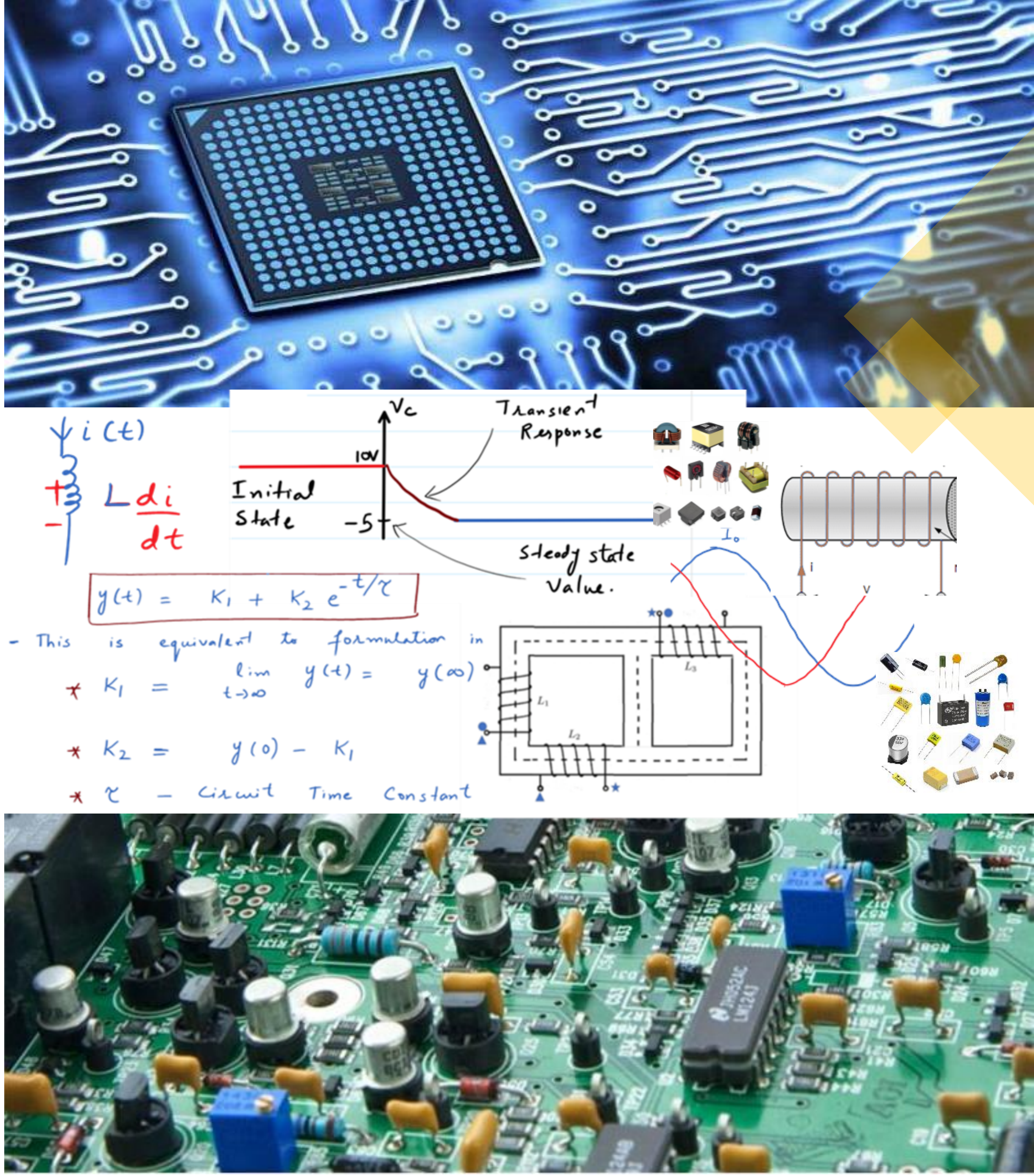
EE 240 Circuits I

Dr. Zubair Khalid

Department of Electrical Engineering
School of Science and Engineering
Lahore University of Management Sciences

https://www.zubairkhalid.org/ee240_2021.html

- Thevenin’s Theorem and Norton’s Theorem
- Equivalent Circuit



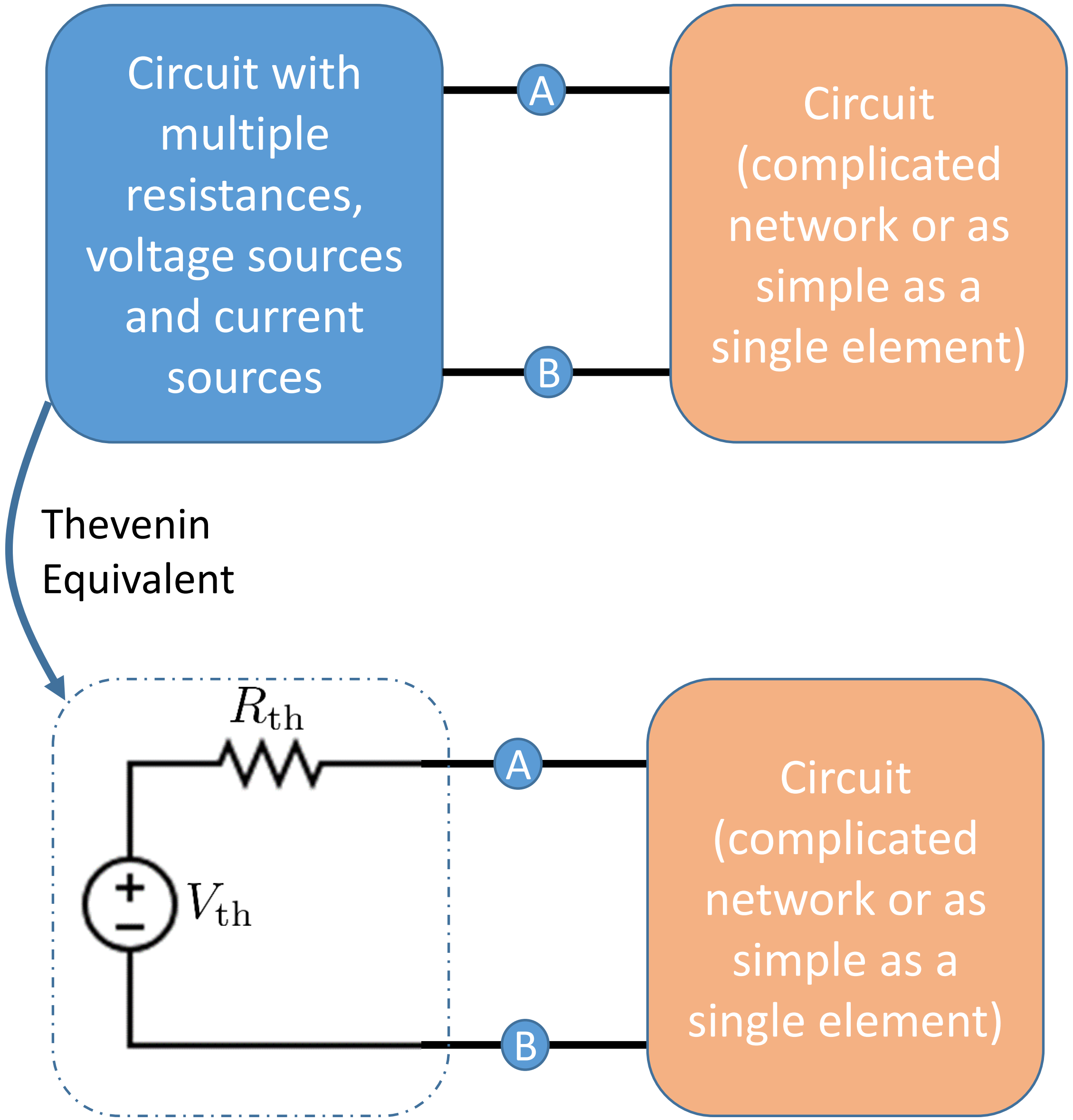
Thevenin's Theorem

Overview

Thevenin theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single voltage source, referred to as Thevenin voltage V_{th} in series with a single resistance, referred to as Thevenin Resistance R_{th}.

