EE 240 Circuits I

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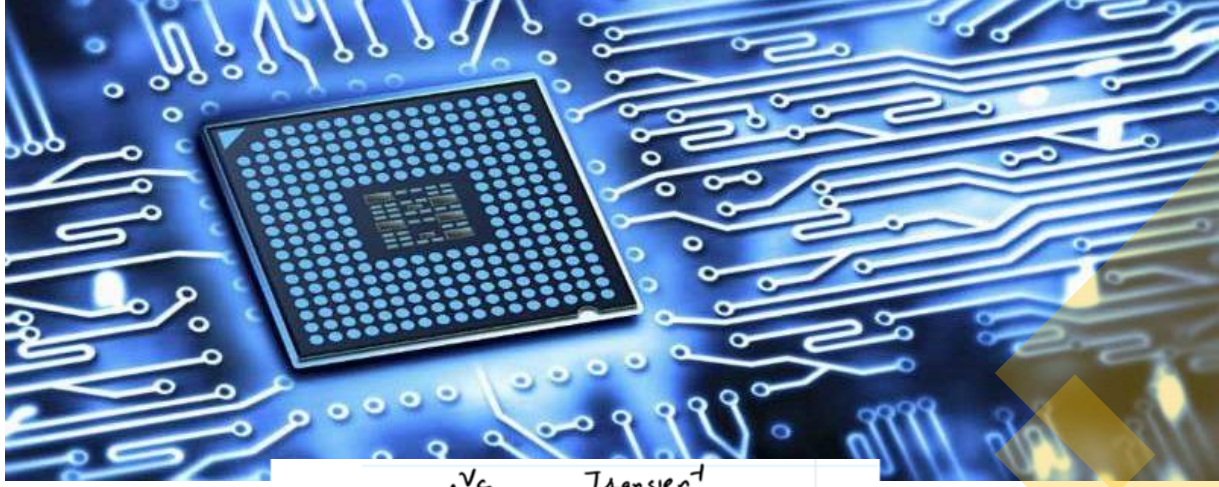
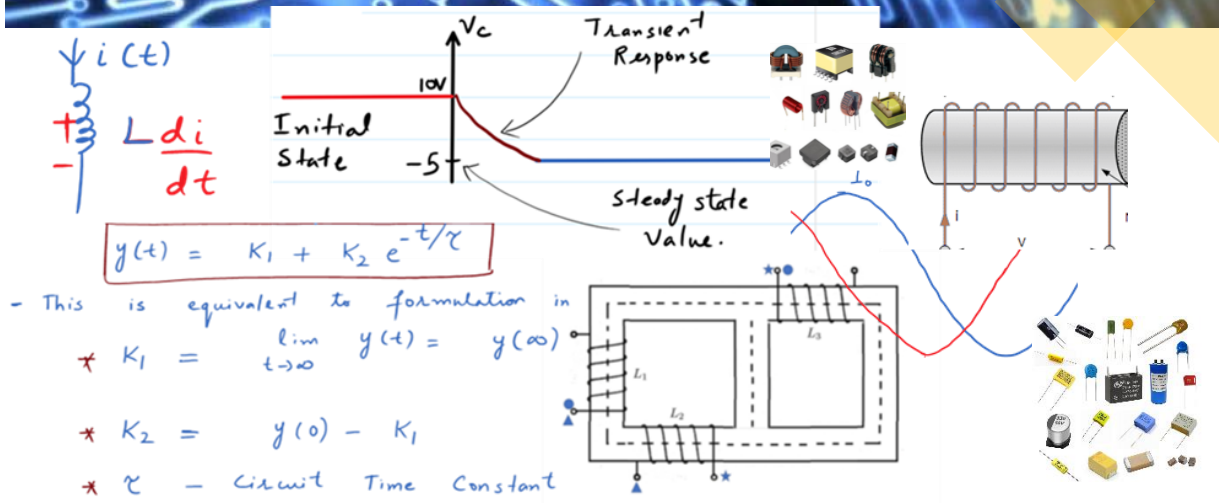
- Duality





Duality in Circuits

Overview

The concept is that the **voltage or current** in a circuit behaves in **analogous** manner as the **current or voltage** in a dual circuit.

