EE240 – Circuits I

Mid Examination (Fall 2021)

November 13, 2021 12:30 pm–02:45 pm

INSTRUCTIONS:

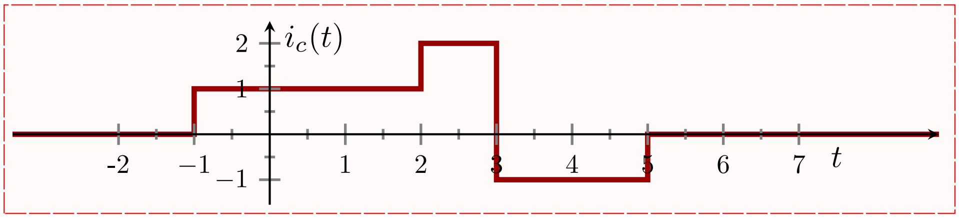
- Reading time: 15 minutes
- Writing time: 2 hours
- We require you to solve the exam in a single time-slot of two hours without any external or electronic assistance.
- We encourage you to solve the exam on A4 paper, use new sheet for each question and write sheet number on every sheet.
- Clearly outline all your steps in order to obtain any partial credit.
- The exam is closed book and notes. You are allowed to have one A4 sheet with you with hand-written notes on both sides. Calculators can be used.
- If you are ready, please proceed to the next page.

Mapping between exam parts and course learning outcomes (CLOs)

- Part 1: R, L, C Basics, Sources and I-V Characteristics (CLO1)
- Part 2: Network Topology, Network Equations and Kirchhoff's Laws (CLO2)
- Part 3: Additional Analysis Techniques (CLO3)

Part 1: R, L, C Basics, Sources and I-V Characteristics

Problem 1. (**10 pts**) The current $i_c(t)$ through the capacitor of capacitance $1F$ is shown in Figure 1 below.



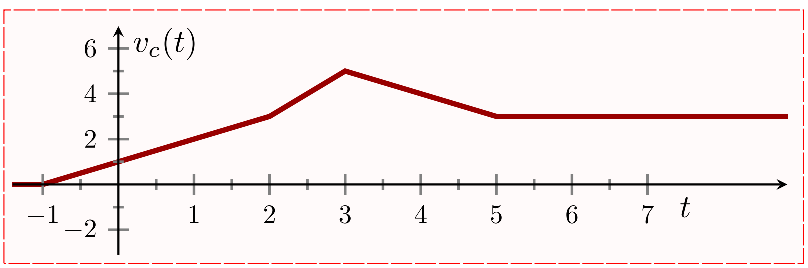
(a) (**1 pts**) Express $i_c(t)$ as piecewise function of time.

$$i_c(t) = \begin{cases} 0 & t < -1 \\ 1 & -1 \le t < 2 \\ 2 & 2 \le t < 3 \\ -1 & 3 \le t < 5 \\ 0 & t \ge 5 \end{cases}$$

(b) (**7 pts**) Assuming that the current is zero for times $t \le -1$ seconds, determine the voltage across the capacitor and **plot** for $-2 \le t \le 7$ seconds.
Let $v_c(t)$ be the voltage across capacitor.

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt$$

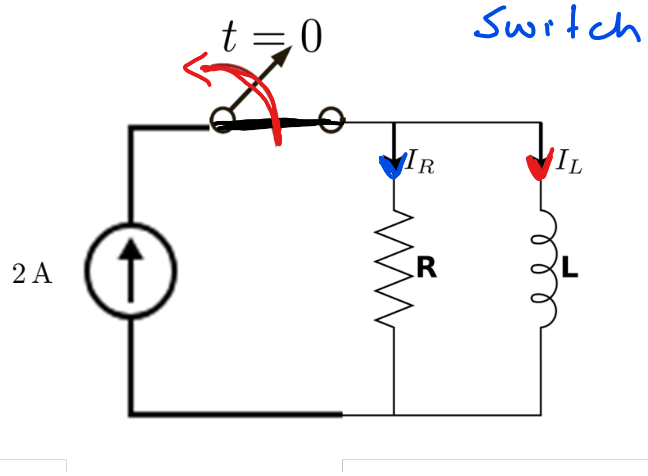
$$v_c(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \le t < 2 \\ 2t-1 & 2 \le t < 3 \\ -t+8 & 3 \le t < 5 \\ 3 & t \ge 5 \end{cases}$$