Thevenin Theorem Statement:

Any circuit containing only resistances, voltage sources, and current sources be replaced at the terminals A-B by an equivalent combination of a voltage source V_{th} in a series connection with a resistance R_{th}.



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Key Idea: Use the concept of equivalence: same current and voltage characteristics across terminals A-B.

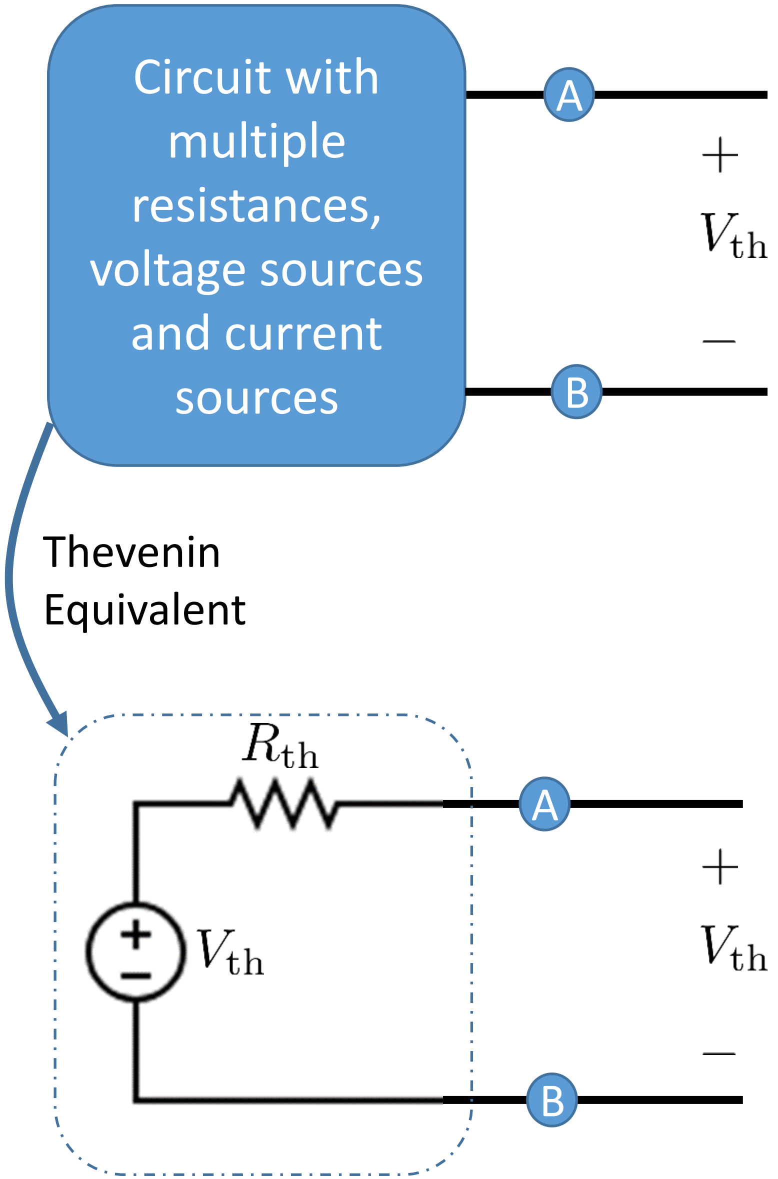
Only keep the circuit for which we want to find the equivalent circuit; disconnect the rest of the circuit.

Determine V_{th}:

As no current is flowing from A to B,
V_{th} is simply a voltage across terminals A-B.

We can determine V_{th} by analysisng the circuit inside blue box and determine the voltage across terminals A-B.

We will learn different methods to obtain Thevenin Resistance.



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Method 1:

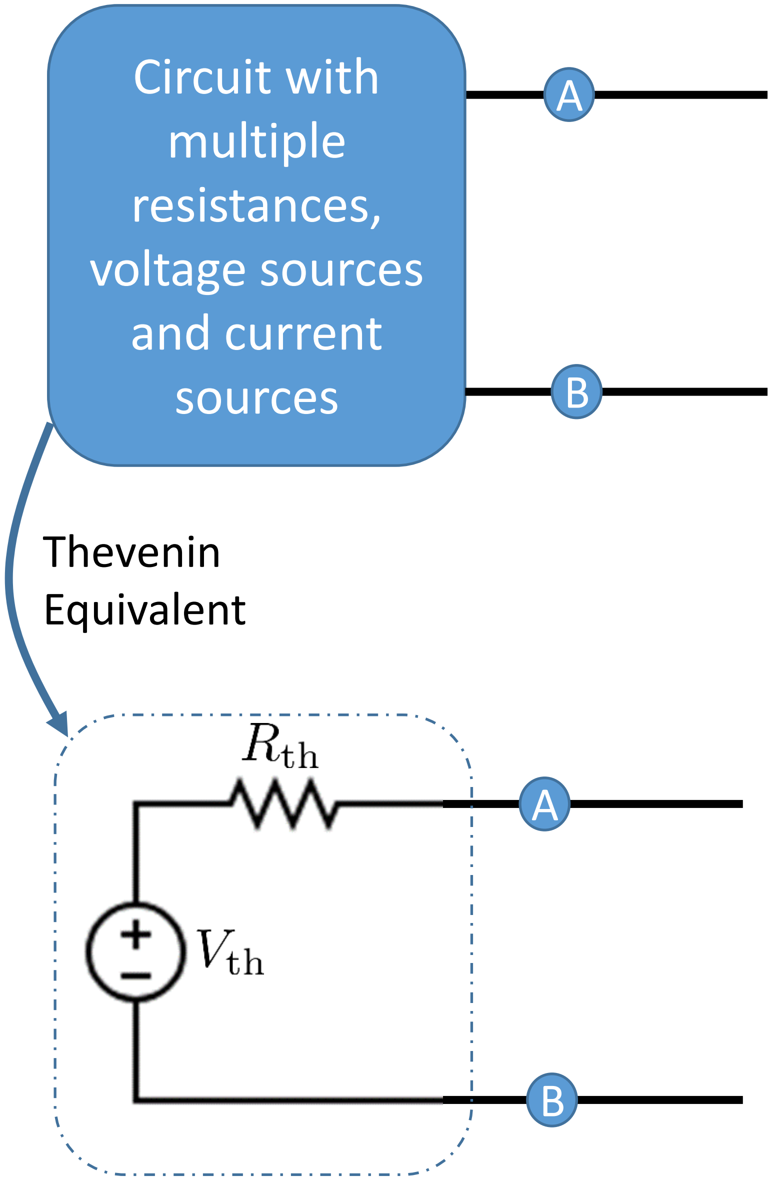
- Switch off the independent sources and determine the equivalent resistance across terminals A and B.

Method 2:

- Short-circuit terminals A and B and determine the current flowing from A to B, referred to as I_{SC}.

- Using this current, we can determine R_{th} as

$$R_{\text{th}} = \frac{V_{\text{th}}}{I_{\text{sc}}}$$



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Method 3:

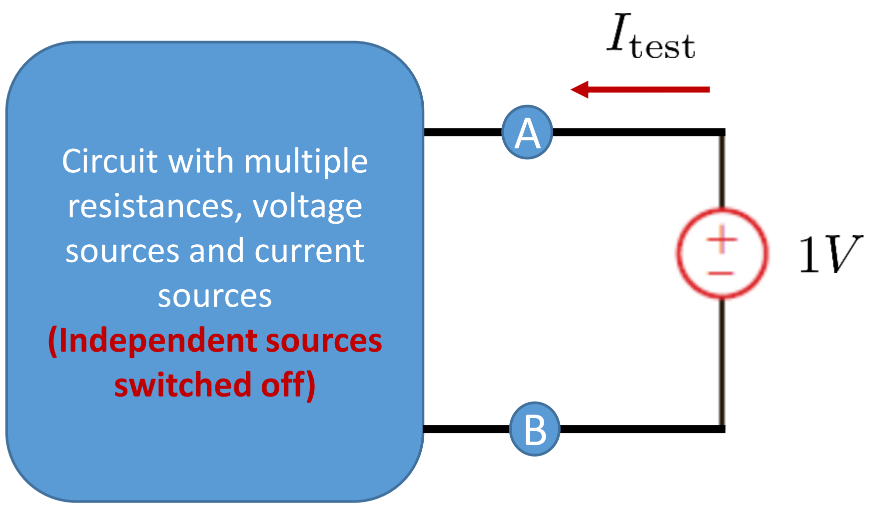
- Switch off the independent sources
- Connect a test source across terminals A-B

- **If 1V (known) voltage test-source is connected:**

  Determine the current I_{test} supplied by voltage source.

