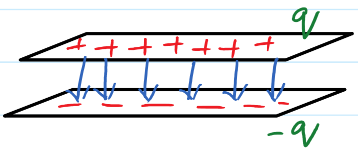
Capacitor

A passive electrical component which stores energy in the form of electric charge.

Capacitors appear in different shapes and sizes but the basic configuration is two conductors carrying equal but opposite charges.

The simplest form of a capacitor consists of two conducting parallel plates of area A and separation d



Field lines

Capacitors in uncharged state do not carry any charge. During the charging phase, charge moves from one conductor to the other; making one conductor positively charged and other negatively charged (net charge on the capacitor is zero).

These charges establish electric field between the conductors and consequently creates the potential difference between the conductors with conductor carrying the positive charge at higher potential.

Symbols:

Normal

Polarized

Capacitance:

The capacitance is a measure of the capacity of capacitor of storing electric charge for a given potential difference between the conductor (plates) of the capacitor.

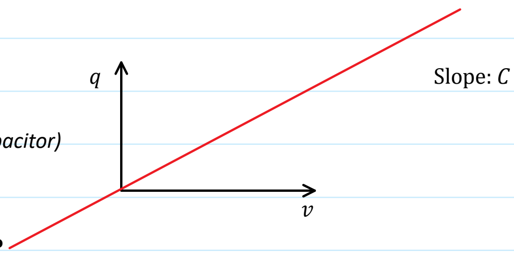
Denoted by **C** and is defined as $C = \frac{q}{V}$

Unit of capacitance is Farad, abbreviated as F. Typical capacitors have the capacitance in the range pF to mF (very large capacitors)

Elastance:
Inverse of capacitance
Measured in Daraf

q-v Characteristics:

Linear characteristics (ideal capacitor)

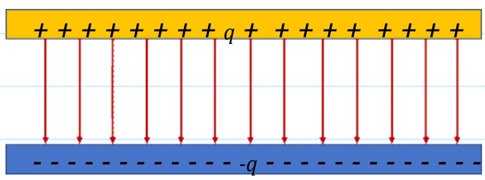


How to Compute Capacitance?

Parallel Plate Capacitor:

Two metallic plates of area A, separated by distance d; each carrying charge q

Ignoring the edge effect, we assume the charge density of each plate is $\frac{q}{A}$



Using Gauss's Law :-

$$\phi_E = \frac{q}{\epsilon_0}$$ (Charge enclosed)

Electric Flux

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$$\phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S E\, dA = EA = \frac{q}{\epsilon_0} \quad \text{— (01)}$$

Potential difference b/w the Plates :-

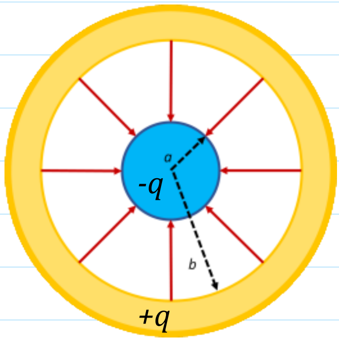
$$\int E \cdot dl = v = Ed = v \Rightarrow E = \frac{v}{d} \quad \text{— (02)}$$

$$\frac{q}{v} = \frac{A\epsilon_0}{d}$$

$$\frac{q}{v} = C \quad \text{(By definition)} \Rightarrow \boxed{C = \frac{A\epsilon_0}{d}}$$

Cylindrical Capacitor

We consider a cylindrical capacitor that includes a hollow/solid cylindrical conductor surrounded by the *concentric* hollow spherical cylinder.