(c) (**2 pts**) Determine the energy stored in the capacitor at $t = 3$ seconds.
$w_c(t) = \frac{1}{2}C\big(v_c(t)\big)^2 \Rightarrow w_c(3) = \frac{1}{2}\big(v_c(3)\big)^2 = 12.5\,J.$

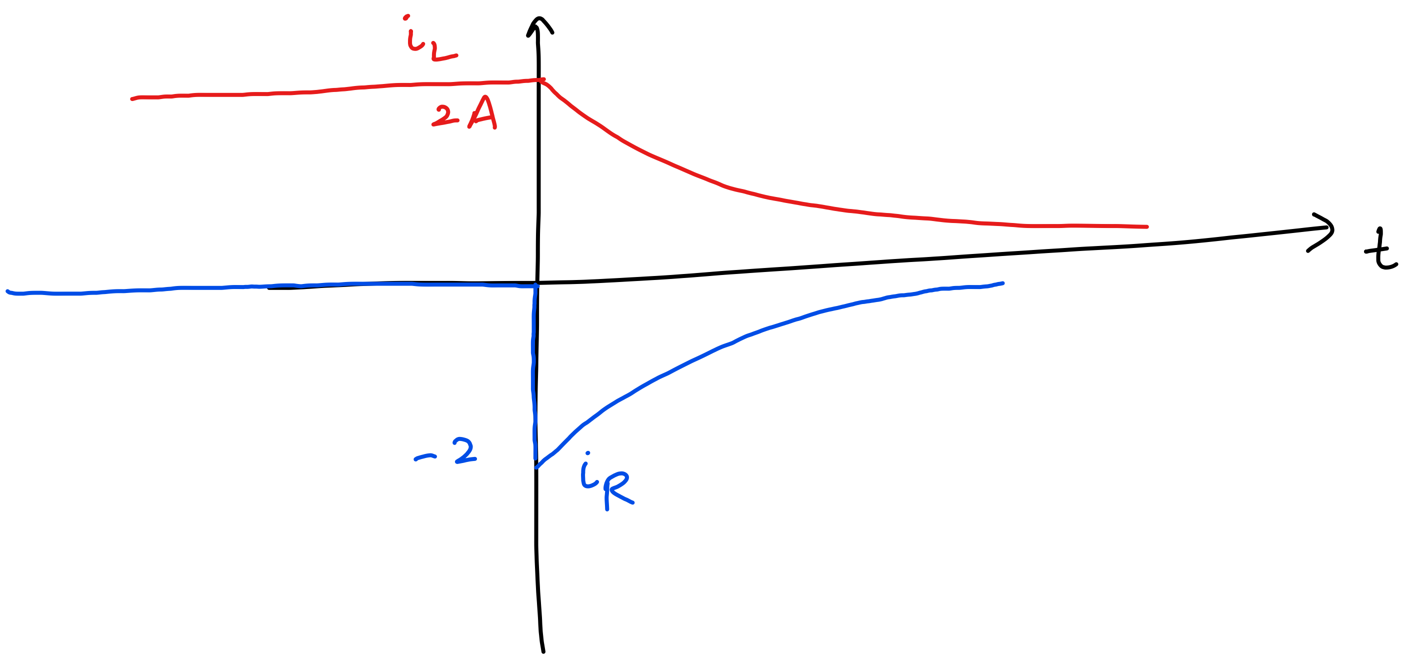
Problem 2. (**8 pts**) Consider a circuit where the DC current source of $2A$ is connected to a parallel combination of $2\ \Omega$ resistor and $0.5H$ inductor through the switch. **Assume that the switch is initially closed and is opened at** $t = 0$.