  We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{1}{I_{\text{test}}}.$$

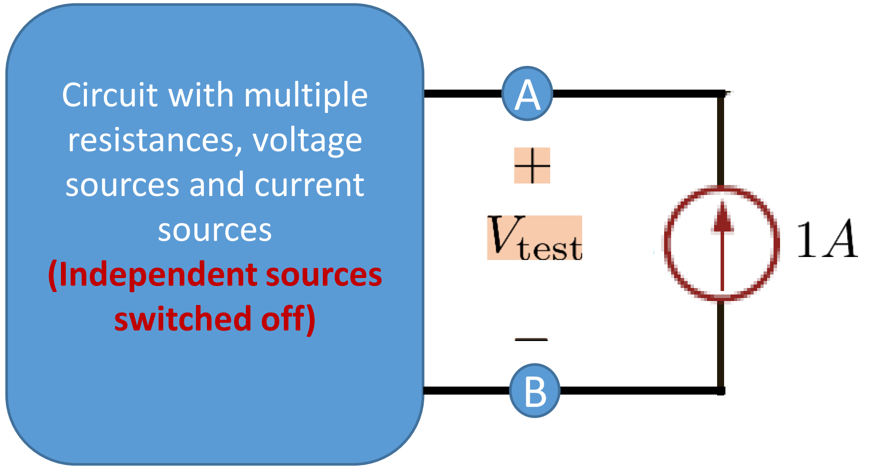


- **If 1A (known) current test-source is connected:**

  Determine the voltage V_{test} developed across the current source.

  We can determine R_{th} as follows:

$$R_{\text{th}} = \frac{V_{\text{test}}}{1}.$$



Thevenin's Theorem

How to Obtain Thevenin Equivalent

Determine R_{th}:

Which method to use?

Independent sources	Dependent sources	Method – can be used	Justification
✓	✗	Methods 1, 2 and 3	
✓	✓	Methods 2 and 3	Method 1 cannot be used due to the presence of dependent sources
✗	✓	Method 3	No independent source driving V_{th} or I_{SC}
✗	✗	Methods 1 and 3	No independent source driving V_{th} or I_{SC}

Note: Equivalent circuit does not have the voltage source if there is no independent source in the circuit.

Norton's Theorem

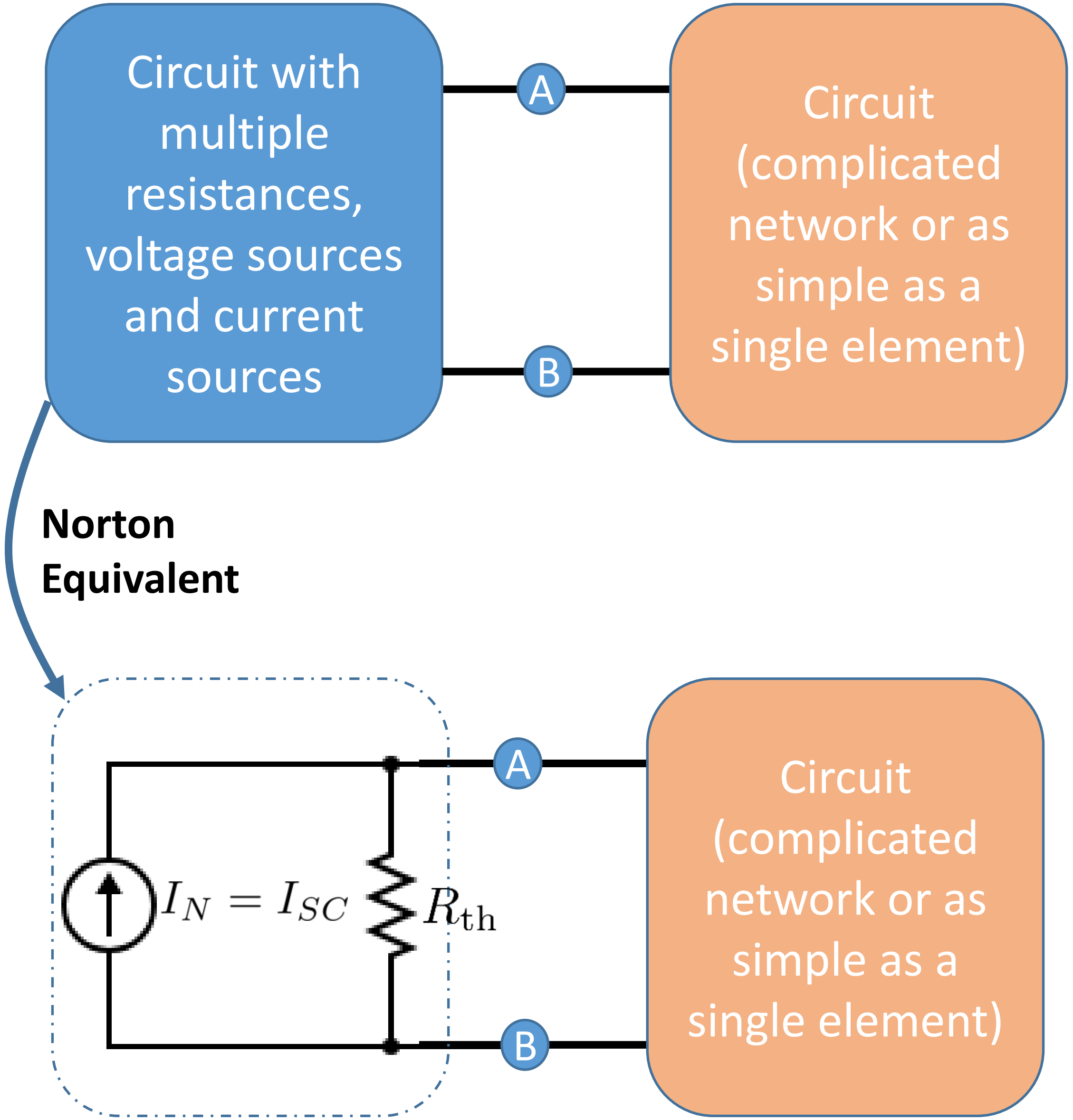
Norton theorem is used to change a complicated circuit into a simple equivalent circuit consisting of a single current source, referred to as Norton current voltage I_N in parallel with a single resistance (the same as Thevenin Resistance R_{th}, explained below).

- The value of the Norton current is one that flows from terminal A to B when the two terminals are shorted together. This is in fact I_{sc}, that is short-circuit current.

- The resistance represents the resistance looking back into the terminals when source is switched off. This is in fact Thevenin Resistance.

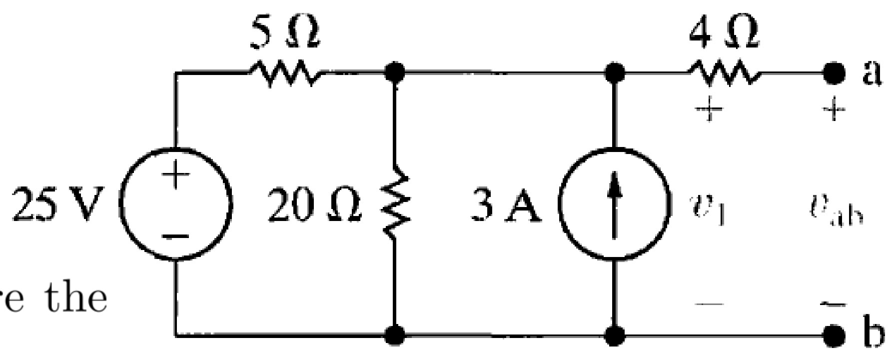
Connection with the Thevenin's Theorem

The source transformation of Thevenin's equivalent yields Norton's equivalent and vice versa.



