EE 240 Circuits I

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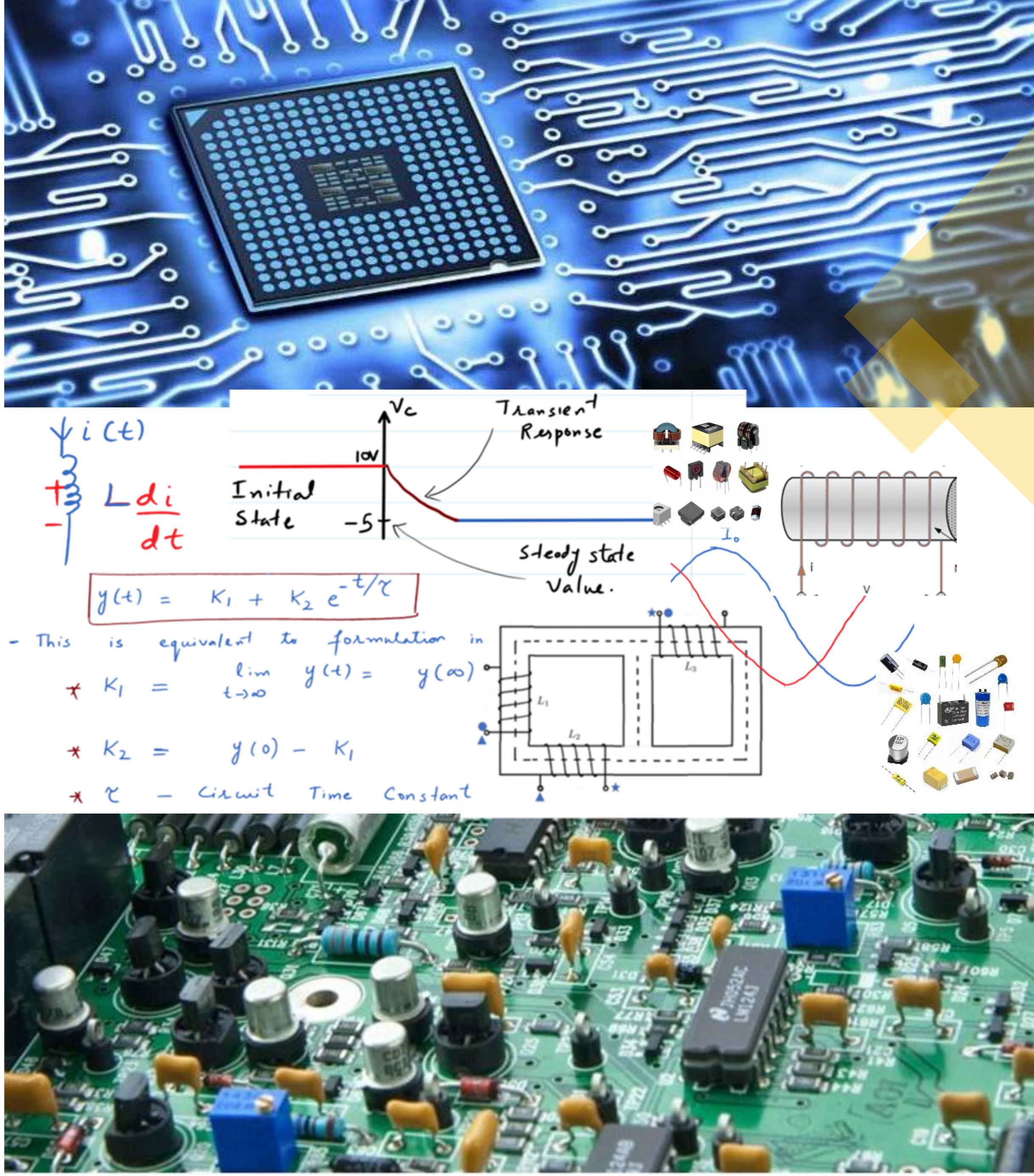
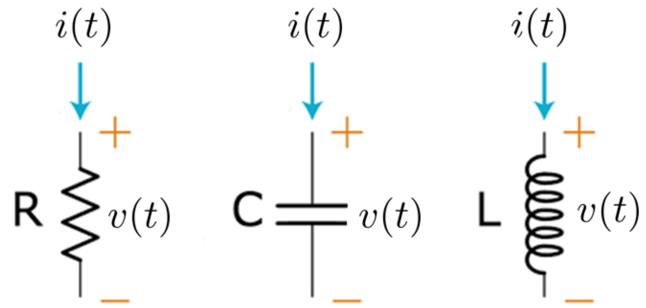
https://www.zubairkhalid.org/ee240_2021.html