- Ohm's law is a simple example, that is, we have v = iR. This is equivalent to i=v(1/R). If we current and voltage are swapped in the first equation and R is replaced with (1/R), we obtain a second equation. We refer to this as 1) voltage and current are dual of each other and 2) (1/R) is a dual of (R) .

- Similarly KCL and KVL are analogous.

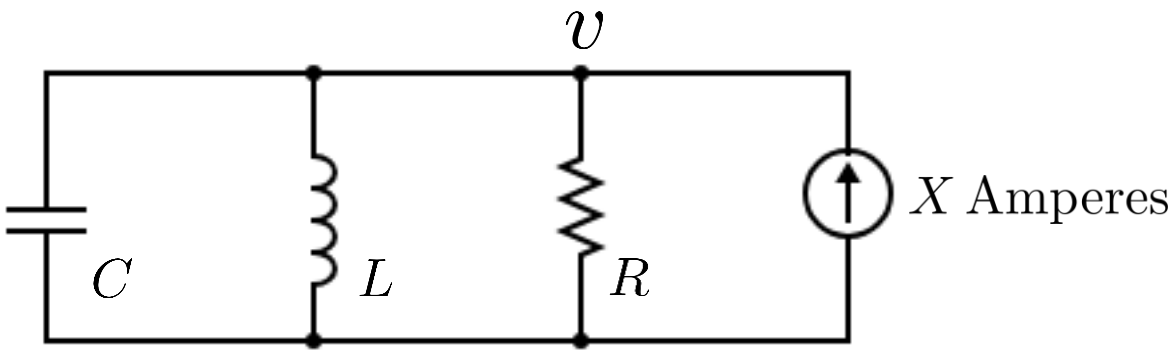
- This similarity is part of large pattern of identical behaviour patterns between voltage and current in the network.

- Let's analyse series RLC and parallel RLC networks.

Duality in Circuits

Example:

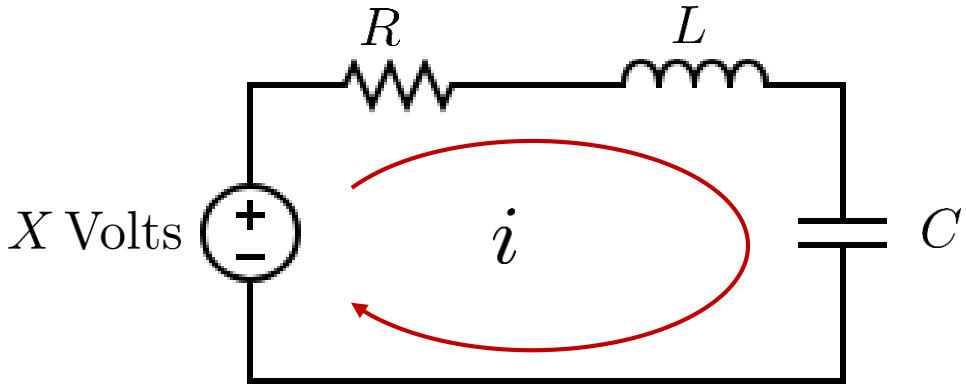
Parallel RLC Circuit



Apply KCL:

$$\frac{v}{R} + C\frac{d}{dt}v + \frac{1}{L}\int v dt - X = 0 \quad (1)$$

Series RLC Circuit



Apply KVL:

$$iR + L\frac{d}{dt}i + \frac{1}{L}\int i dt - X = 0 \quad (2)$$

- Note the similarity between the resulting integro-differential equation.
- If we know the solution of one circuit, it can be utilized to readily obtain the solution of the other circuit.

- The solution of (1) yields v(t) in terms of R,L,C and X.
- The solution of (2) yields i(t) in terms of R,L,C and X.
- If we replace v(t) with i(t), C with L, L with C, R with (1/R) in equation (1), we obtain equation (2).

- Noting this, we can make these subsititutions in the solution v(t) to determine the solution i(t).