Thevenin's Theorem

Example 1: Find the Thevenin's equivalent circuit for the following circuit across terminals a-b.

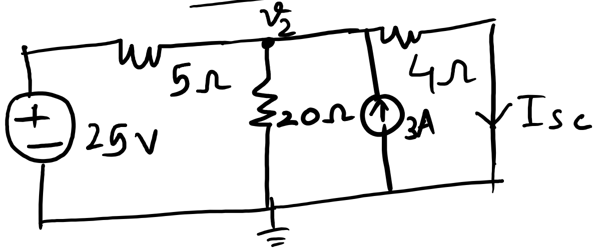


Taking bottom node as ground node and applying KCL at a node where the voltage is v_1

$$* \quad \frac{v_1 - 25}{5} + \frac{v_1}{20} = 3$$

$$\frac{v_1}{5} + \frac{v_1}{20} = 8 \Rightarrow \boxed{v_1 = 32V}$$

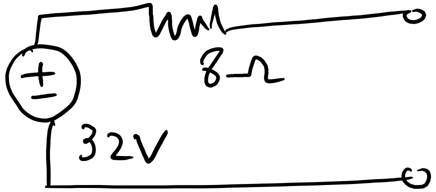
R_{th} : Find I_{sc} :



$$-3 + \frac{v_2 - 25}{5} + \frac{v_2}{20} + \frac{v_2}{4} = 0$$

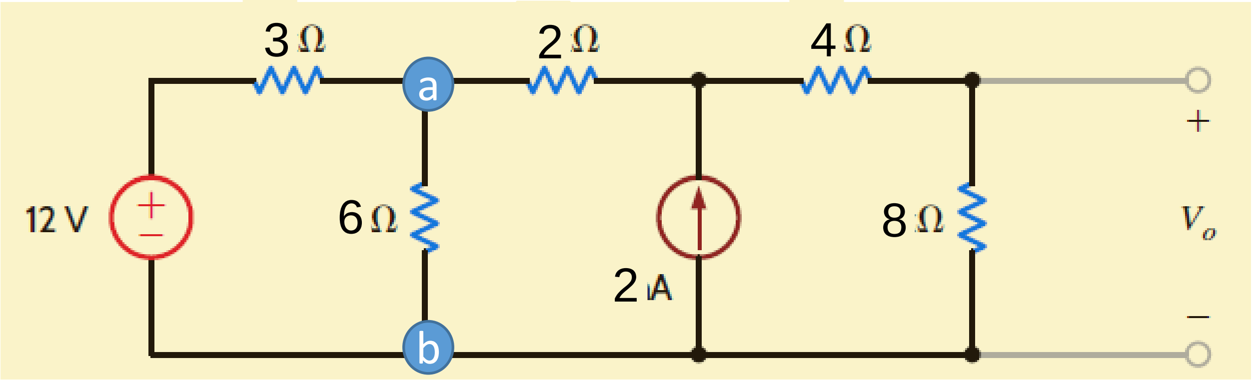
$$v_2 = 16V \Rightarrow \boxed{I_{sc} = 4A}$$

$$R_{th} = \frac{v_{th}}{I_{sc}} = 8\Omega$$



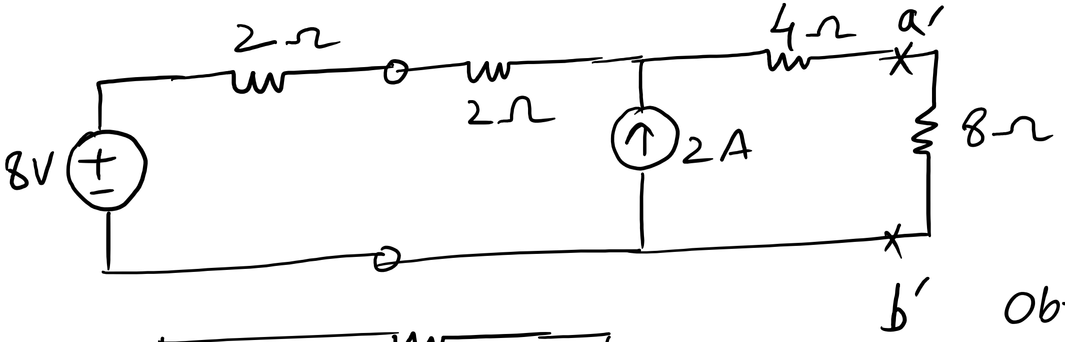
Thevenin’s Theorem

Example 2: Find V_o using Thevenin’s theorem.

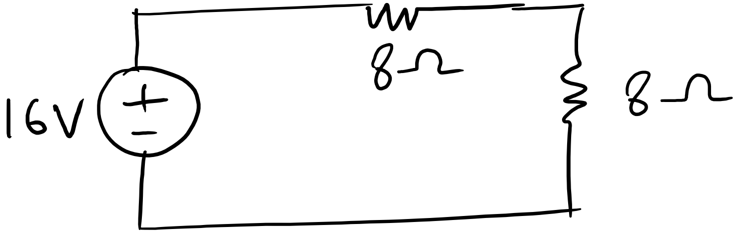


We will use Thevenin’s theorem twice here.

Obtaining Thevenin equivalent across terminals a-b:



Obtaining Thevenin equivalent across terminals a’-b’:

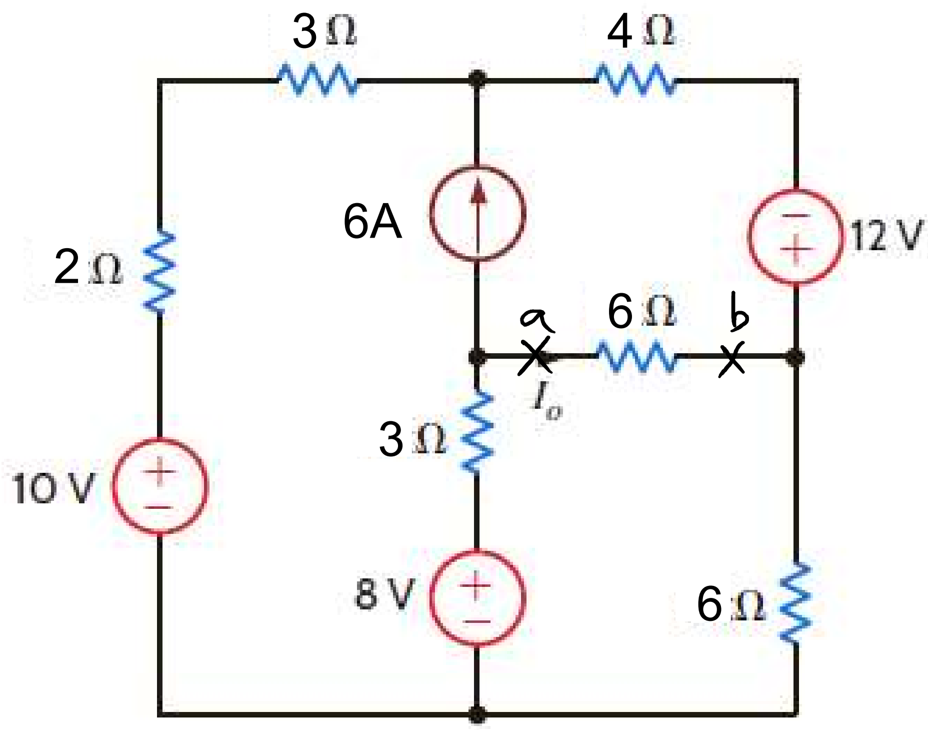


$$V_o = 16 \times \frac{8}{16} = \boxed{8V}$$

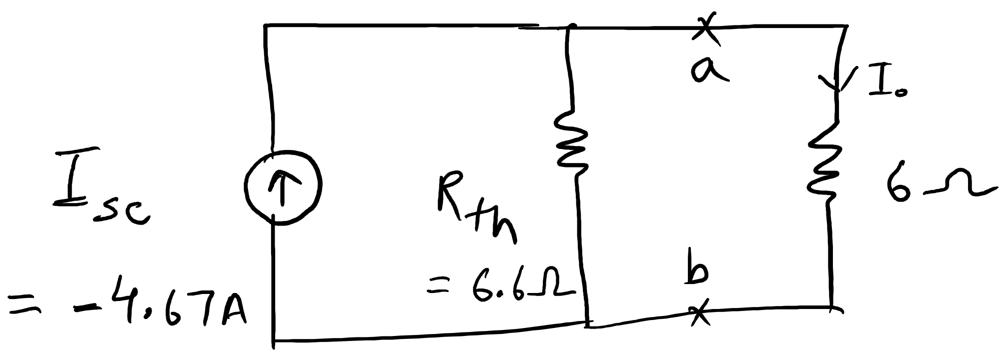
Thevenin's and Norton's Theorems

Problems – In class

Example 3: Find I_o using Thevenin's or Norton's theorem



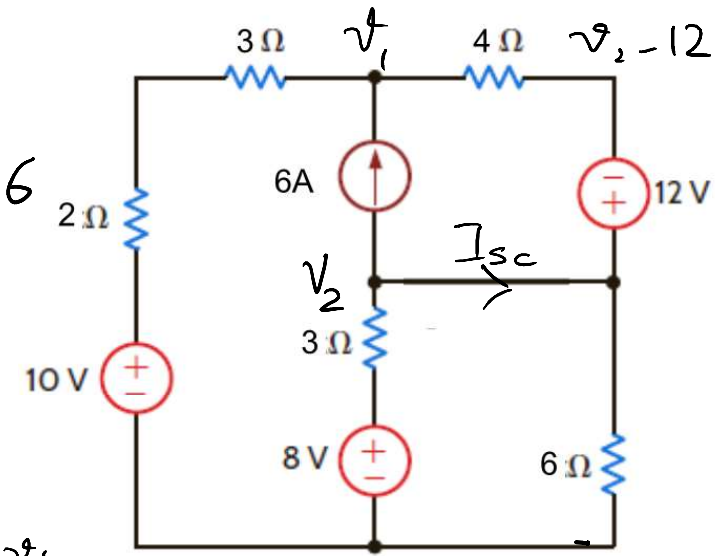
Equivalent: (computed on next page)



$$I_o = \frac{6.6}{12.6} \times (-4.67) = -2.44A$$

Thevenin's and Norton's Theorems

Problems – In class



$$\frac{v_1 - 10}{5} + \frac{v_1 - (v_2 - 12)}{4} = 6$$

$$\boxed{9v_1 - 5v_2 = 100}$$

$$\frac{v_2 - 8}{3} + \frac{v_2}{6} + \frac{v_2 - 12 - v_1}{4} = -6$$

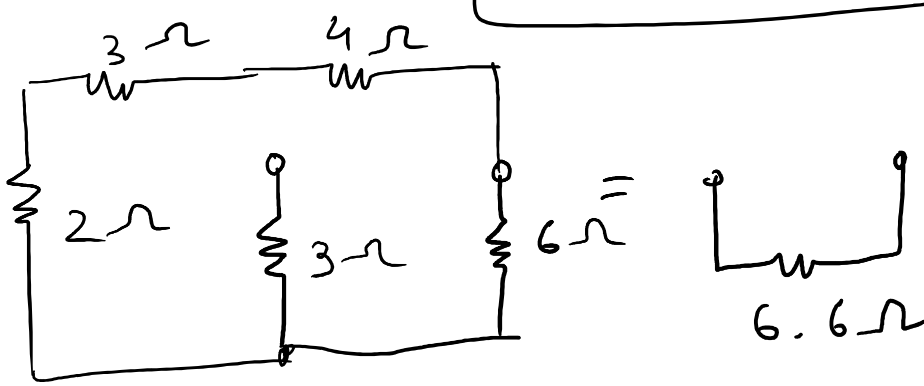
$$\Rightarrow \frac{4v_2 - 24 + 2v_2 + 3v_2 - 36 - 3v_1}{12} = -6$$

$$\boxed{-3v_1 + 9v_2 = -4} \Rightarrow -9v_1 + 27v_2 = -12$$

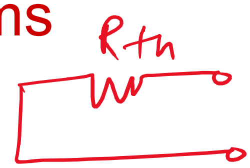
$$\Rightarrow 22v_2 = 88 \Rightarrow \boxed{v_2 = 4V}$$

$$\frac{v_2 - 8}{3} + 6 + I_{sc} = 0 \Rightarrow \boxed{I_{sc} = -4.667\,A}$$

R_{th}:

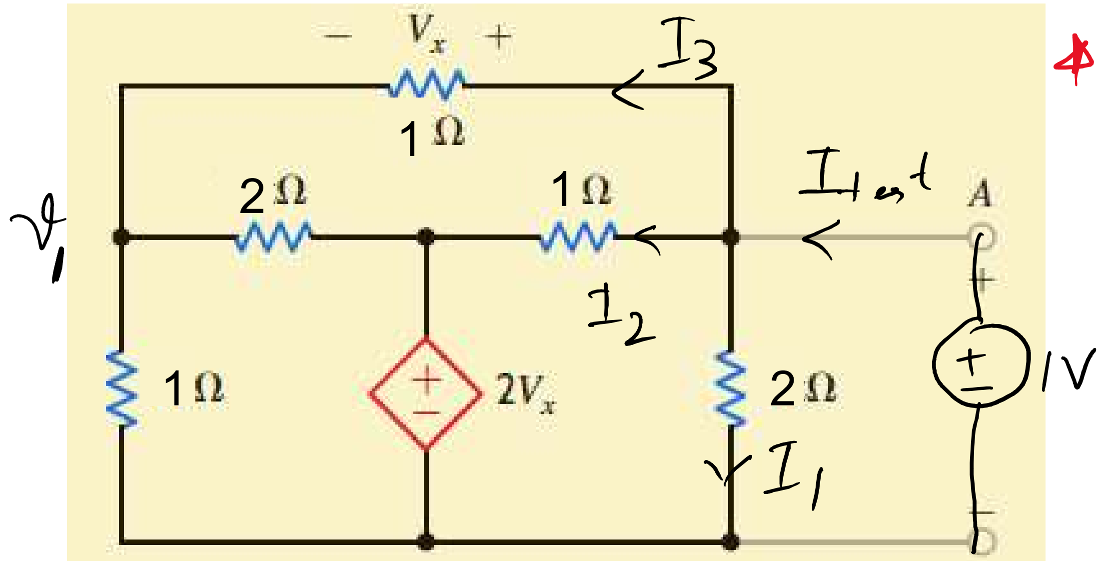


Thevenin's and Norton's Theorems



Problems – In class

Example 4: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – Example 5.8)