Topics:

- R,L,C summary
- Series, parallel connection



Resistor, Capacitor, Inductor (R,C,L)



Summary:

R,C and L are passive elements

Resistor, R

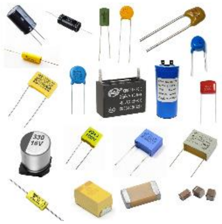
$v = iR$

$i = \frac{v}{R}$

$w(t) = \int_{\tau=-\infty}^{t} p(\tau)d\tau$

$p = vi = i^2R = \frac{v^2}{R}$

Capacitor, C



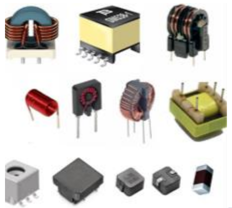
$v(t) = \frac{1}{C}\int_{\tau=-\infty}^{t} i(\tau)d\tau$

$i = C\frac{dv}{dt}$

$w = \frac{1}{2}Cv^2$

$q = Cv$

Inductor, L



$v = L\frac{di}{dt}$

$i(t) = \frac{1}{L}\int_{\tau=-\infty}^{t} v(\tau)d\tau$

$w = \frac{1}{2}Li^2$

$\Psi = Li$

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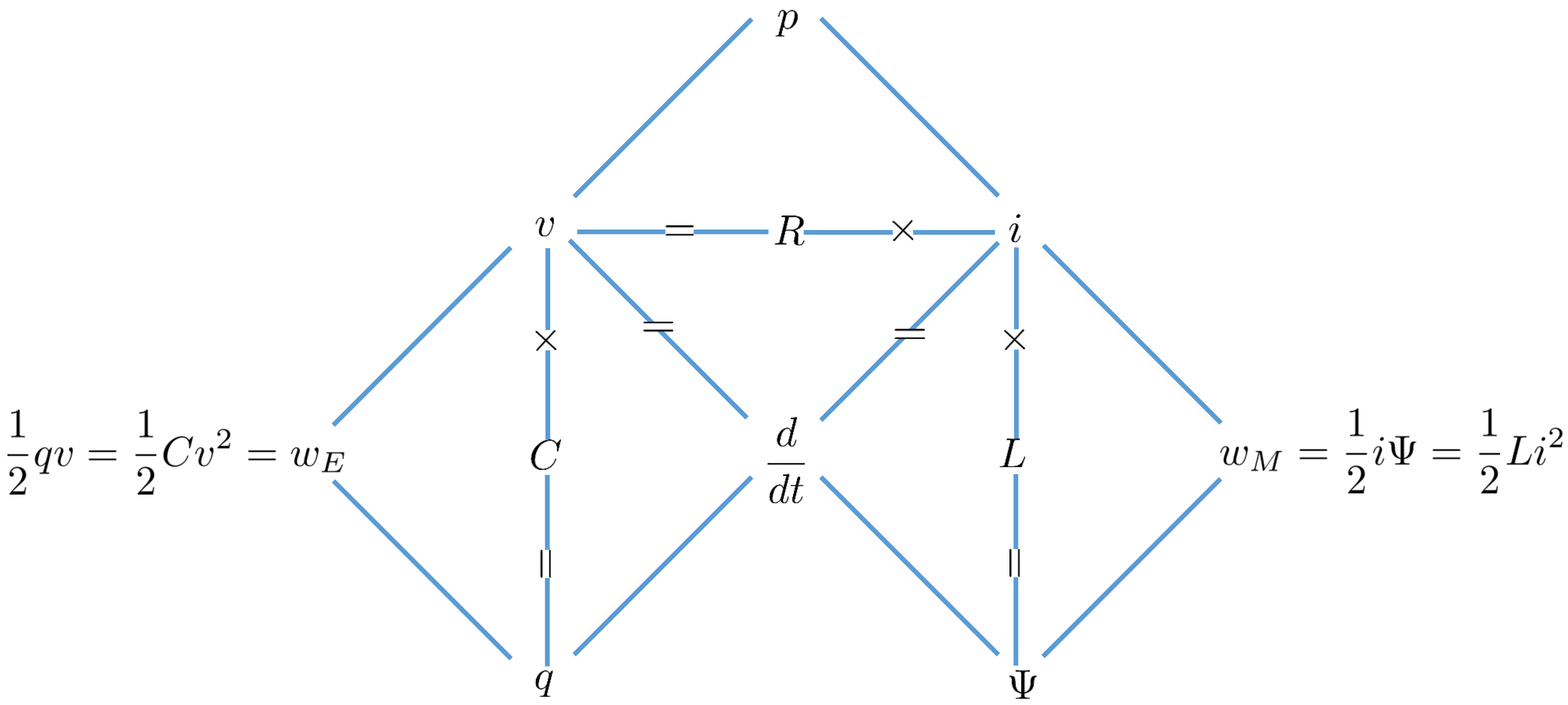
Resistor, Capacitor, Inductor (R,C,L)