(a) (**2 pts**) Draw the circuit and indicate the voltage $v(t)$ across the resistor and the current $i_R(t)$ and $i_L(t)$ through the resistor and the inductor respectively.



Switch is opened at $t = 0_-$

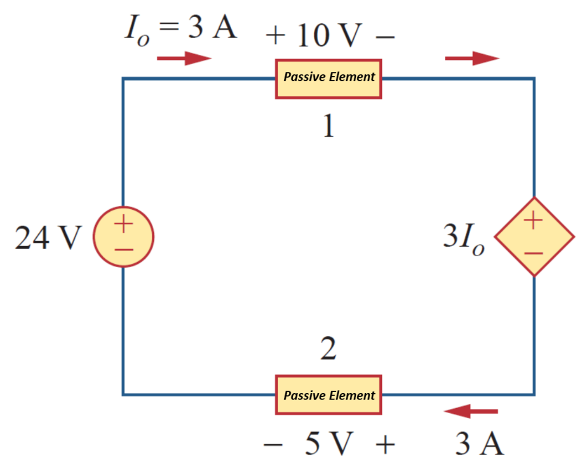
(b) (**6 pts**) Plot the waveforms (not to the scale but label the axes and indicate intercepts) of the currents $i_R(t)$ and $i_L(t)$.



$i_R = 0 \quad t < 0$

$i_R = -i_L \quad t \geq 0$

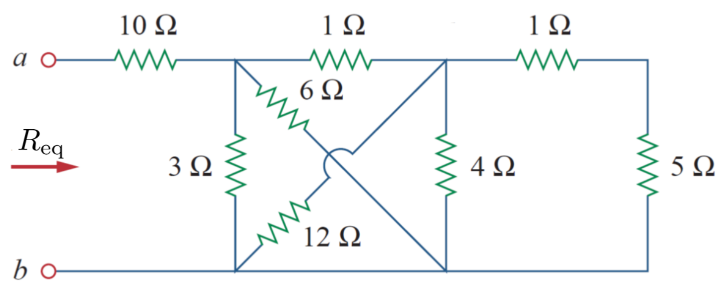
Problem 3. (**4 pts**) Calculate the power **supplied** by each element in the circuit given below.



Solutions:

Power supplied by 24V voltage source: $24 \times 3 = 72\,\mathrm{W}$
Power supplied by Passive element 1: -10 $\times 3 = -30\,\mathrm{W}$
Power supplied by controlled voltage source: -$3I_o \times I_o = -27\,\mathrm{W}$
Power supplied by Passive element 2: -5 $\times 3 = -15\,\mathrm{W}$

Problem 4. (**4 pts**) For a network of resistors, calculate the equivalent resistance R_eq across terminals a and b.