* Cylinder length - L

* Again, using Gauss's law

01;
$$\oint\!\!\oint_S E \cdot dA = \frac{q}{\epsilon_0}$$

$$\underbrace{E\,(2\pi r)L}_{\text{Surface area of cylinder of radius } r.} = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{2\pi\epsilon_0 L r}$$

02;
$$v = \int E \cdot dl \Rightarrow v = \int_{r=a}^{b} \frac{q}{2\pi\epsilon_0 L r}\,dr = \frac{q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r}$$

$$\Rightarrow v = \frac{q}{2\pi\epsilon_0 L}\left(\ln b - \ln a\right) = \frac{q}{2\pi\epsilon_0 L}\ln\frac{b}{a}$$

$$\Rightarrow \frac{q}{v} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Role of Dielectric:

Dielectric, electrically insulating material that does not conduct electric current, is used between the metallic plates of t he capacitor. The use of dielectric enables

- Separation between the plates; we can increase the capacitance by keeping smaller plate separations
- Increases the capacitance by reducing the strength of electric field. This requires explanation; dielectric under the influence of electric field creates its own electric field that is in opposite direction and thus allow the capacitor to same amount of charge at a lower voltage. Mathematically,ϵ_o is replaced by $k\epsilon_o$ *, where k>1 (k*=1 for vaccum, 1.00059 for air) is referred to as the dielectric constant. The capacitance is increased by the value of dielectric constant.

Commonly used dielectric includes air, paper (k=4-7), glass (4-10), mica (3-6) and water (81).

Capacitors in Electric Circuits:

Applications of a Capacitor:

Tons of applications; *to name a few*

- Storing energy
- Delaying voltage changes
- Filtering
- Resonant circuits
- Voltage divider (frequency dependent)

Relationship between Voltage and Current:

For a capacitor, we have

$q = C\,v$

Using the relationship between charge and the current, we can write

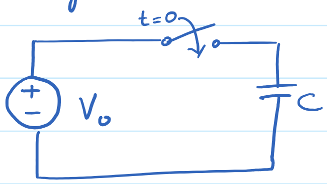
$$i = \frac{dq}{dt} = \frac{d}{dt}(C\,v) = C\,\frac{dv}{dt} + v\frac{dC}{dt}$$

$$i = C\,\frac{dv}{dt} \qquad \text{if} \qquad \frac{dC}{dt} = 0$$

A couple of examples to further elaborate

Example 01

Capacitor is uncharged and connected to DC voltage source through a switch that is closed at $t=0$



$+$ v $-$

$$v = \begin{cases} V_0 & t \geq 0 \\ 0 & t < 0 \end{cases} = V_0\,u(t)$$

Interpretation: voltage across capacitor changes instantaneously!

* What about charge on capacitor?

At $t = 0^-$; charge $q = 0$; Charge is transferred in zero time (infinitely small)

$t = 0^+$; $q = C V_0$

$$\boxed{q(t) = C V_0\, u(t)}$$

* What about current through the capacitor?

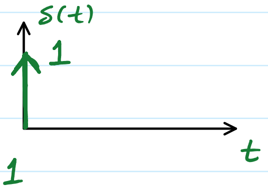
Intuitively: infinite amount of current flows in zero time; because circuit behaves as short circuit (zero resistance, no voltage) at $t=0$. This rush current is short-lived (only for infinitely small duration) because when charge is accumulated on the capacitor, we have voltage V_0 across capacitor and no potential difference between capacitor and source implies no current.

Mathematically : $i(t) = \frac{dq}{dt}$ OR $i(t) = C\frac{dv}{dt}$

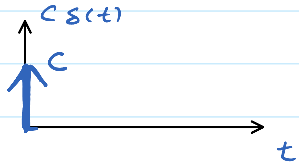
$$\Rightarrow \quad i(t) = C\frac{d}{dt}\left(u(t)\right) = C\,\underset{\substack{\downarrow \\ \text{Impulse Function (Derivative of } u(t))}}{\delta(t)}$$

Graphically $\delta(t)$ is given by

* Arrow upward ; infinitely large height
  ; infinitely small width
  ; $\underbrace{(\text{width} \times \text{height})}_{\text{area}}$ is 1



$i(t) = C\,\delta(t)$
(Impulse at $t=0$ of area C)

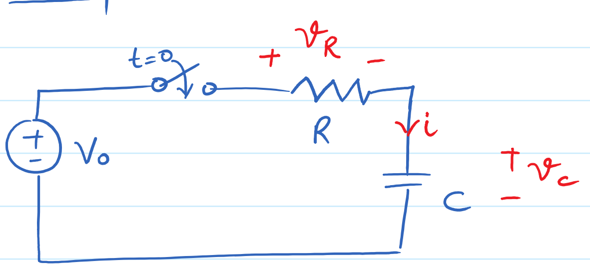


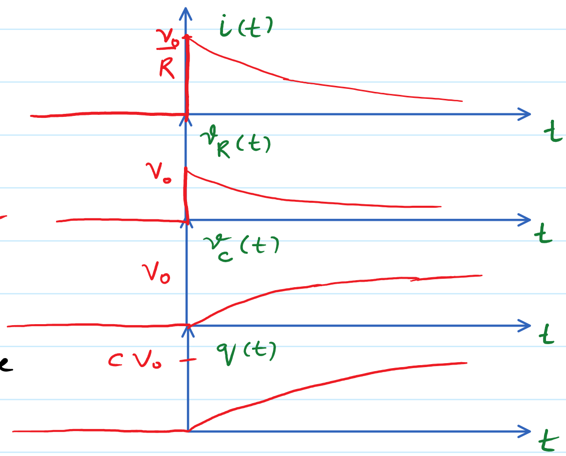
Most important :- * Capacitor does not allow instantenous change in voltage as it requires infinite amount of current.

* This stems from $i(t) = C\frac{dv}{dt}$

* This infinite current would always generate a spark around switch
* In practice, we use R in series to charge capacitor.
* Let's analyze this in the next example.

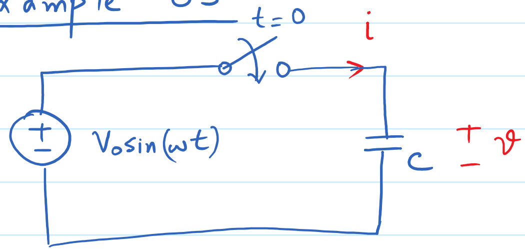
Example 02





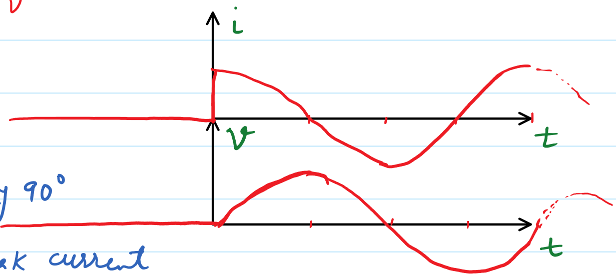
* Using simple intuition, we have plotted i, v_R, v_c and q
* These plots have been explained in detail during the session.
* Later in the course; we will carry out mathematical analysis of this circuit.

Example 03



$v = V_o \sin(\omega t)$

$i = C \dfrac{dV_c}{dt} = C V_o \omega \cos(\omega t)$



* Observations

- i and v; phase shifted by 90°
- Ratio of peak voltage to peak current

$$\frac{v_{max}}{i_{max}} = \frac{V_0}{C V_0 \omega} = \frac{1}{C\omega_0}$$ (This quantity represents some type of resistance)

* When $\omega = 0$; $i = 0$ (DC is blocked by capacitor) You must have heard this before.

* When ω is large; i is large.

Energy in Capacitors:

We know

$$v = \frac{dw}{dq}$$

The work done to move the total charge Q is

$$dw = \int_0^Q v\, dq = \int_0^Q \frac{q}{C}\, dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \qquad\qquad Q = CV$$

Alternatively;

$$w = \int_o^t p(\tau)d\tau = \int_0^t v(\tau)\, i(\tau)d\tau = \int_0^t v(\tau)\frac{dq}{d\tau}d\tau = \int_0^Q v(q)\, dq = \int_0^Q \frac{q}{C}\, dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$