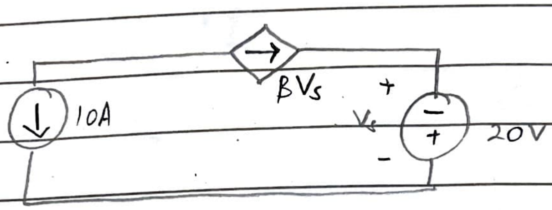
EE-240
Assignment 2.

Q1:



$V_s = -20V$.

We know, a DC current source maintains constant current in a branch.

direction

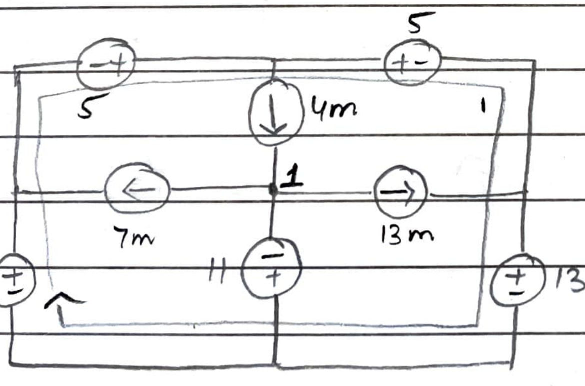
$\therefore \beta V_s = -10A$

$\beta(-20) = -10$

$\therefore \boxed{\beta = \frac{1}{2} \text{ or } 0.5}$

Thus, interconnection is valid for just 1 value of β i.e. 0.5.

Q2:



~~that interconnection is not valid~~

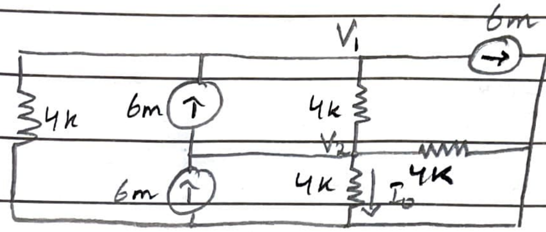
Many ways to show! Here is one way:

apply KVL on the bigger loop.

$-5+5+13-7 = 6 \neq 0$ (violates KVL)

Thus, interconnection is not valid!

Q3:



Applying KCL on node labelled V_1:-

$$\frac{V_1}{4} + \frac{(V_1 - V_2)}{4} + 6 = 6.$$

or $$V_1 + V_1 = V_2$$

$$\therefore \quad 2V_1 = V_2 \quad — ①$$

Applying KCL on node labelled V_2:-

$$\frac{V_2}{4} + \frac{V_2 - V_1}{4} + \frac{V_2}{4} + 6 = 6$$

$$\Rightarrow \quad 3V_2 - V_1 = 0$$

$$\therefore \quad V_1 = 3V_2 \quad — ②$$

note: dropping kilo & milli during mathematical analysis. Do not forget them in final ans!

① & ② :-

$$2V_1 - V_2 = 0$$
$$V_1 - 3V_2 = 0$$

This system of equations can have infinite solutions.

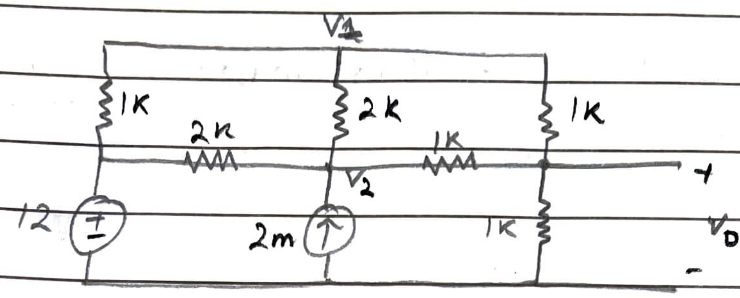
Also, $$I_o = \frac{V_2}{4k}.$$

Let $V_1 = 0 \Rightarrow V_2 = 0$

$$\therefore \boxed{I_o = 0}$$

KING'S NOTES

Q5 :