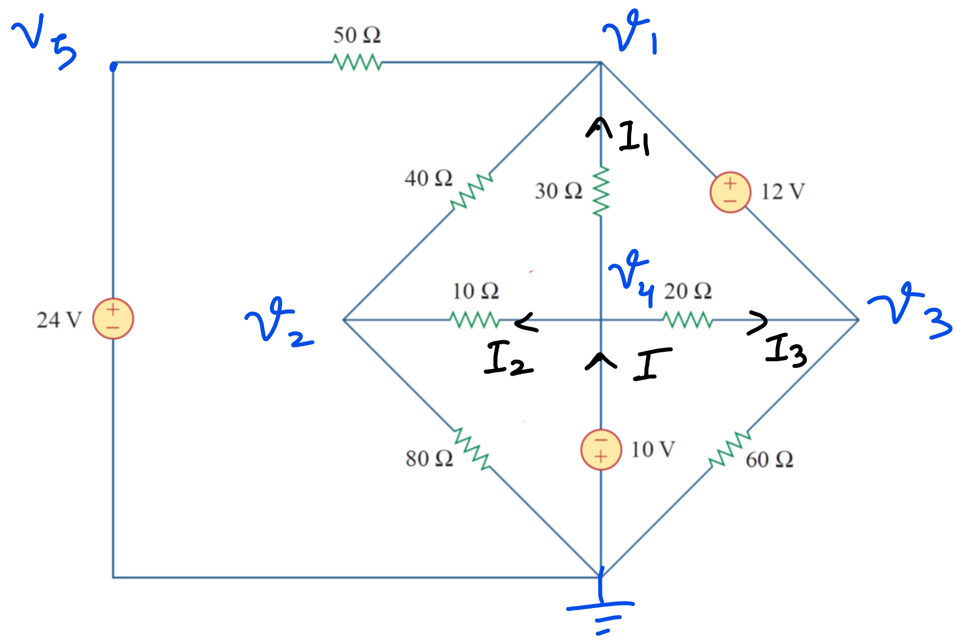


Solutions:

$\big((5+1)||4||12 \quad + \quad 1\big)||6||3 + 10 = 3||6||3 + 10 = \frac{6}{5} + 10 = \frac{56}{5} =$ ~~5.6 Ω~~ 10.2 Ω

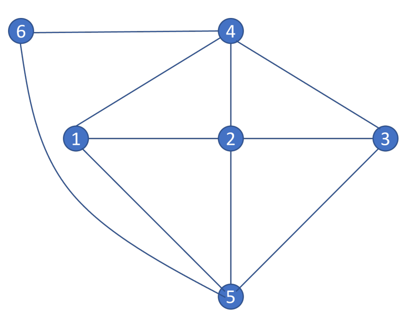
Part 2: Network Topology, Network Equations and Kirchhoff's Laws

Problem 5. (**18 pts**) Consider the circuit given below.

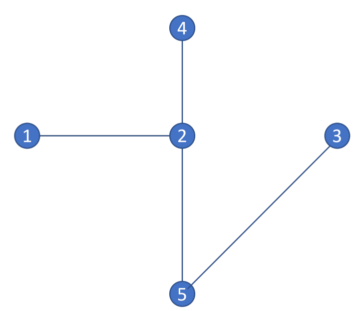


(a) (**4 pts**) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in the circuit (or graph).

Solutions:



Graph



Tree

Number of branches = b = 10
Number of nodes = n =6

(b) (**2 pts**) Determine the number of network equations required for carrying out i) nodal analysis and ii) loop analysis.

Solutions:

For loop analysis, we need $n - 1 = 5$ equations. For nodal analysis, we need $b - n + 1 = 5$ equations.

(c) (**9 pts**) Carry out the nodal analysis, that is, identify and determine the nodal voltages.

Node voltages indicated on circuit

$v_4 = -10V$, $v_5 = 24V$

* Equation for node 2: $11V_2 - 2V_1 = -80$

* Equation for (node 1 and node 3) supernode:

$$47V_1 - 15V_2 + 40V_3 = -212$$

* Supernode equation: $V_1 - V_3 = 12$

Solving for V_1, V_2 and V_3:

$V_1 = 6.9169V$

$V_2 = -6.0151V$

$V_3 = -5.0831V$

(d) (**3 pts**) Determine the power delivered by the 10 V voltage source.

