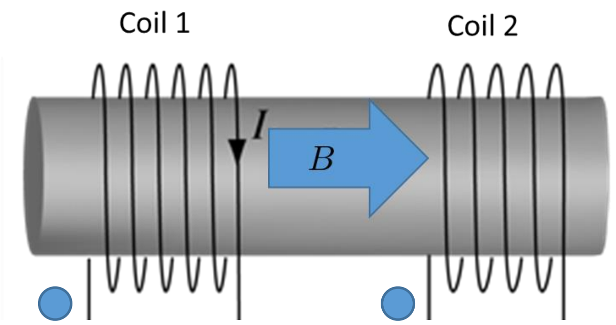




● *Dot adopted to indicate the coupling*

# Coupled Inductors

**Dot Convention:**

- ***How to mark the dots given the construction (core, windings directions)?***
  - *Place the dot arbitrarily on the one winding.*
  - *Determine the direction of the magnetic field ($B_1$) for the current entering the dotted terminal.*
  - *Place the dot on the second winding on the terminal such that when current enters (or leaves) the dotted terminal, it produces a magnetic field in the direction that enhances (or opposes) $B_1$.*




***Note:*** *If there are more than two coupled inductors, a separate mark is used for each pair of windings.*

# Graphical Representation of Electric Circuits (Networks)

Before we start our analysis of electric circuits, we quickly review the concept of network topology or graphical representation of electric circuits as this will help us in our analysis later in the course. We use graph of the circuit to study its topological properties.

**Topology** is the branch of Geometry (helps us in analyzing t he circuit).

**Graph** is a collection of nodes and lines connecting the nodes.

- Nodes are also called as vertices.
- Lines are also called edges or branches.
- If the path from one node to the other node is uni-directional, such path is represented by an arrow and the graph is referred to as directed graph or oriented graph.

For a given electrical circuit, we construct graph by replacing network elements with branches.

# Graphical Representation of Electric Circuits (Networks)

In our circuit analysis later, we will be interested in node pairs and loops (or meshes)

**Node pair:** Two nodes with potential difference.

**Loop (Mesh):** A closed path in a graph formed by number of connected branches.

**Tree:** Type of subgraph obtained by removing branches from the original graph such that the nodes remain connected and no loops in the resulting subgraph.

There could be many trees for a given graph but we always have a very important property for a tree

$$\text{number of nodes in a tree} = \text{number of branches in a tree} + 1$$

The number of branches removed from a graph of 'b' branches and 'n' nodes to make it a tree is given by number of chords;

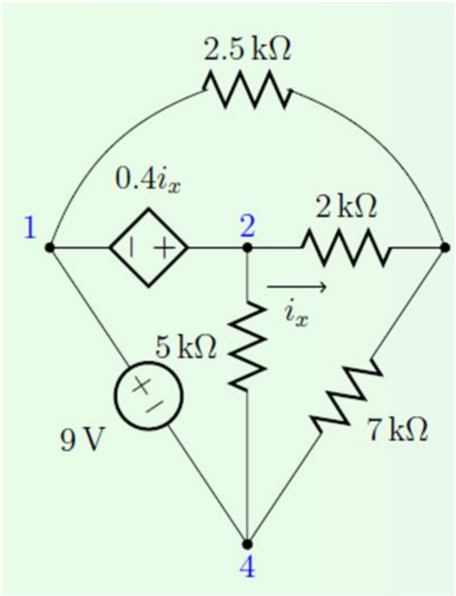
$$\text{number of chords} = b-(n-1) = b-n+1$$

# Graphical Representation of Electric Circuits (Networks)

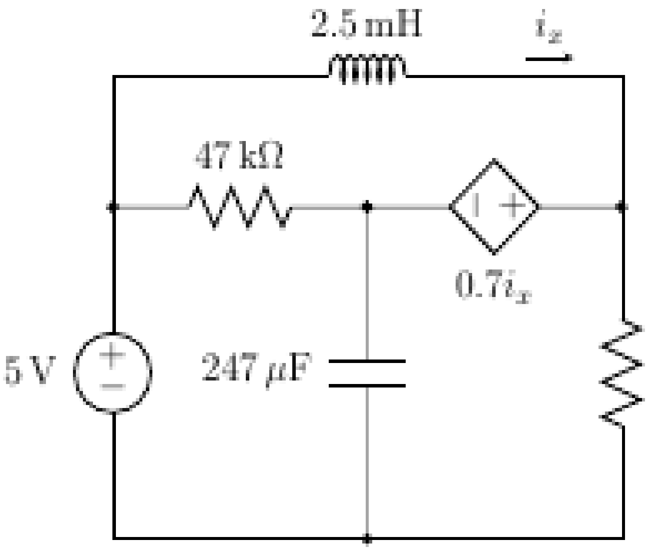
**Example:**

*Both circuits have same graphs*

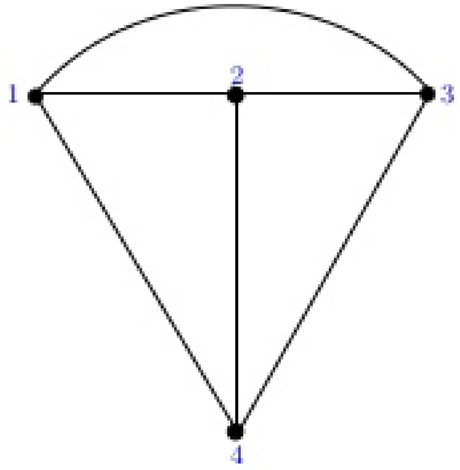
*Graph can be twisted or oriented to obtain an equivalent graph*



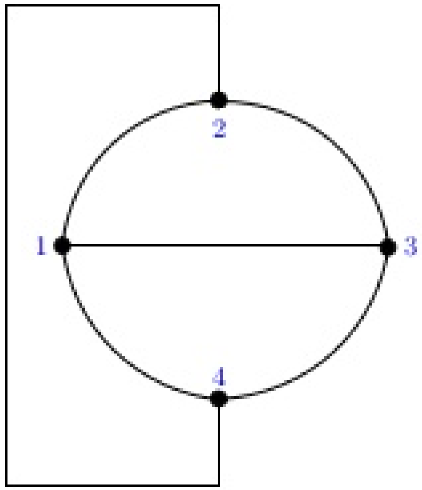

Circuit 1



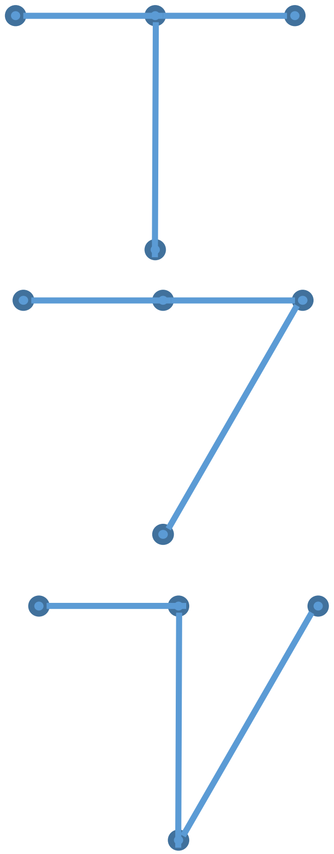

Circuit 2




Graph




Graph Equivalent



Trees

*b=6,*
*n = 4*
*b-n+1 = 3 = number of chords to be removed to obtain a tree of the graph*

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