Applying KCL on node 1 :-

$$\frac{V_1 - 12}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_o}{1} = 0.$$

$$2V_1 + \frac{V_1}{2} - \frac{V_2}{2} - V_o = 12$$

$$\frac{5}{2}V_1 - \frac{1}{2}V_2 - V_o = 12. \quad — 1$$

Applying KCL on node 2 :-

$$\frac{V_2 - 12}{2} + \frac{V_2 - V_1}{2} + \frac{V_2 - V_o}{1} = 2$$

$$-\frac{1}{2}V_1 + 2V_2 - V_o = 8 \quad — 2$$

Applying KCL on node 0 :- $\to V_o$

$$\frac{V_o}{1} + \frac{V_o - V_2}{1} + \frac{V_o - V_1}{1} = 0.$$

$$-V_1 - V_2 + 3V_o = 0 \quad — 3.$$

In Matrix form :- (from 1, 2, 3)

$$\underbrace{\begin{bmatrix} 5/2 & -1/2 & -1 \\ -1/2 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 12 \\ 8 \\ 0 \end{bmatrix}}_{B}$$

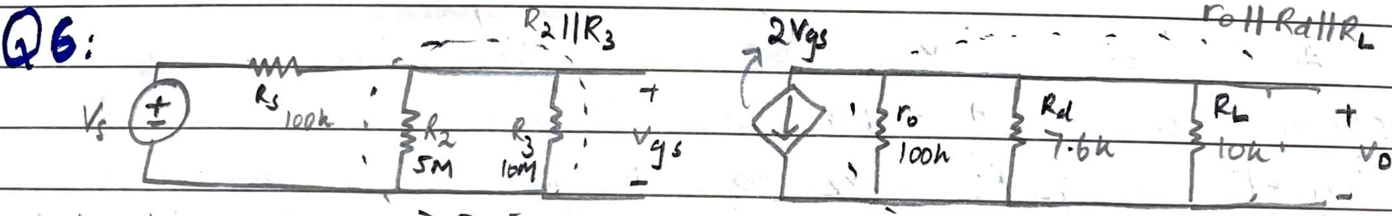
★ Remember, A matrix formed should always be symmetric i.e. $A = A^T$; in case of independent sources.

Solve for $\begin{bmatrix} v_1 \\ v_2 \\ v_o \end{bmatrix}$ using calculator or find inverse of by reduce-echelon and then → $X = A^{-1} B$.

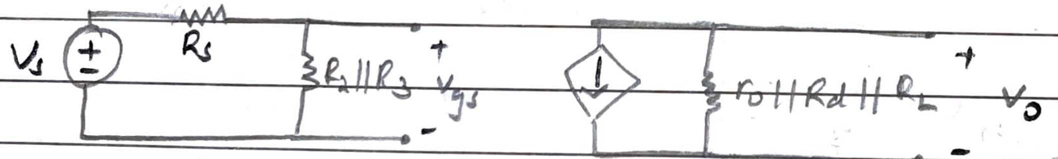
$$\therefore \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 64/7 \\ 328/35 \\ 216/35 \end{bmatrix}$$

$$\Rightarrow \boxed{V_o = \frac{216}{35} \approx 6.17\text{ V}}$$

Q6:



We can redraw the circuit as



(a) $\frac{V_{gs}}{V_s}$

simple voltage divider

$$\Rightarrow V_{gs} = \frac{R_2 || R_3}{(R_2||R_3) + R_s} V_s$$

$$\therefore \frac{V_{gs}}{V_s} = \frac{R_2 || R_3}{(R_2||R_3) + R_s}$$

$$\boxed{\frac{V_{gs}}{V_s} = \frac{3.33M}{3.33M + 100k} = 0.971}$$

$$R_2 || R_3 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_2 || R_3 = \frac{1}{0.2 + 0.1} = \frac{10}{3}\text{M}\Omega \approx 3.33\text{M}\Omega$$

$$r_o || R_d || R_L = \frac{1}{\frac{1}{r_o} + \frac{1}{R_d} + \frac{1}{R_L}}$$

$$= \frac{1}{0.01 + \frac{5}{38} + 0.1} \approx 4.139\Omega$$

KING'S NOTES

(b) $\frac{V_o}{V_{gs}}$

$V_o = (-2V_{gs})(r_o \| R_d \| R_L)$ (from $V = IR$)

$\Rightarrow \frac{V_o}{V_{gs}} = (-2)(r_o \| R_d \| R_L)$

$\therefore \boxed{\frac{V_o}{V_{gs}} = -8.278}$

$\frac{V_o}{V_s} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_s}$

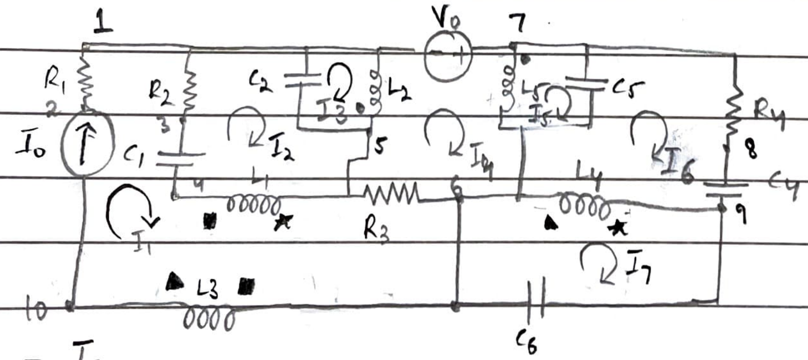
$\frac{V_o}{V_s} = -8.278 \times 0.971$

$\boxed{\frac{V_o}{V_s} \approx -8.038}$

Note: As you can observe, if we apply a specific V_s, it will appear ~8 times larger at output, inverted.

Thus, what we just solved was the gain of a simplified amplifier.

Q7:



L_1 & $L_3 = M_a$

L_1 & $L_4 = M_b$

L_2 & $L_5 = M_c$

L_3 & $L_4 = M_d$

$I_1 = I_0$

Loop 1:-

$$R_1 I_0 + R_2(I_0 - I_2) + \frac{1}{C_1}\int (I_0 - I_2)\,dt + L_1 \frac{d(I_0 - I_2)}{dt}$$

$$+ R_3(I_0 - I_4) + L_3 \frac{dI_0}{dt} + M_a \frac{d(I_0 - I_2)}{dt} + M_a \frac{dI_0}{dt} + M_b \frac{d(I_7 - I_6)}{dt}$$

$$- M_d \frac{d(I_7 - I_6)}{dt} = 0$$

Loop 2 :-

$$R_2(I_2-I_0)+\frac{1}{C_2}\int(I_2-I_3)\,dt+L_1\frac{d(I_2-I_0)}{dt}+\frac{1}{C_1}\int(I_2-I_0)\,dt$$

$$-M_a\frac{dI_0}{dt}-M_b\frac{d(I_7-I_6)}{dt}=0.$$

Loop 3 :-

$$\frac{1}{C_2}\int(I_3-I_2)\,dt+L_2\frac{d(I_3-I_4)}{dt}+M_c\frac{d(I_5-I_4)}{dt}=0.$$

Loop 4 :-

$$L_5\frac{d(I_4-I_5)}{dt}+R_3(I_4-I_1)+L_2\frac{d(I_4-I_3)}{dt}=V_0$$

$$+M_c\frac{d(I_4-I_3)}{dt}-M_c\frac{d(I_5-I_4)}{dt}$$

Loop 5 :-

$$\frac{1}{C_5}\int(I_5-I_6)\,dt+L_5\frac{d(I_5-I_4)}{dt}-M_c\frac{d(I_4-I_3)}{dt}=0.$$