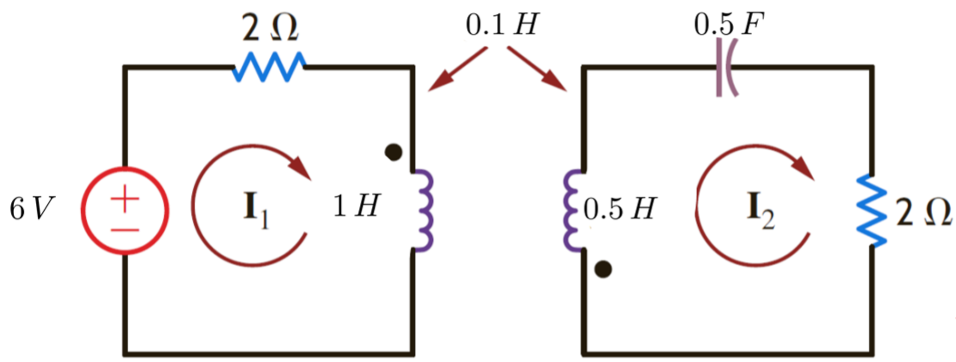
* In order to find power, we determine the current I (indicated on the circuit).

$$I = I_1 + I_2 + I_3 = \frac{-10 - V_1}{30} + \frac{-10 - V_2}{10} + \frac{-10 - V_3}{20}$$

* Since I is leaving -ve terminal

Power delivered = $(10)(-I)$

Problem 6. (**06 pts**) Formulate the network equations in terms of loop currents indicated in the circuit given below.