$V_{th} = 0V$

We have

$$v_x + v_1 = 1V \Rightarrow v_1 = 1 - v_x$$

KCL ;

$$\frac{v_1}{1} + \frac{v_1 - 2v_x}{2} + \frac{v_1 - 1}{} = 0$$

$$\Rightarrow 1 - v_x + \frac{1 - 3v_x}{2} - v_x = 0$$

$$\Rightarrow 3 = 7v_x \Rightarrow \boxed{v_x = \frac{3}{7} V}$$

$$I_1 = \frac{1}{2} A, \quad I_2 = \frac{1 - 2(3/7)}{1} = 1/7 A$$

$$I_3 = 3/7 A$$

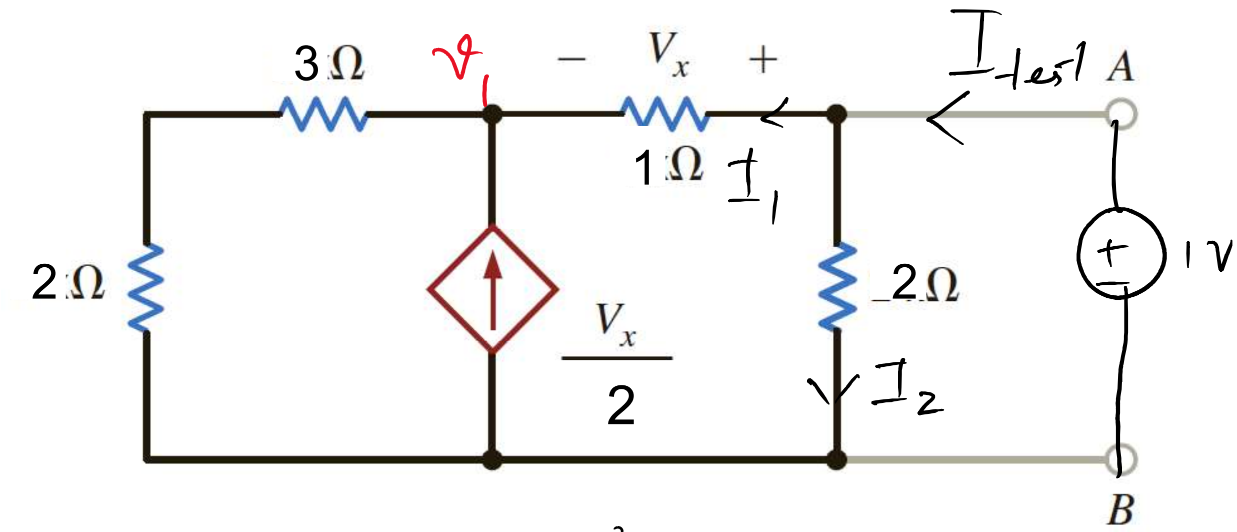
$$I_{test} = I_1 + I_2 + I_3 = \boxed{\frac{15}{14} A}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{14}{15} \Omega$$

Thevenin's and Norton's Theorems

Problems – In class

Example 5: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals AB (Irwin – E 5.13)



$$\frac{v_1}{5} + \frac{v_1 - 1}{1} = \frac{v_x}{2}$$

$$v_x + v_1 = 1 \Rightarrow v_x = 1 - v_1$$

$$\Rightarrow \boxed{v_1 = \frac{15}{17}\ V}$$

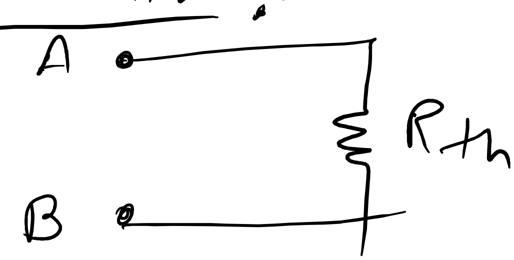
$$I_{test} = I_1 + I_2 = \frac{1}{2} + \frac{2}{17} = \frac{21}{34}\ A$$

$$I_1 = \frac{1 - v_1}{1} = \frac{2}{17}\ A$$

$$I_2 = \frac{1}{2}$$

$$\Rightarrow R_{th} = \frac{1}{I_{test}} = \frac{34}{21}\ \Omega$$

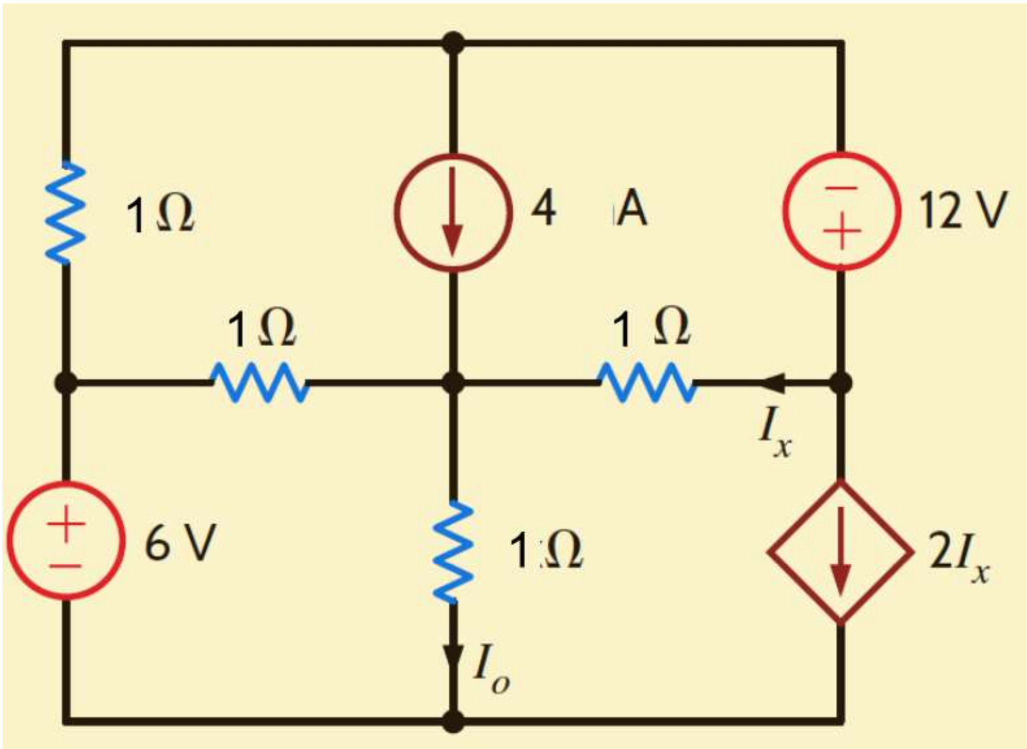
Equivalent :-



Thevenin’s and Norton's Theorems

Problems – In class

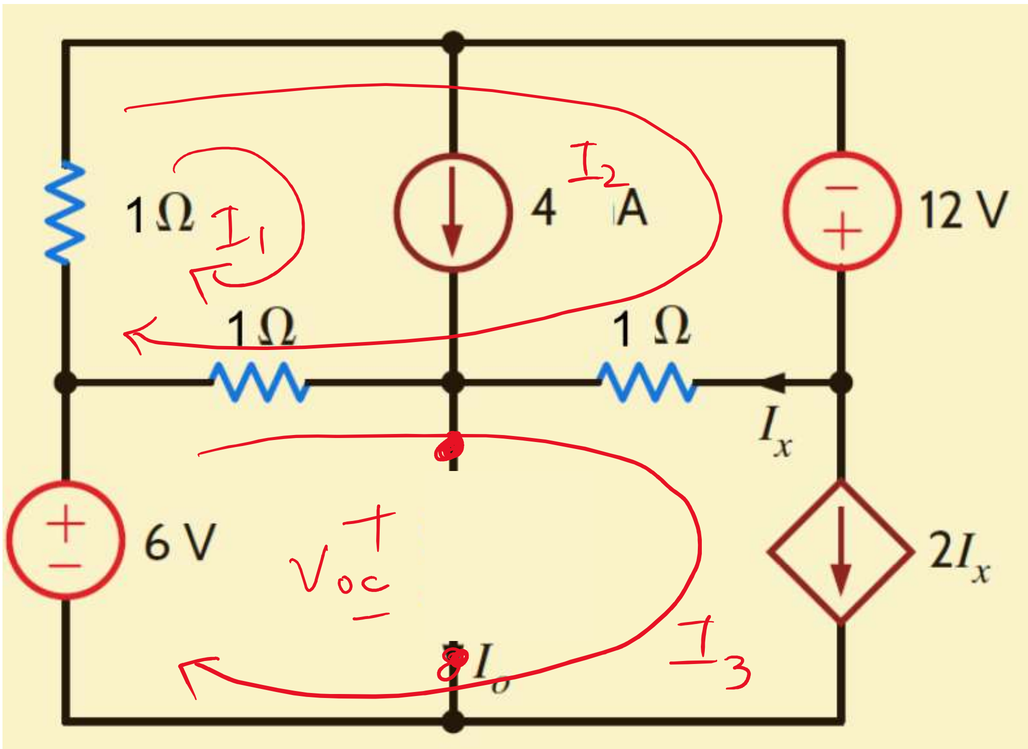
Example 6: Find I_o using Thevenin’s theorem or Norton’s theorem



