EE240 HW1-Solution

Problem 1:

(a):

Circuit fig.(a): V = -20 V, I = - 4 A.
Circuit fig.(b): V = -20 V, I = 4 A.
Circuit fig.(c): V = 20 V, I = - 4 A.
Circuit fig.(d): V = 20 V, I = 4 A.

Compare each figure and the statement given to identify the correct polarities of voltage and current.

(b):

Absorbing because the terminal 2 is at higher potential and the current enters the same terminal as described by the statement.

(c):

P = V*I = 80 W.

Problem 2:

(a):

$0 \text{ s} \le t < 1 \text{ s}$:

$v = 5 \text{ V}; \quad i = 20t \text{ A}; \quad p = 100t \text{ W}$

$1 \text{ s} < t \le 3 \text{ s}$:

$v = 0 \text{ V}; \quad i = 20 \text{ A}; \quad p = 0 \text{ W}$

$3 \text{ s} \le t < 5 \text{ s}$:

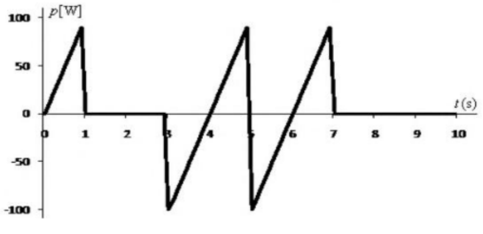
$v = -5 \text{ V}; \quad i = 80 - 20t \text{ A}; \quad p = 100t - 400 \text{ W}$

$5 \text{ s} < t \le 7 \text{ s}$:

$v = 5 \text{ V}; \quad i = 20t - 120 \text{ A}; \quad p = 100t - 600 \text{ W}$

$t > 7 \text{ s}$:

$v = 0 \text{ V}; \quad i = 20 \text{ A}; \quad p = 0 \text{ W}$



(b):

Calculate the area under the curve from zero up to the desired time:

$$
\begin{aligned}
w(1) &= \tfrac{1}{2}(1)(100) = 50 \text{ J} \\
w(6) &= \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) + \tfrac{1}{2}(1)(100) - \tfrac{1}{2}(1)(100) = 0 \text{ J} \\
w(10) &= w(6) + \tfrac{1}{2}(1)(100) = 50 \text{ J}
\end{aligned}
$$

Problem 3:

(a):

(i):

$$\frac{1}{2} * C * V^2 = 46$$

Putting V = 200,

$$C = 2.3mF$$

(ii):

$$C = \frac{\varepsilon A}{d}$$

Making A the subject,

$$A = \frac{[(2.3 * 10^{-3}) * (1 * 10^{-2})]}{\varepsilon}$$

(iii):

Lets call the new capacitance due to quartering the distance as,

$$C_{\frac{d}{4}} = \frac{\varepsilon A}{0.25 * 10^{-2}}$$

Using area from previous part,

$$C_{\frac{d}{4}} = 9.2 * 10^{-3}\ F = 9.2mF$$

i.e. the capacitance is quadrupled.

Lets call the capacitance due to half the area as,

$$C_{A/2} = 1.15mF$$

i.e. the capacitance is halved.

(iv):

For quarter the distance:

$$q_{discharged} = (C_{\frac{d}{4}} * V) - (C * V)$$

$$q_{discharged} = (9.2 * 10^{-3} * 200) - (2.3 * 10^{-3} * 200) = 1.380\ C$$

$$i_{discharged} = \frac{1.380}{t}$$

For half the area:

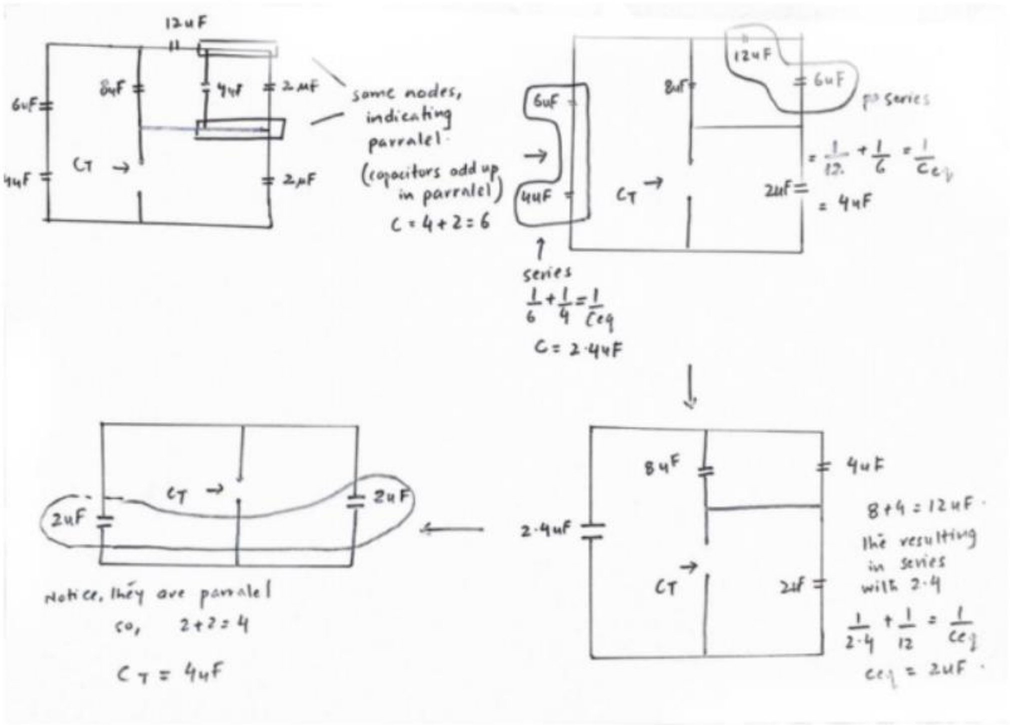
$$q_{discharged} = (C * V) - (C_{\frac{A}{2}} * V)$$

$$q_{discharged} = (2.3 * 10^{-3} * 200) - (1.15 * 10^{-3} * 200) = 0.230\ C$$

$$i_{discharged} = \frac{0.230}{t}$$

(b):

$$C_T = 4 * 10^{-6} F$$



Problem 4:

(a):

$60 \| 20 = 1200/80 = 15\,\Omega$	$12 \| 24 = 288/36 = 8\,\Omega$
$15 + 8 + 7 = 30\,\Omega$	$30 \| 120 = 3600/150 = 24\,\Omega$
$R_{ab} = 15 + 24 + 25 = 64\,\Omega$	

(b):

$35 + 40 = 75\,\Omega$	$75 \| 50 = 3750/125 = 30\,\Omega$
$30 + 20 = 50\,\Omega$	$50 \| 75 = 3750/125 = 30\,\Omega$
$30 + 10 = 40\,\Omega$	$40 \| 60 + 9 \| 18 = 24 + 6 = 30\,\Omega$
$30 \| 30 = 15\,\Omega$	$R_{ab} = 10 + 15 + 5 = 30\,\Omega$

(c):

$50 + 30 = 80\,\Omega$	$80 \| 20 = 16\,\Omega$
$16 + 14 = 30\,\Omega$	$30 + 24 = 54\,\Omega$
$54 \| 27 = 18\,\Omega$	$18 + 12 = 30\,\Omega$
$30 \| 30 = 15\,\Omega$	$R_{ab} = 3 + 15 + 2 = 20\,\Omega$

Problem 5:

(a):

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t)\text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}\text{ J}$$

(b):

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25\text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.13), by writing

$$w\Big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2\times 5^3)^2 - 0 = 156.25\text{ kJ}$$

as obtained before.

Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6\times\frac{t^3}{3} = 2t^3\text{ A}$$

(c):

The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

$$\frac{7\times 42}{7+42} = 6\text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18\text{ H}$$

(d):

$$w = L\int_{-\infty}^{t} i\,dt = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

$$= \frac{1}{2}\text{x}80\text{x}10^{-3}\text{x}\left(60\text{x}10^{-3}\right)^2 - 0$$

$= \underline{\mathbf{144\ \mu J}}$

(e):

$$v = L\frac{di}{dt} = 12x10^{-3}x4(100)\cos 100t$$
$$= \underline{\mathbf{4.8\ cos\ 100t\ V}}$$

$$p = vi = 4.8 \text{ x } 4 \sin 100t \cos 100t = 9.6 \sin 200t$$

$$w = \int_o^t pdt = \int_o^{11/200} 9.6 \sin 200t$$
$$= -\frac{9.6}{200}\cos 200t\Big|_o^{11/200} \text{J}$$
$$= -48(\cos\pi - 1)\text{mJ} = \underline{\mathbf{96\ mJ}}$$

Problem 6:

Recall that current is related to charge by $i(t) = \frac{dq(t)}{dt}$. The current is equal to the slope of the charge waveform.

$i(t) = 0$	$0 \le t \le 1$ ms
$i(t) = \frac{3\times10^{-3} - 1\times10^{-3}}{2\times10^{-3} - 1\times10^{-3}} = 2$ A	$1 \le t \le 2$ ms
$i(t) = 0$	$2 \le t \le 3$ ms
$i(t) = \frac{-2\times10^{-3} - 3\times10^{-3}}{5\times10^{-3} - 3\times10^{-3}} = -2.5$ A	$3 \le t \le 5$ ms
$i(t) = 0$	$5 \le t \le 6$ ms
$i(t) = \frac{2\times10^{-3} - (-2\times10^{-3})}{9\times10^{-3} - 6\times10^{-3}} = 1.33$ A	$6 \le t \le 9$ ms
$i(t) = 0$	$t \ge 9$ ms

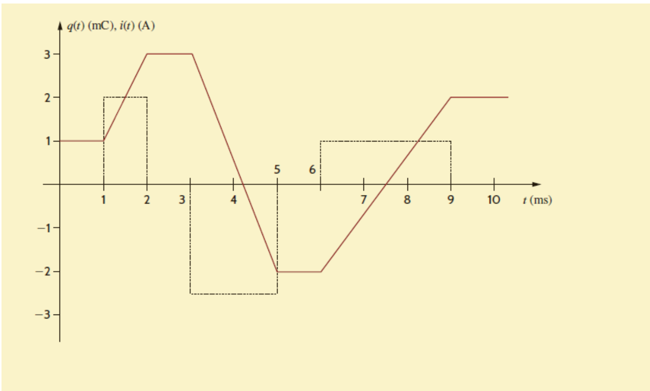
The current is plotted with the charge waveform in Fig. 1.21. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

The power absorbed by the BOX is $12 \times i(t)$.

$p(t) = 12(0) = 0$	$0 \le t \le 1$ ms
$p(t) = 12(2) = 24$ W	$1 \le t \le 2$ ms
$p(t) = 12(0) = 0$	$2 \le t \le 3$ ms
$p(t) = 12(-2.5) = -30$ W	$3 \le t \le 5$ ms
$p(t) = 12(0) = 0$	$5 \le t \le 6$ ms
$p(t) = 12(1.33) = 16$ W	$6 \le t \le 9$ ms
$p(t) = 12(0) = 0$	$t \ge 9$ ms

The power absorbed by the BOX is plotted in Fig. 1.22. For the time intervals, $1 \le t \le 2$ ms and $6 \le t \le 9$ ms, the BOX is absorbing power. During the time interval $3 \le t \le 5$ ms, the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.

(a):



(b):