Q1) (5 marks)

Consider the circuit below and answer the following questions:

a) What value of v_g is required for the interconnection to be valid.

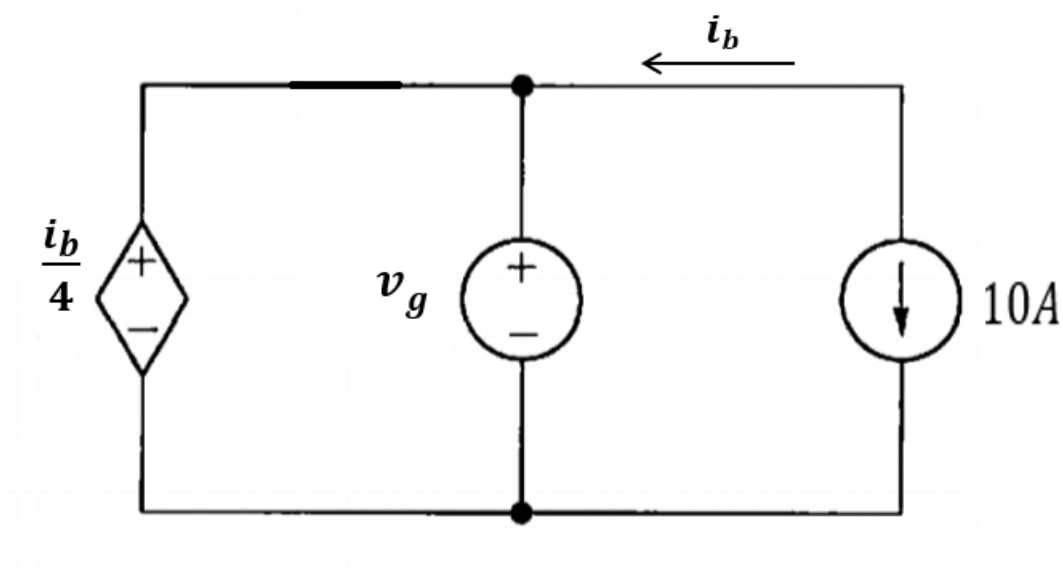
Answer: $i_b = -10A$

$\frac{10}{4} = v_g = 2.5$

b) For this value of v_g find the power associated with the 10A source.