Duality in Circuits

Dual Quantities or Elements:

$v \longleftrightarrow i$

Open circuit $\longleftrightarrow$ Short circuit

$R \longleftrightarrow \frac{1}{R}$

Node pair $\longleftrightarrow$ Loop

$L \longleftrightarrow C$

Voltage source and element in series $\longleftrightarrow$ Current source and element in parallel (Source Transformation)

$\int v dt \longleftrightarrow \int i dt$

KVL $\longleftrightarrow$ KCL

- The two circuits are dual of each other if they have the same network equations with dual quantities replaced. For a given circuit, we can determine the dual circuit. Let's learn the graphical construction of the dual circuit.

Graphical Construction of a Dual Circuit

Step 01:

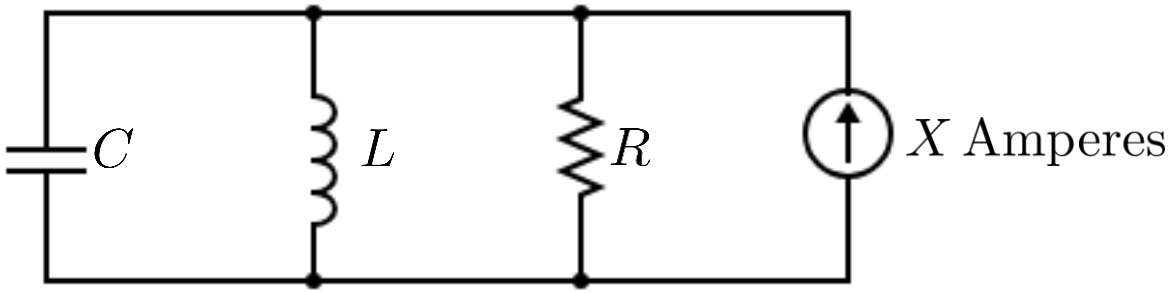
- In a given circuit, place a node inside each loop and place and add an add one additional node (a datum node) outside the circuit.
- On a separate space, arrange the same numbered node on a separate sheet.

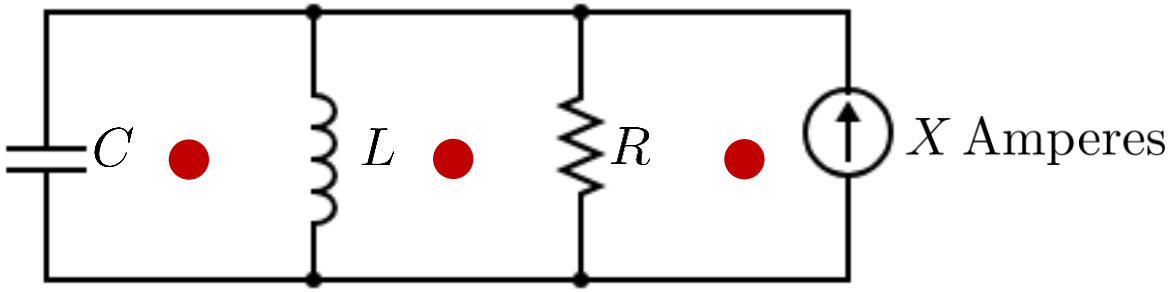
Step 02:

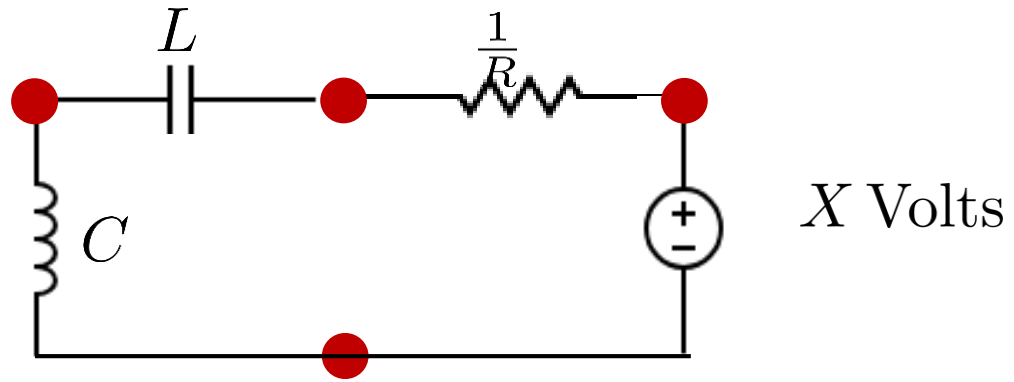
- Draw lines from node to node through elements in the original network only traversing one element at one time.
- For each element traversed from one node to the other in the original network, connect the dual element form the list between a pair of nodes in the dual network.
- Continue this process until the number of possible paths through single element is exhausted.