Loop 6 :-

$$R_4I_6+\frac{1}{C_4}\int I_6\,dt+L_4\frac{d(I_6-I_7)}{dt}+\frac{1}{C_5}\int(I_6-I_5)\,dt+M\frac{dI_0}{dt}$$

$$-M_b\frac{d(I_0-I_2)}{dt}=0.$$

Loop 7 :-

$$L_4\frac{d(I_7-I_6)}{dt}+\frac{1}{C_6}\int I_7\,dt-M_d\frac{dI_0}{dt}+M_b\frac{d(I_0-I_2)}{dt}=0.$$

Q8:

(a) No. of nodes = 10 = n (labelled as #s in prev. Q's diag)

(b) No. of nodal equations required = n - 1

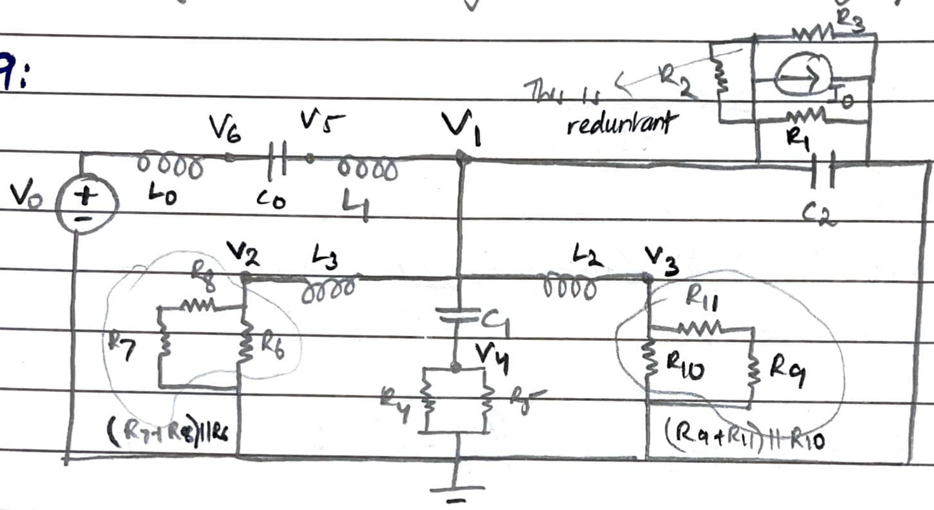
= 9.

(c) No. of branches b = 16 (simply count the no. of elements).

(d) & (e)

Any graph having 10 nodes and proper interconnection in accordance with the original circuit would suffice. Similarly, construct any tree from the graph obtained.

Q9:



Node 1 :-

$$\frac{1}{L_1}\int(V_1-V_5)\,dt + \frac{1}{L_2}\int(V_1-V_3)\,dt + \frac{1}{L_3}\int(V_1-V_2)\,dt + C_1\frac{d(V_1-V_4)}{dt}$$

$$+ C_2\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{V_1}{R_3} = -I_0$$

Node 2 :-

$$\frac{V_2}{(R_7+R_8)||R_6} + \frac{1}{L_3}\int(V_2-V_1)\,dt = 0.$$

Node 3 :-

$$\frac{V_3}{(R_9+R_{11})||R_{10}} + \frac{1}{L_2}\int(V_3-V_1)\,dt = 0.$$

Node 4 :-

$$\frac{V_4}{(R_4 \parallel R_5)} + C_1 \frac{d(V_4 - V_1)}{dt} = 0$$

Node 5 :-

$$\frac{1}{L_1} \int (V_5 - V_1)\, dt + C_0 \frac{d(V_5 - V_6)}{dt} = 0 .$$

Node 6 :-

$$\frac{1}{L_0} \int (V_6 - V_0)\, dt + C_0 \frac{d(V_6 - V_5)}{dt} = 0 .$$