$P = IV$

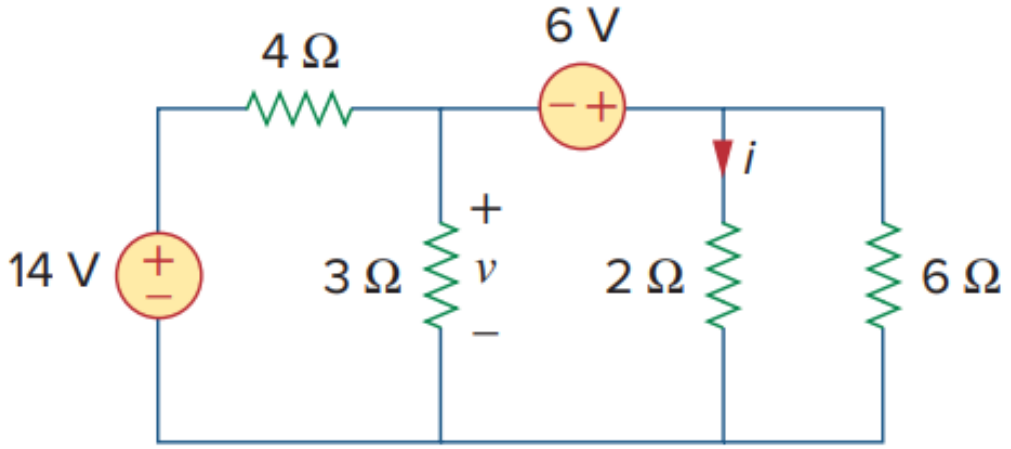
$10 \times 2.5 = 25W$

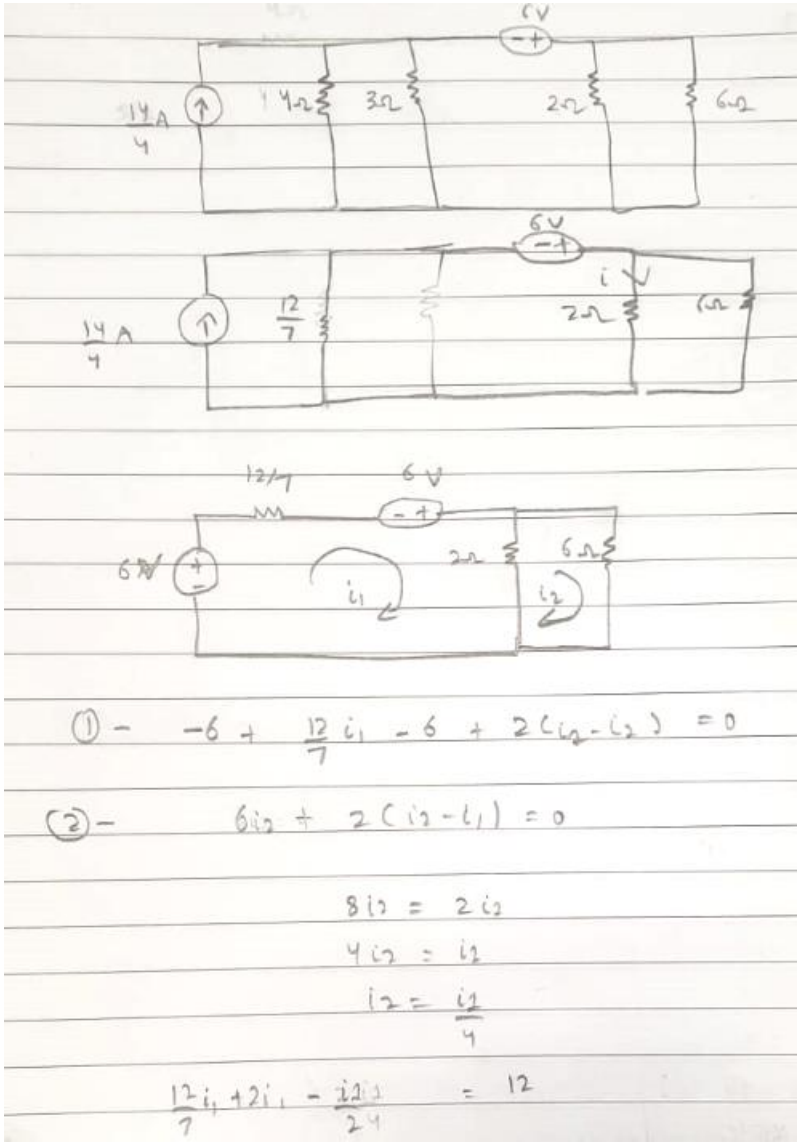


Q2) (10 marks)

Given this circuit below, find the values of i by using source transformation:

(Hint: Use Kirchoff voltage law to find the currents flowing in the loop first)