Graphical Construction of a Dual Circuit

Example:



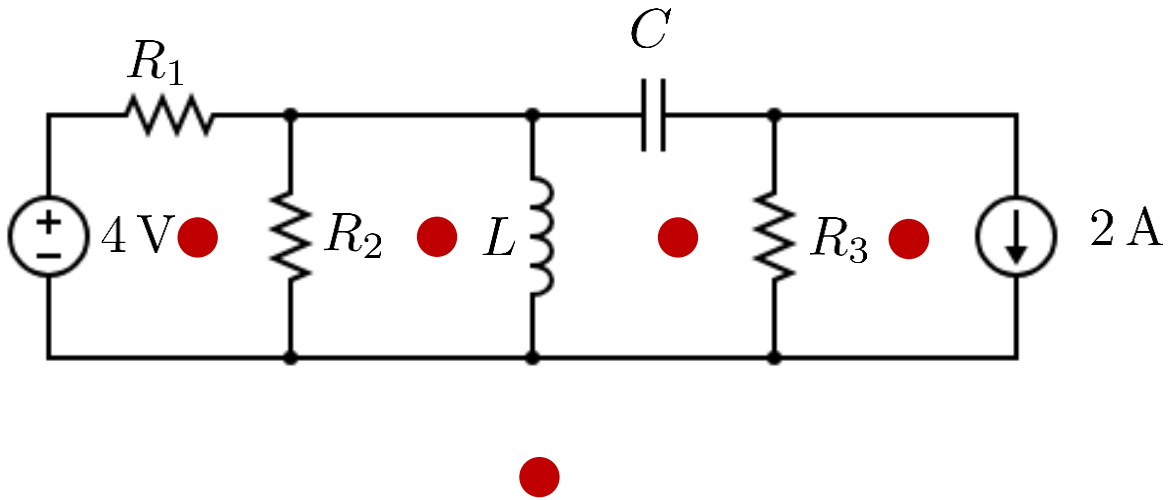


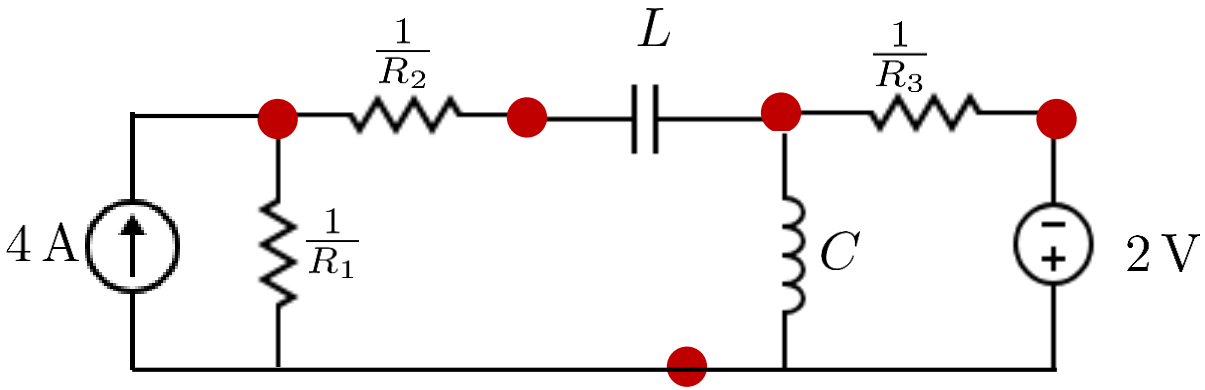


Dual Circuit

Graphical Construction of a Dual Circuit

Example:





Dual Circuit