Thevenin's and Norton's Theorems

Problems – In class

Determine V_{oc} first:



$I_1 = 4A, \quad I_3 = 2I_x$

Loop 2 $\quad -12 + 1(I_2 - I_3) + 1(I_1 + I_2 - I_3) + (I_1 + I_2)1 = 0$

$I_x = I_2 - I_3 \Rightarrow \boxed{3I_x = I_2}$

Solving

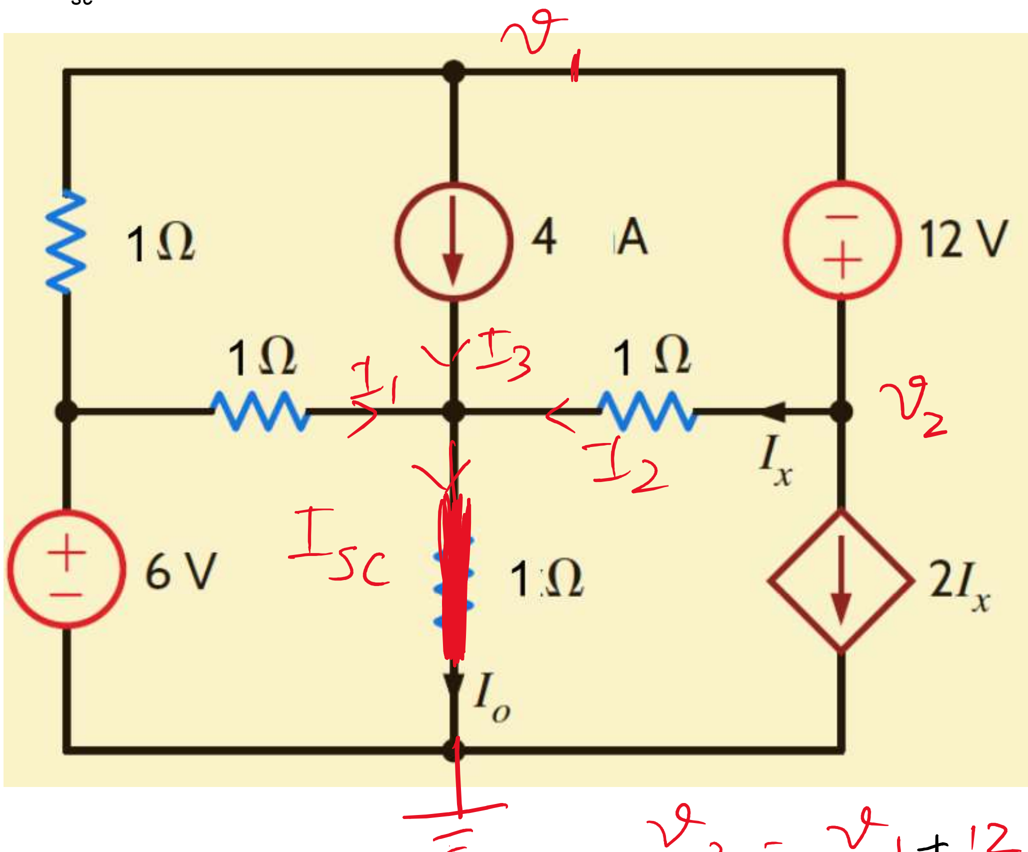
$I_3 = \frac{8}{5}A, \quad I_2 = \frac{12}{5}A, \quad I_x = \frac{4}{5}A$

$V_{oc} = 6 + (I_1 + I_2 - I_3)1 = \frac{54}{5}V$

Thevenin's and Norton's Theorems

Problems – In class

Determine I_{sc}



$v_2 = v_1 + 12$ (Super Node)

$$\frac{v_1 - 6}{1} + 4 + \frac{v_2}{1} + 2I_x = 0 \qquad I_x = \frac{v_2}{1}$$

$$\Rightarrow v_1 - 6 + 4 + 3v_1 + 36 = 0$$

$$\Rightarrow v_1 = -\frac{17}{2}\,V, \quad v_2 = 7/2$$

$$I_{sc} = \frac{6}{1} + \frac{7}{2} + 4 = \frac{27}{2}\ A$$

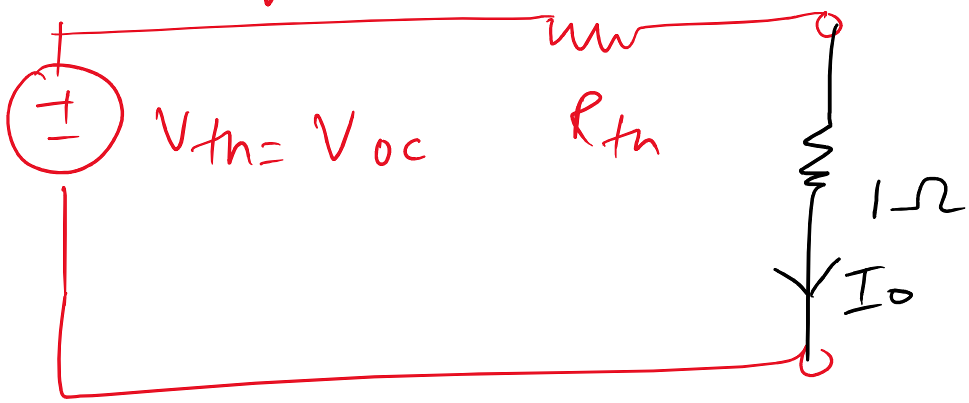
$$\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{54/5}{27/2} = \frac{4}{5}\,\Omega$$