① − $-6 + \frac{12}{7} i_1 - 6 + 2(i_2 - i_1) = 0$

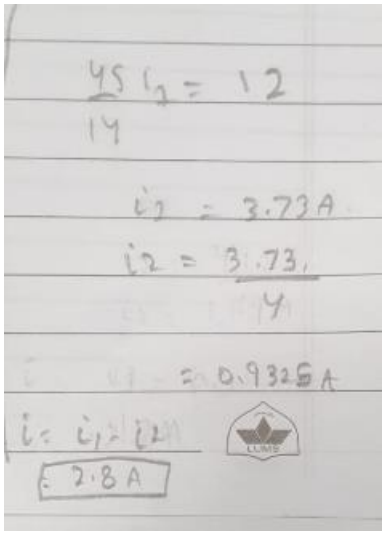
② − $6 i_2 + 2(i_2 - i_1) = 0$

$8 i_2 = 2 i_1$

$4 i_2 = i_1$

$i_2 = \frac{i_1}{4}$

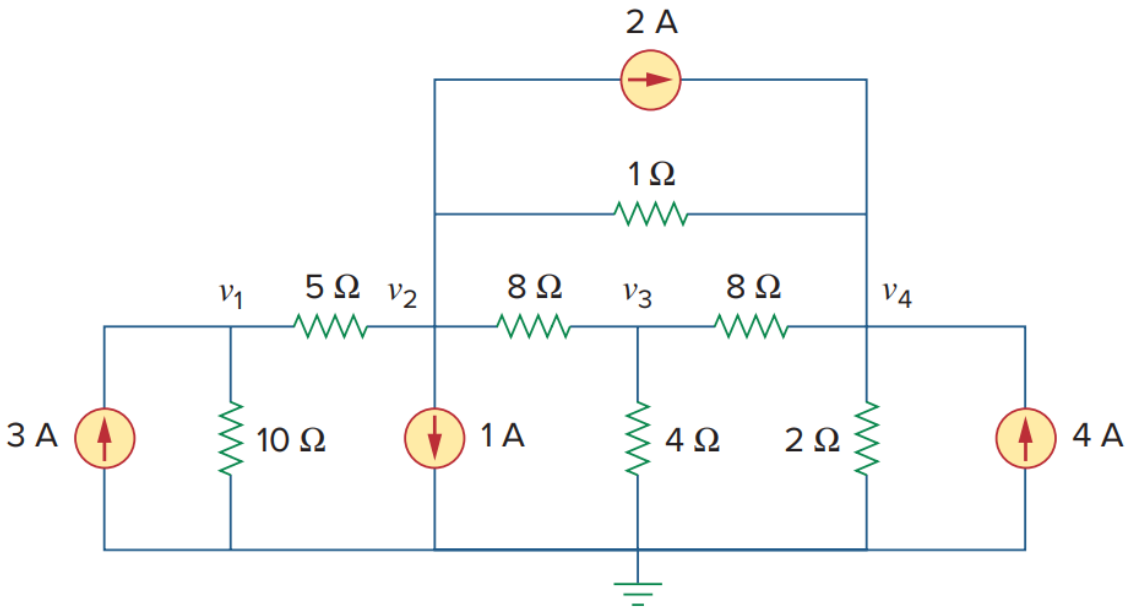
$\frac{12}{7} i_1 + 2 i_1 - \frac{2 i_1}{4} = 12$



Q3) (15 marks)

Consider this circuit and answer the following parts:

a) Formulate network equations
b) Convert it into matrix form and then solve for the nodal voltages



The network equations:

There are 4 nodes so there will be 4 equations:

Node 1:

$$\frac{v_2 - v_1}{5} + \frac{v_1}{10} - 3 = 0$$

Node 2:

$$\frac{-(v_2 - v_1)}{5} + \frac{v_2 - v_3}{8} + 1 + \frac{v_2 - v_4}{1} + 2 = 0$$

Node 3:

$$\frac{-(v_2 - v_{3)}}{8} + \frac{v_3 - v_4}{8} + \frac{v_3}{4} = 0$$

Node 4:

$$\frac{-(v_3 - v_4)}{8} + \frac{v_4}{2} - 4 + \frac{-(v_2 - v_{4)}}{1} - 2 = 0$$

In Matrix Form:

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

$$v_1 = 13.8966 \quad v_2 = 5.845 \quad v_3 = 3.348 \quad v_4 = 7.547$$

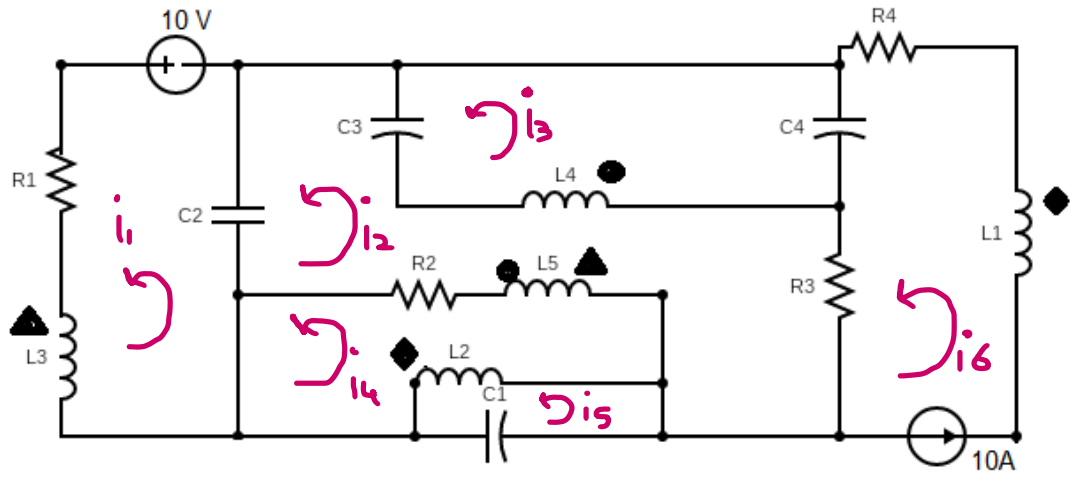
Q4) (20 marks)

Consider the circuit drawn below. Formulate the loop equations by considering the current in each loop in anticlockwise direction. You must make use of the dot convention while considering the mutual inductances which are given below:

$$\boldsymbol{L1\ and\ L2 = Ma}$$

$$\boldsymbol{L3\ and\ L5 = Mb}$$

$$\boldsymbol{L4\ and\ L5 = Mc}$$



Loop 6

$$\boldsymbol{i_6 = 10A}$$

Loop 5

$$\frac{\mathbf{1}}{\boldsymbol{C1}}\int \boldsymbol{i_5 dt} + \boldsymbol{L2}\frac{\mathrm{d(i_5 - i_4)}}{\mathrm{dt}} - \boldsymbol{Ma}\frac{\mathrm{di_6}}{\mathrm{dt}} = \mathbf{0}$$

Loop 4

$$\boldsymbol{R2}(\boldsymbol{i_4} - \mathrm{i_2}) + \boldsymbol{L2}\frac{\mathrm{d(i_4 - i_5)}}{\mathrm{dt}} - \boldsymbol{Ma}\frac{\mathrm{di_6}}{\mathrm{dt}} + \boldsymbol{L5}\frac{\mathrm{d(i_4 - i_2)}}{\mathrm{dt}} - \boldsymbol{Mc}\frac{\mathrm{d(i_3 - i_2)}}{\mathrm{dt}} + \boldsymbol{Mb}\frac{\mathrm{di_1}}{\mathrm{dt}} = \mathbf{0}$$

Loop 2

$$\boldsymbol{R2}(\boldsymbol{i_2} - \mathrm{i_4}) + \boldsymbol{L4}\frac{\mathrm{d(i_2 - i_3)}}{\mathrm{dt}} - \boldsymbol{L5}\frac{\mathrm{d(i_4 - i_2)}}{\mathrm{dt}} - \boldsymbol{Mc}\frac{\mathrm{d(i_3 - i_2)}}{\mathrm{dt}} + \boldsymbol{Mb}\frac{\mathrm{di_1}}{\mathrm{dt}} + \boldsymbol{R3}(\boldsymbol{i_2} - \mathrm{i_6}) + \frac{\mathbf{1}}{\boldsymbol{C2}}\int (\boldsymbol{i_2} - \mathrm{i_1})\boldsymbol{dt} = \mathbf{0}$$