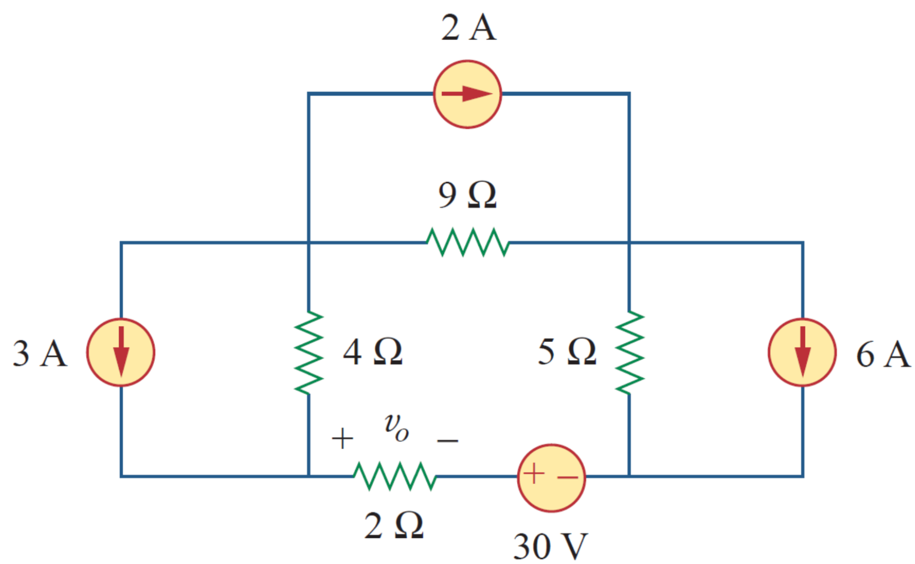
coupling terms

$$* \quad 2I_1 + L\frac{dI_1}{dt} + 0.1\frac{dI_2}{dt} = 6$$

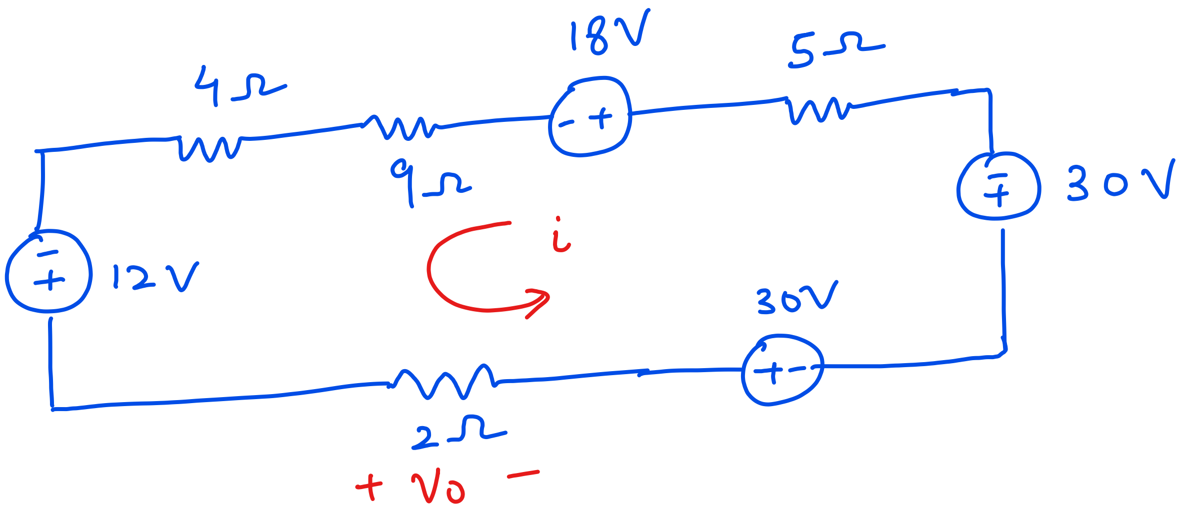
$$* \quad 2\int I_2\,dt + 2I_2 + \frac{1}{2}\frac{dI_2}{dt} + 0.1\frac{dI_1}{dt} = 0$$

Part 3: Additional Analysis Techniques

Problem 7. (**06 pts**) Use source transformation to find v_o in the circuit given below.