Thevenin's and Norton's Theorems

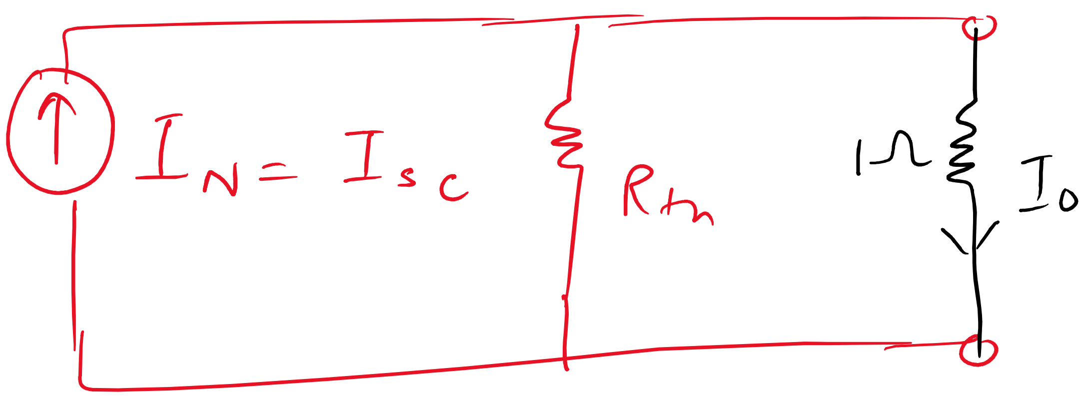
Problems – In class

Using equivalent circuits to determine I_o

Thevenin Equivalent



Norton Equivalent

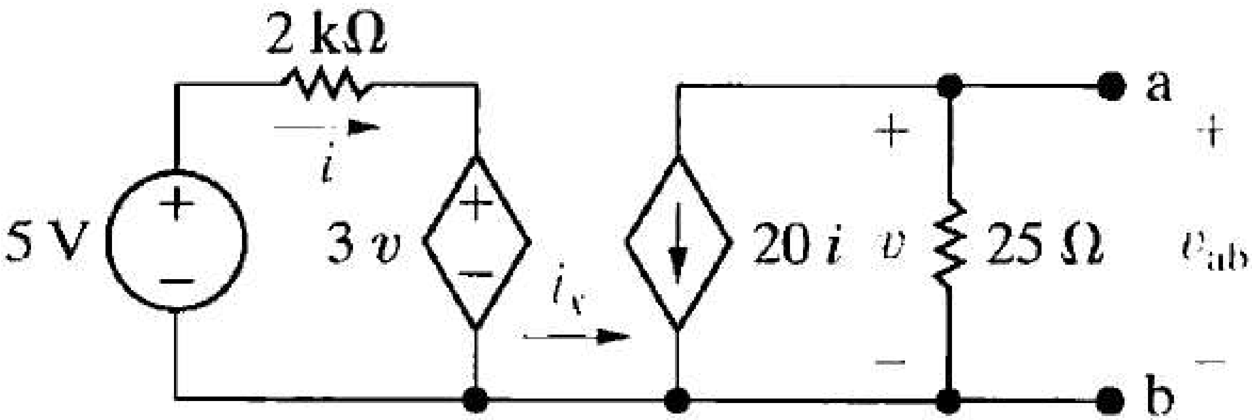


$$I_o = \frac{54/5}{1 + \frac{4}{5}} = \boxed{6\ A}$$

Thevenin's and Norton's Theorems

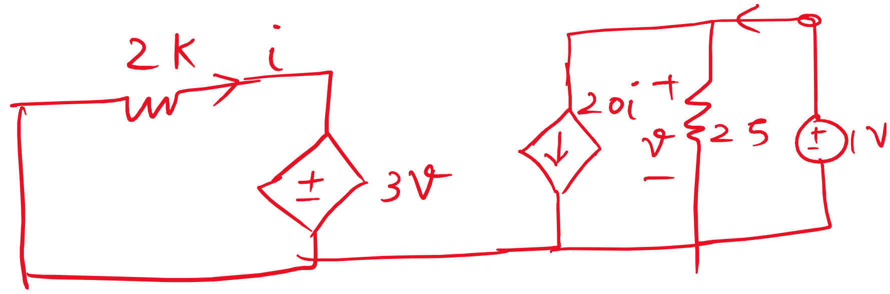
Problems – In class

Example 7: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



First find R_{th} :-

switch off sources and connect 1V test source



$v = 1V$

$3v = 3V$

$i = -3/2k$

$\Rightarrow \quad I_{test} = \frac{1}{25} - (20)(i) = 0.04 - 0.03 = 0.01A$

$\Rightarrow \quad \boxed{R_{th} = 100\Omega} \leftarrow \frac{1}{I_{sc}}$

V_{th} :- $V_{ab} = v = v_{th}$

$V_{th} = -20i \times 25$ — ①

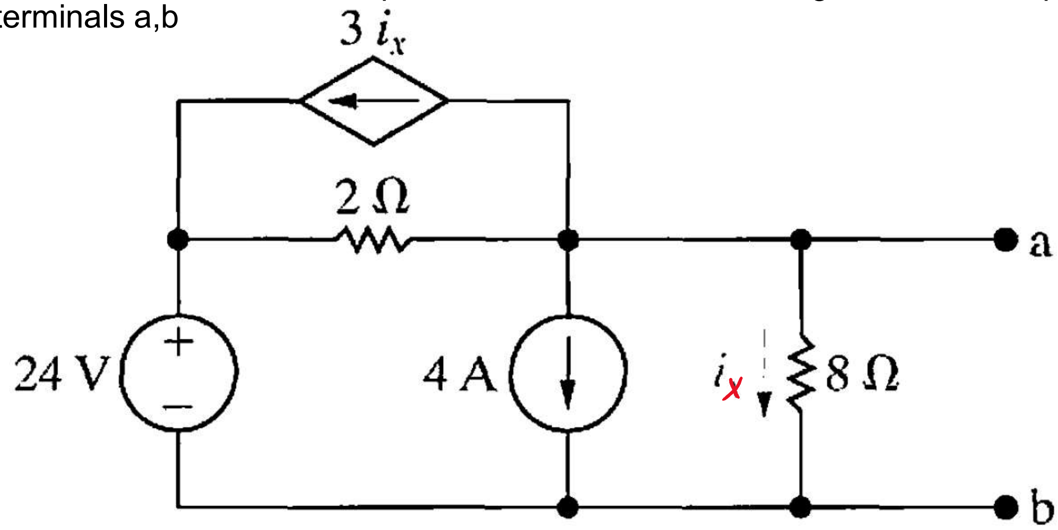
$i = \frac{5 - 3V_{th}}{2k}$ — ②

① + ② $\Rightarrow$ $\boxed{V_{th} = -5V}$

Thevenin's and Norton's Theorems

Problems – In class

Example 8: Find the Thevenin equivalent circuit for the following circuit with respect to the terminals a,b



* Circuit contains both dependent and Independt sources; we can use either of the following techniques

1) Determine V_{ab} and I_{sc}
2) Determine V_{ab} ; Determine R_{Th} by switching off independent sources and applying test current (or voltage) source at a–b.

Let's apply 2)

V_{ab} : Using KCL;

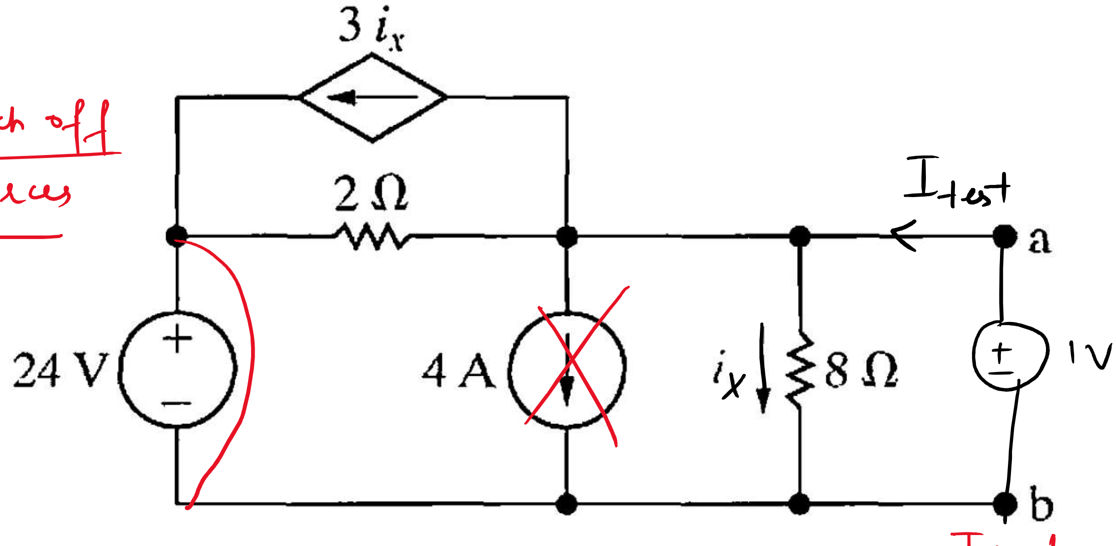
$$\frac{V_{ab}}{8} + \frac{V_{ab}-24}{2} + 3\,i_x + 4 = 0$$

where $i_x = \dfrac{V_{ab}}{8} \Rightarrow \boxed{V_{ab} = 8\,i_x}$

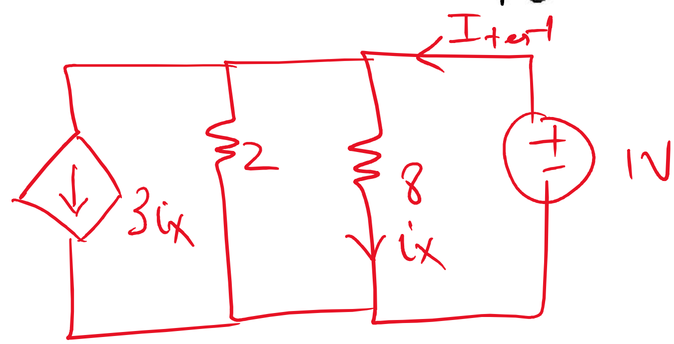
Solving $\boxed{V_{ab} = 8V}$

R_{th} : Apply 1V voltage source :

Switch off Sources



$i_x = \frac{1}{8}$ A



$I_{test} = \frac{1}{8} + \frac{1}{2} + \frac{3}{8}$

$= 8/8 = 1A$

$\Rightarrow R_{th} = \frac{1V}{I_{test}} = \boxed{1\,\Omega}$