Loop 3

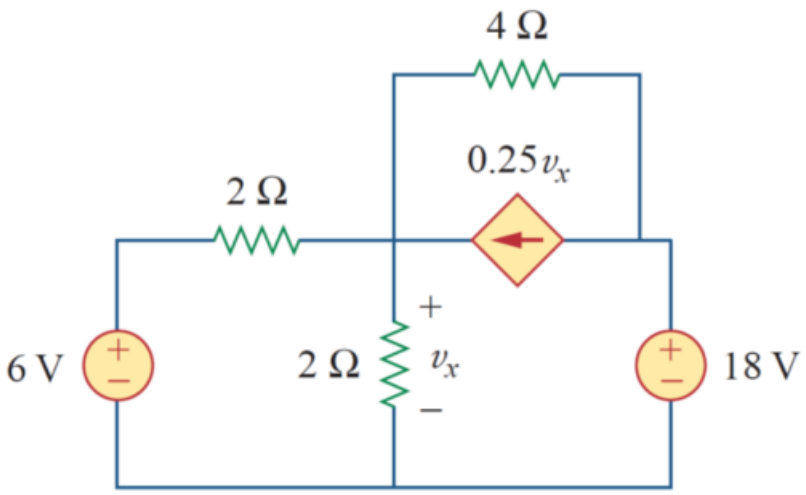
$$\frac{\mathbf{1}}{\boldsymbol{C3}}\int (\boldsymbol{i_3} - \mathrm{i_2})\boldsymbol{dt} + \frac{\mathbf{1}}{\boldsymbol{C4}}\int (\boldsymbol{i_3} - \mathrm{i_6})\boldsymbol{dt} + \boldsymbol{Mc}\frac{\mathrm{d(i_2 - i_3)}}{\mathrm{dt}} = \mathbf{0}$$

Loop 1

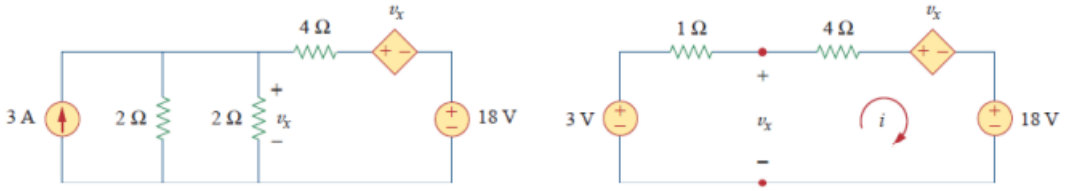
$$\frac{\mathbf{1}}{\boldsymbol{C2}}\int (\boldsymbol{i_1} - \mathrm{i_2})\boldsymbol{dt} + \boldsymbol{R1 i_1} + \boldsymbol{L3}\frac{\mathrm{di_1}}{\mathrm{dt}} + \boldsymbol{Mb}\frac{\mathrm{d(i_4 - i_2)}}{\mathrm{dt}} = \mathbf{10}$$

Q5) (10 marks)

Determine the voltage v_x in the following circuit using the source transformation technique.



Applying source transformation:



Applying KVL:

$$-v_x + 4i + v_x + 18 = 0 \quad \Rightarrow \quad i = -4.5 \text{ A}$$

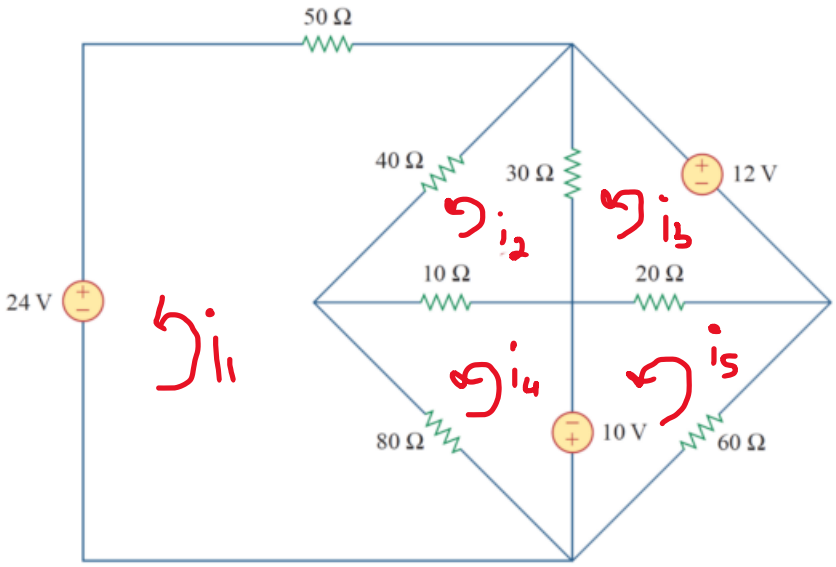
Relating v_x and i:

$$-3 + 1i + v_x = 0 \quad \Rightarrow \quad v_x = 3 - i$$

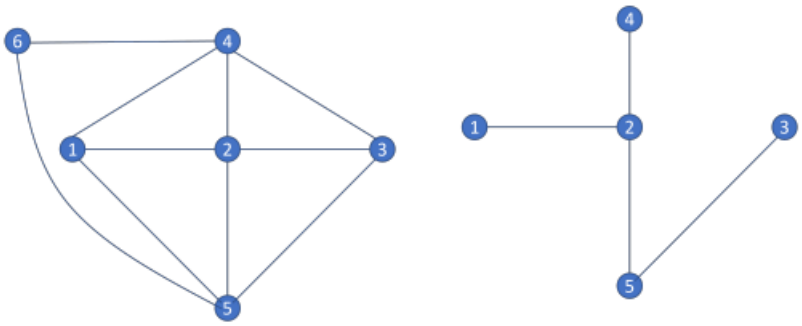
$$v_x = 3 - i = 7.5 \text{ V}.$$

Q6) (20 marks)

Consider the circuit given below and answer the following parts:



a) Draw the graph and one tree of the circuit. Determine the number of nodes and number of branches in the circuit (or graph).



Graph Tree

Number of branches = b = 10
Number of nodes = n =6

b) Determine the number of network equations required for carrying out
i) nodal analysis
ii) loop analysis.

For loop analysis, we need $n - 1 = 5$ equations. For nodal analysis, we need $b - n + 1 = 5$ equations.

c) Carry out the loop analysis, that is, identify and determine the loop currents.

Loop 1:

$$24 + 50i_1 + 40(i_1 - i_2) + 80(i_1 - i_4) = 0$$

Loop 2:

$$40(-i_1 + i_2) + 30(i_2 - i_3) + 10(i_2 - i_4) = 0$$

Loop 3:

$$-12 + 30(-i_{2+}i_3) + 20(i_{3-}i_5) = 0$$

Loop 4:

$$10 + 10(i_{4-}i_2) + 80(i_{4-}i_{1-}) = 0$$

Loop 5:

$$-10 + 60i_5 + 20(-i_{3+}i_5) = 0$$

Solve this equation :

$$\begin{pmatrix} 170 & -40 & 0 & -80 & 0 \\ -40 & 80 & -30 & -10 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -10 & 0 & 90 & 0 \\ 0 & 0 & -20 & 0 & 60 \end{pmatrix} \begin{pmatrix} i1 \\ i2 \\ i3 \\ i4 \\ i5 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \\ 12 \\ -10 \\ 10 \end{pmatrix}$$

$i_1 = -0.4310$ $i_2 = -0.1990$ $i_3 = 0.2161$ $i_4 = -0.5163$ $i_5 = -0.2387$

NOTE ALL THE CURRENTS ARE TAKEN ANTICLOCKWISE