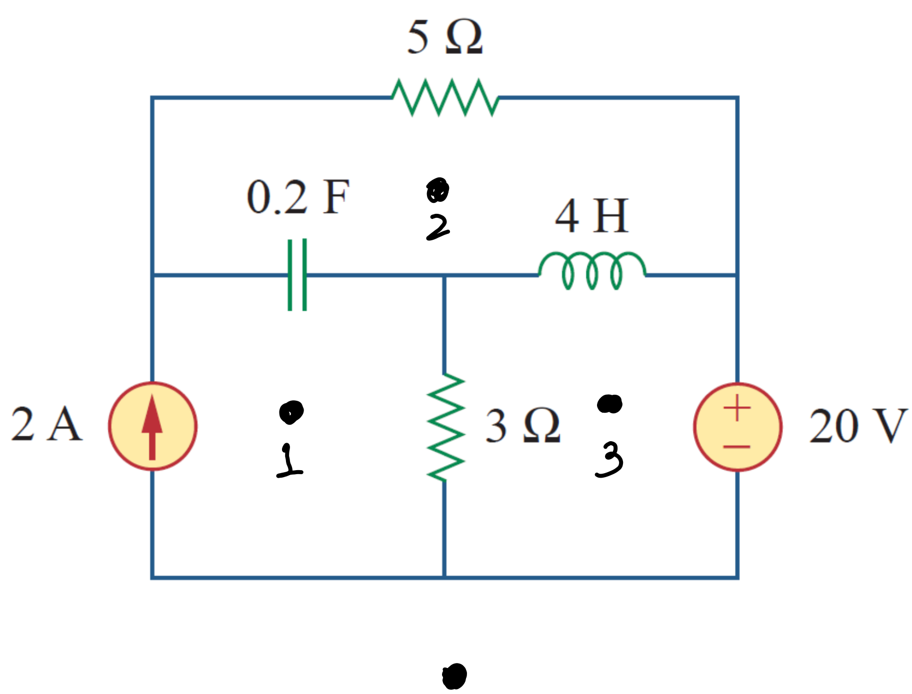
Source transformation yields

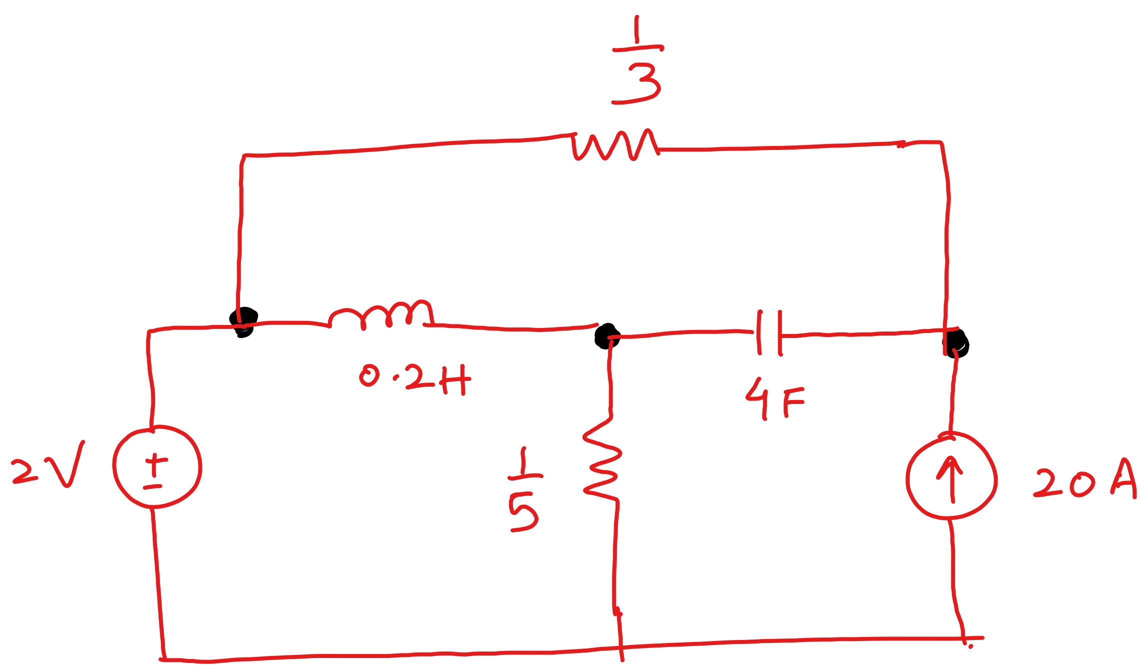


$$20i + 18 + 30 + 30 - 12 = 0 \Rightarrow i = -\frac{66}{20} = -3.3\,A$$

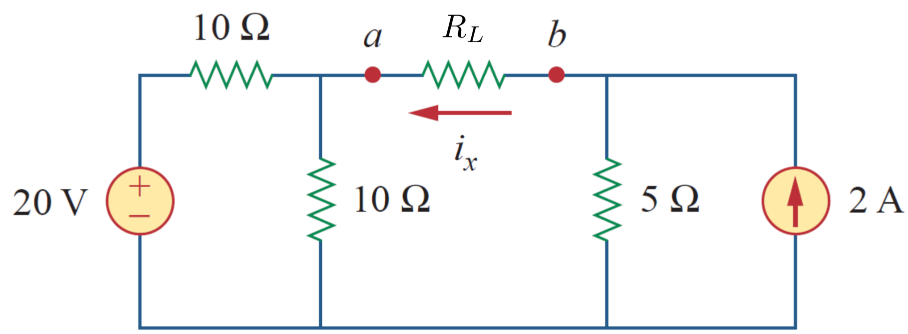
$$V_o = (-3.3)(2) = \boxed{-6.6V}$$

Problem 8. (**06 pts**) Obtain a dual circuit for the circuit given below.





Problem 9. (**10 pts**) Consider a circuit given below.



