LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE240 Circuits I
Quiz 09

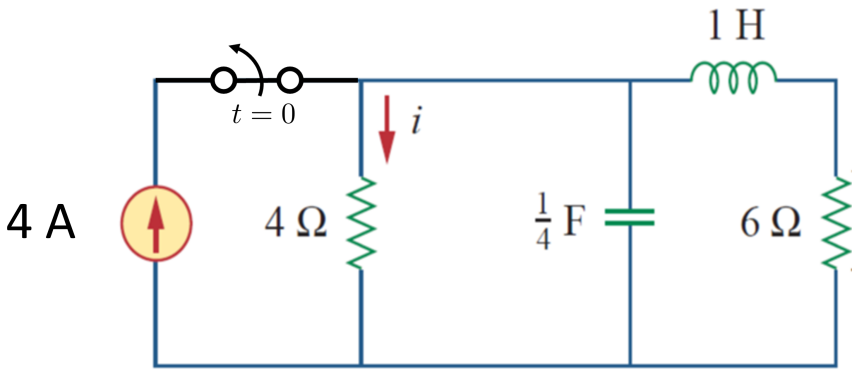
Name: _____

Campus ID: _____

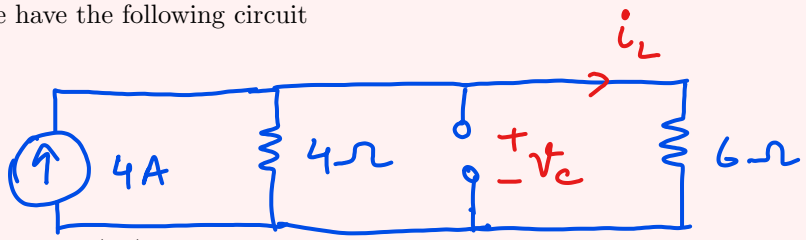
Total Marks: 10
Time Duration: 20 minutes

Question 1 (10 marks)

For the following second-order circuit, determine $i(t)$ for all times. The switch is initially closed and is opened at $t = 0$.

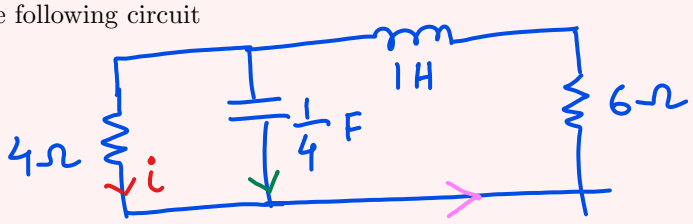


Solution: At $t = 0^-$, we have the following circuit



$i_L = 0^- = \frac{4}{10} \times 4 = 1.6A$ and $v_C(0^-) = 9.6\,V$.

At $t = 0^+$, we have the following circuit



The voltage across 4Ω resistor and capacitor is $4\,i$ and therefore the current through capacitor is $\frac{1}{4}\frac{d}{dt}(4i) = \frac{di}{dt}$. Now the current through inductor is $i + \frac{di}{dt}$ anti-clockwise. Writing the equation of outer loop yields

$$4i + 6\left(i + \frac{di}{dt}\right) + \frac{d}{dt}\left(i + \frac{di}{dt}\right) = 0$$

$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 10i = 0$$

We can formulate characteristic equation as

$$s^2 + 7s + 10 = 0,$$

for which we have $s_1 = -2, s_2 = -5$ and therefore we have

$$i(t) = K_1 e^{-2t} + K_2 e^{-5t}, \quad t \geq 0$$

Initial Conditions: Since $v_C(0^+) = 9.6\,V$ and therefore $i(0^+) = 2.4A$. Furthermore, the current through inductor is $-(i + \frac{di}{dt})$ which yields $\frac{di}{dt}(0^+) = -1.6 - i(0^+) = -4\,A/s$. Solving for K_1 and K_2 as

$$K_1 + K_2 = 2.4 - 2K_1 - 5K_2 = -4$$

yields $K_1 = \frac{28}{15}$ and $K_2 = \frac{8}{15}$.