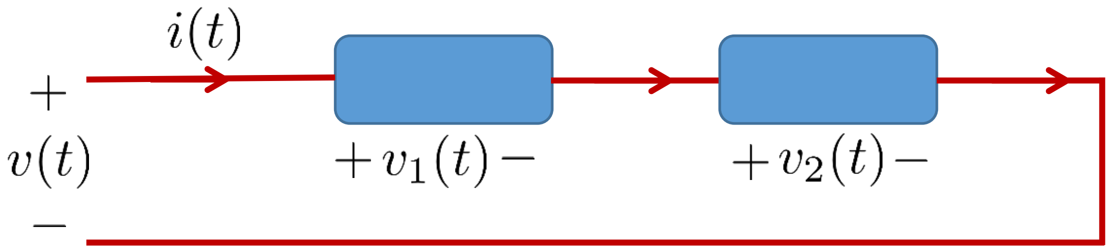
Encapsulated:



Elements and Sources in Series/Parallel

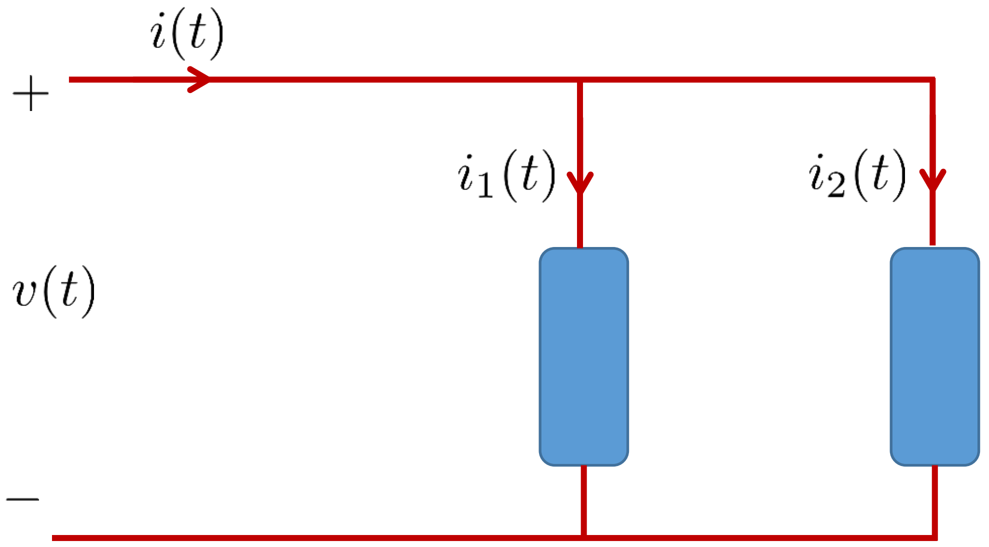
Series Connection:

Idea: same current, voltage is divided

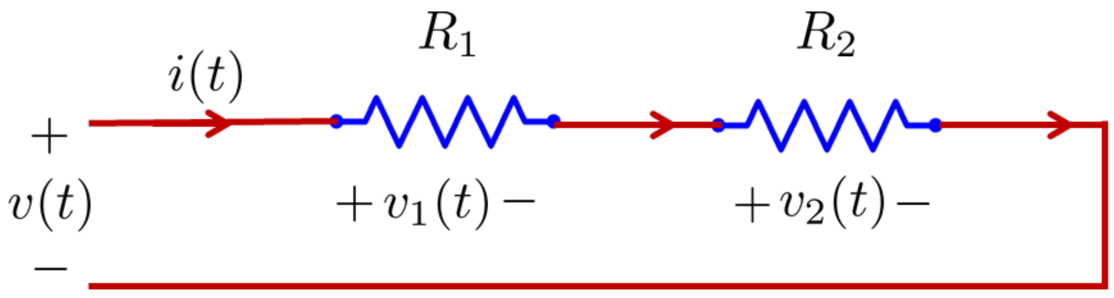


Parallel Connection:

Idea: same voltage, current is divided



Resistors in Series or Parallel



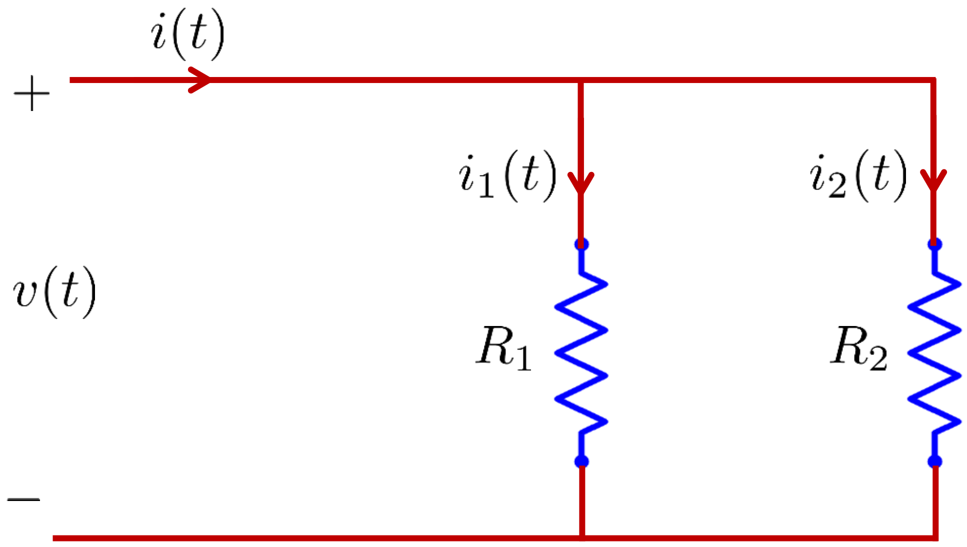
$v_1(t) = i(t) R_1$

$v_2(t) = i(t) R_2$

$v(t) = v_1 + v_2 = i(t)(R_1 + R_2)$

$v(t) = i(t) R_{eq}$

$\Rightarrow R_{eq} = R_1 + R_2$



$i_1(t) = \frac{v(t)}{R_1}, \quad i_2 = \frac{v(t)}{R_2}$

$i(t) = i_1 + i_2 = v(t)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

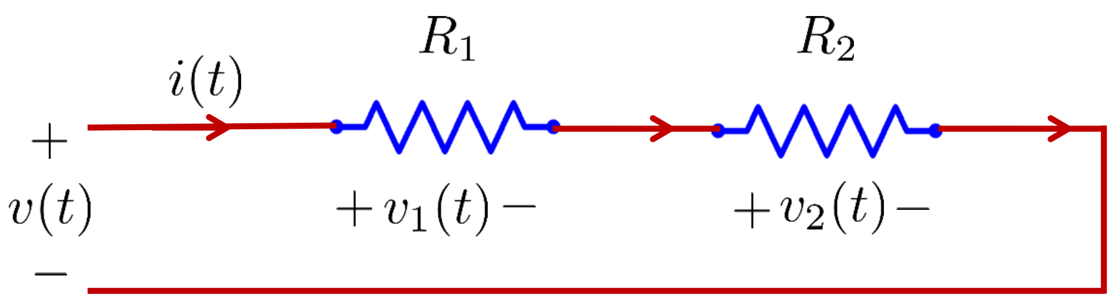
$i(t) = \frac{v(t)}{R_{eq}}$

$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Example: Voltage and Current Divider Rules

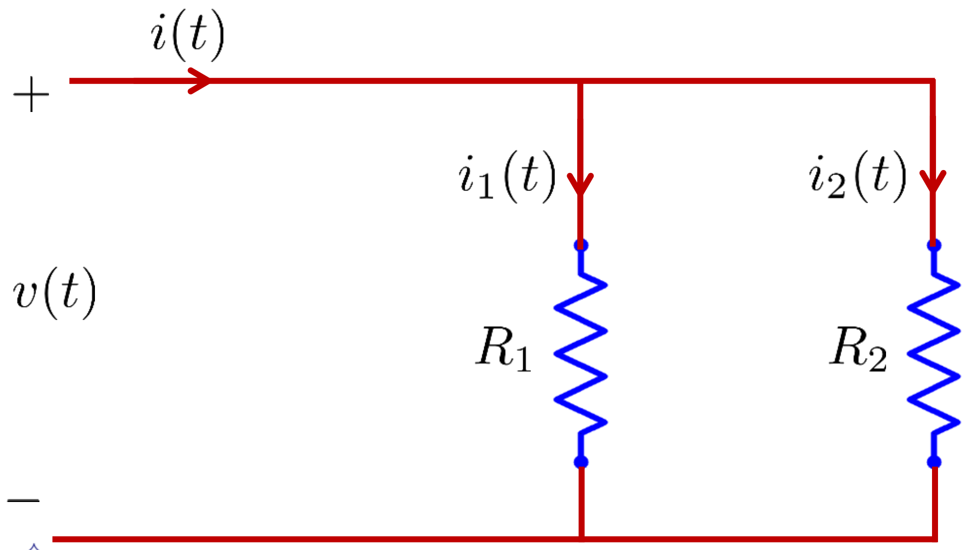
Voltage Divider:



$$v_1(t) = \frac{R_1}{R_1 + R_2} v(t)$$

$$v_2(t) = \frac{R_2}{R_1 + R_2} v(t)$$

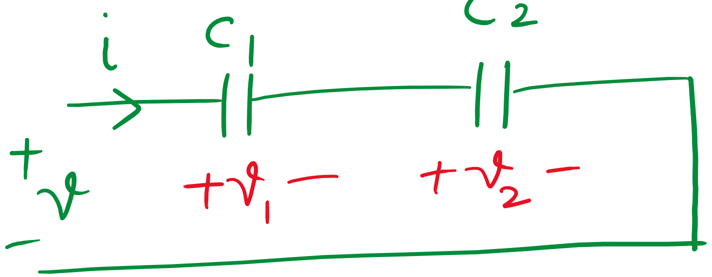
Current Divider:



$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t)$$

$$i_2(t) = \frac{R_1}{R_1 + R_2} i(t)$$

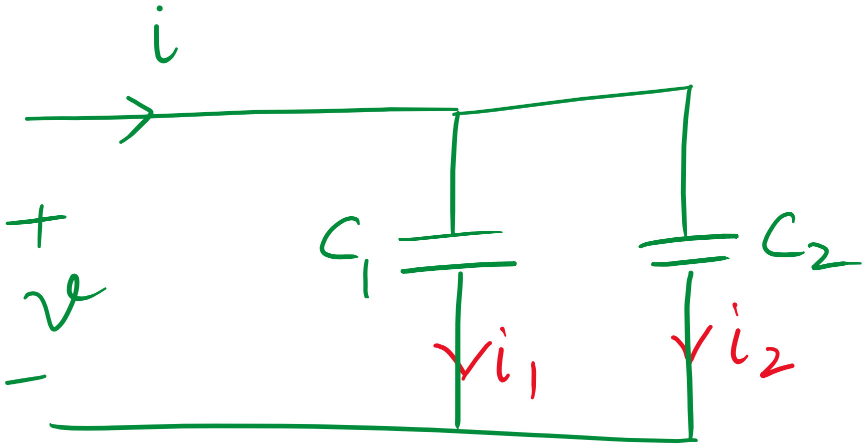
Capacitors in Series or Parallel



Same current ⟷ same charge ⇒ $i \leftrightarrow q$

$v_1 = \frac{q}{C_1}$, $v_2 = \frac{q}{C_2}$ ⇒ $v = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

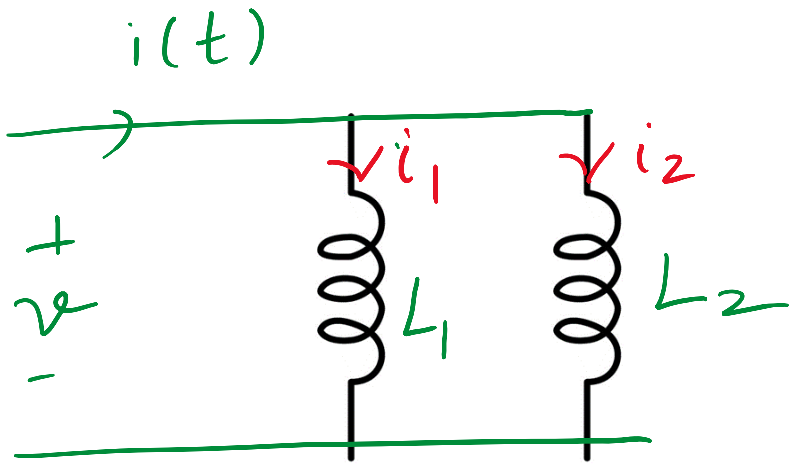
$v = \frac{q}{C_{eq}} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$



Similarly

$C_{eq} = C_1 + C_2$

Inductors in Series or Parallel



* Since flux ∝ current in inductor, we can use flux to determine equivalent inductance (L_{eq}).
* Let's determine L_{eq} using

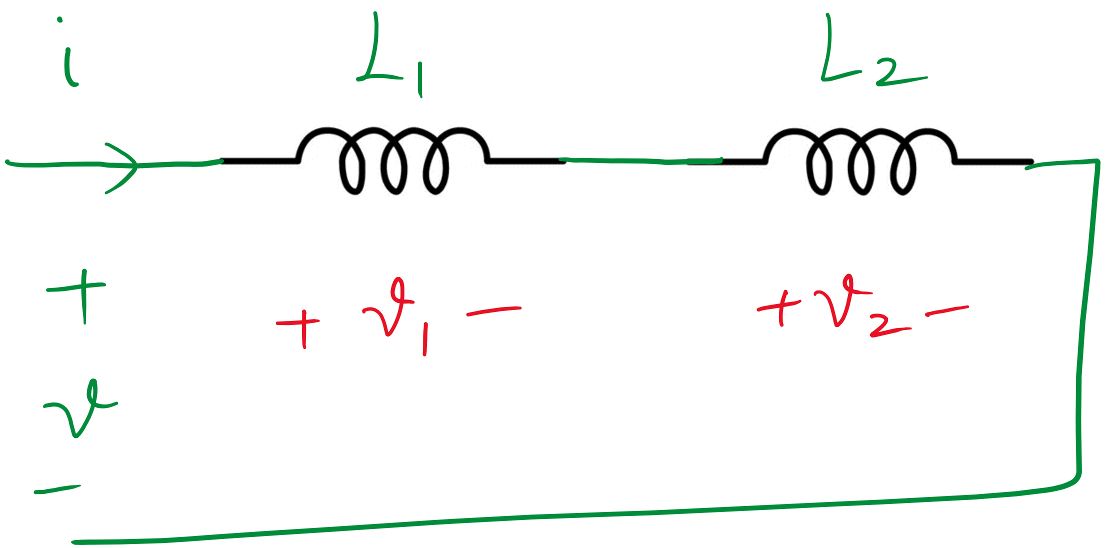
$$v = L_{eq} \frac{di}{dt}$$

Since $i = i_1 + i_2 \Rightarrow v = L_{eq}\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$

$\frac{di_1}{dt} = \frac{v}{L_1}$, $\frac{di_2}{dt} = \frac{v}{L_2} \Rightarrow v = L_{eq}\left(\frac{1}{L_1} + \frac{1}{L_2}\right) v$

$$\Rightarrow \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}} \quad \text{OR} \quad \boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

Inductors in Series or Parallel



$$v = v_1 + v_2$$

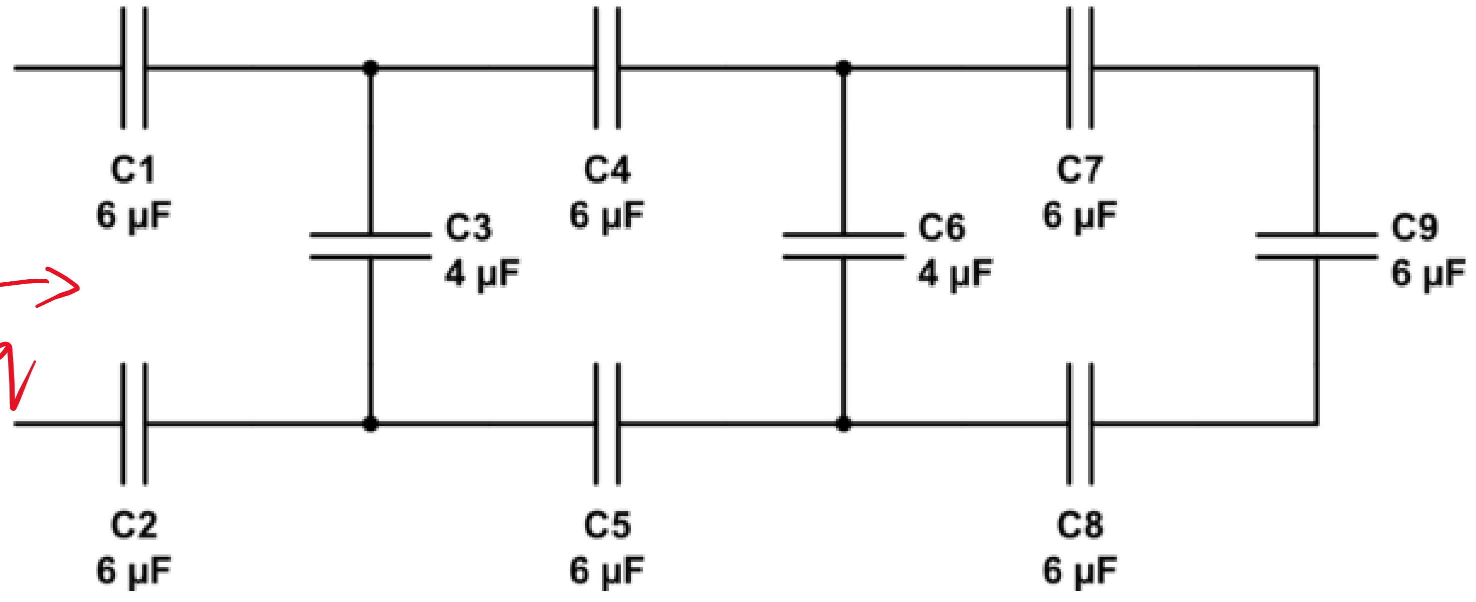
$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$\Rightarrow v = \underbrace{(L_1 + L_2)}_{L_{eq}} \frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2}$$

Example: Equivalent Capacitance

Determine Equivalent Capacitance:



* $C_7 - C_8 - C_9 = 2\,\mu F$ (series)
* $C_6 \parallel C_7 C_8 C_9$
  $= 2\mu + 4\mu = 6\,\mu F$
* $C_4 C_5 (C_6 \parallel C_7 C_8 C_9) = 2\,\mu F$
* $C_3 \parallel (\quad) = 6\,\mu F$

$C_{eq} = 2\,\mu F$