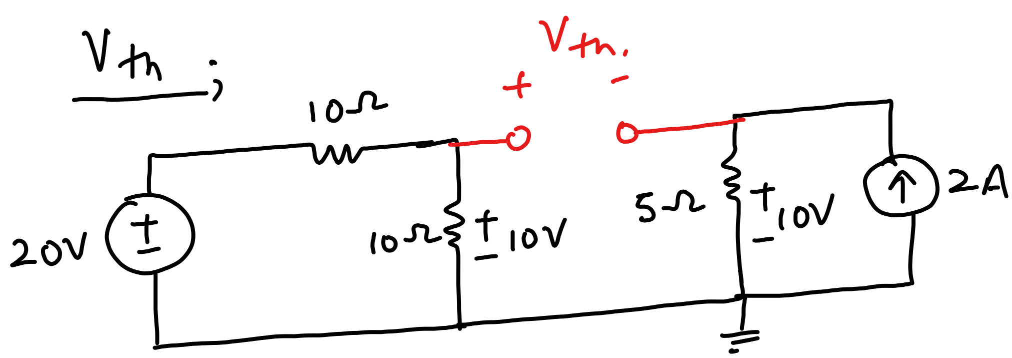
(a) (**4 pts**) Determine the value of R_L for maximum power transfer to R_L using Thevenin's Theorem.

(b) (**4 pts**) Obtain Thevenin equivalent of the circuit across terminals a and b.

(c) (**2 pts**) Determine the power absorbed by R_L

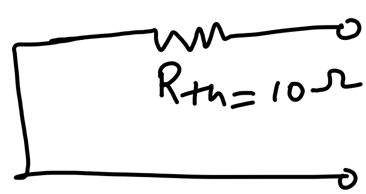
(a) $R_{th} = 10 \| 10 + 5 = 10\,\Omega$, $R_L = R_{th}$

(b) $\underline{V_{th}}$;



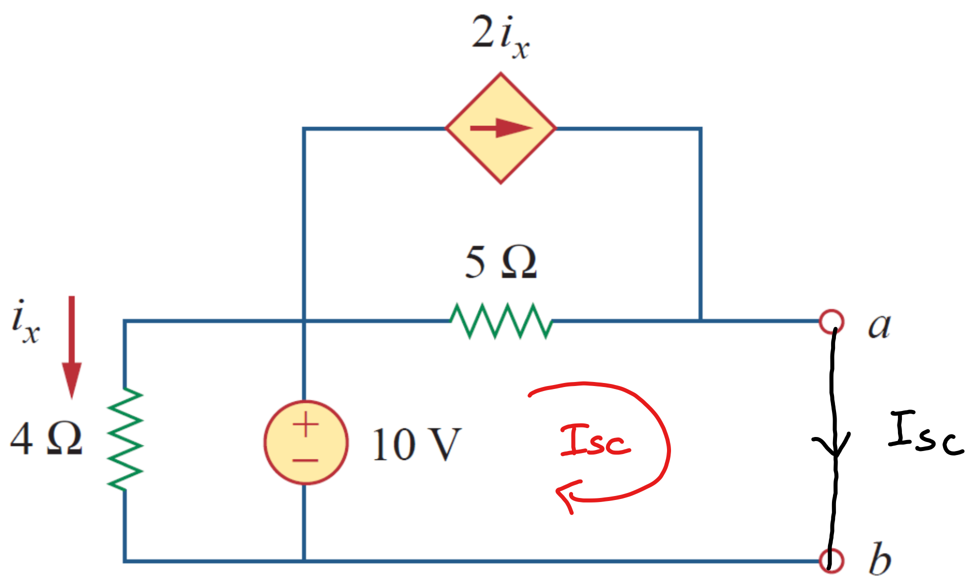
$$V_{th} = 0$$

Thevenin Equivalent Circuit:



(c) $P = 0$ since $V_{th} = 0$.

Problem 10. (**08 pts**) Find the Norton equivalent circuit of the circuit given below



$\underline{I_{sc}}$:

* $i_x = \frac{10}{4} = 2.5A$

$5(I_{sc} - 2i_x) - 10 = 0$

$I_{sc} = \frac{10 + 10i_x}{5} = \frac{35}{5} = 7A$

$\underline{V_{oc}}$:- (V_{th}) * open circuit voltage across a and b

$i_x = 2.5A$

$V_{oc} = (5)(2i_x) + 10 = 35V$

$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{35}{7} = \boxed{5\Omega}$

Norton Equivalent: