EE 240 Circuits I

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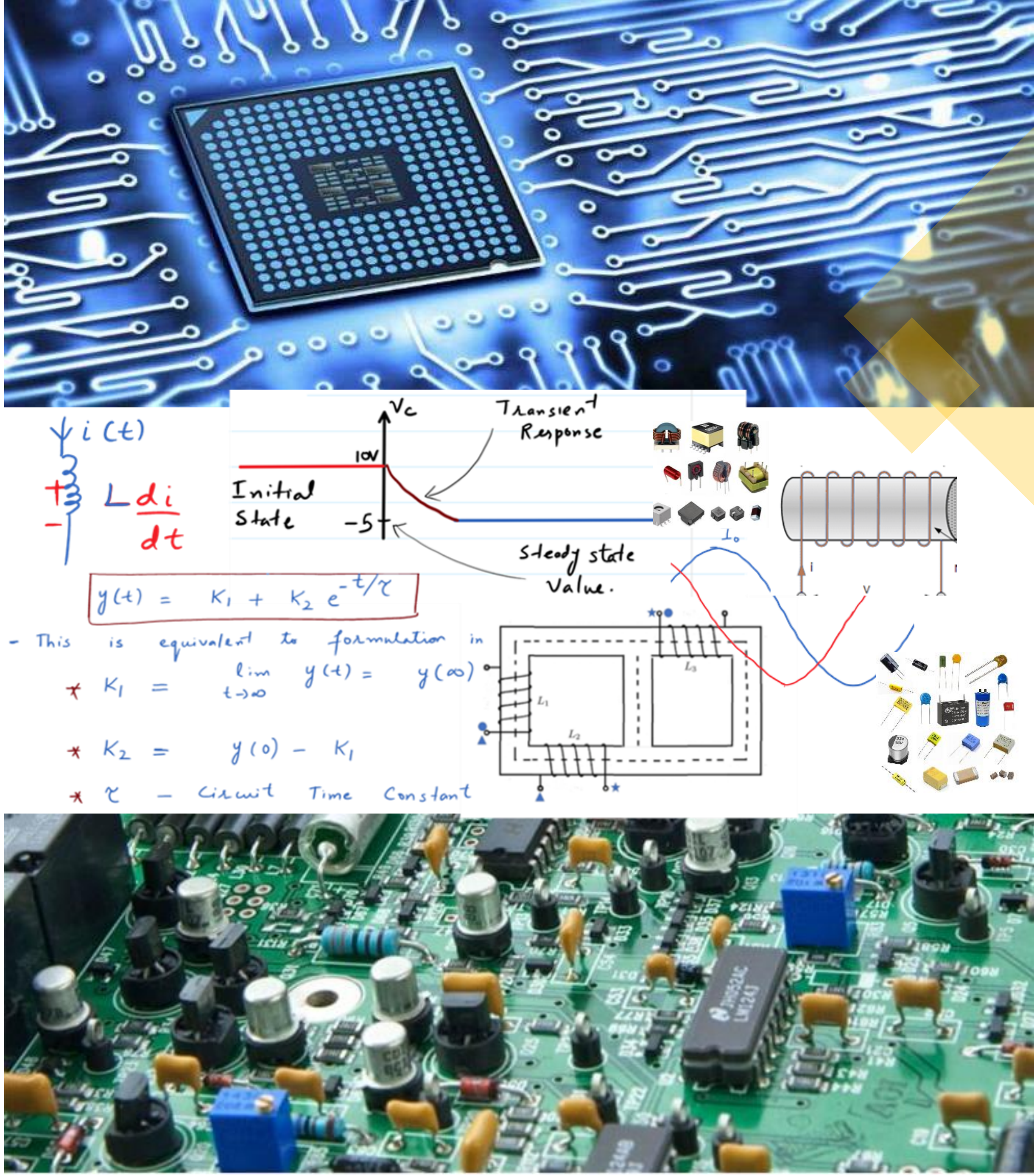
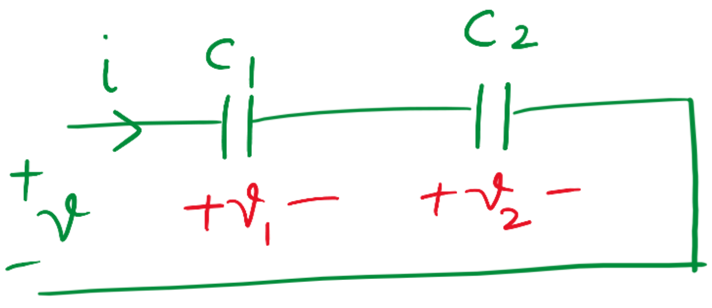
- Capacitors, Inductors in Series and Parallel
- Mutual Inductance



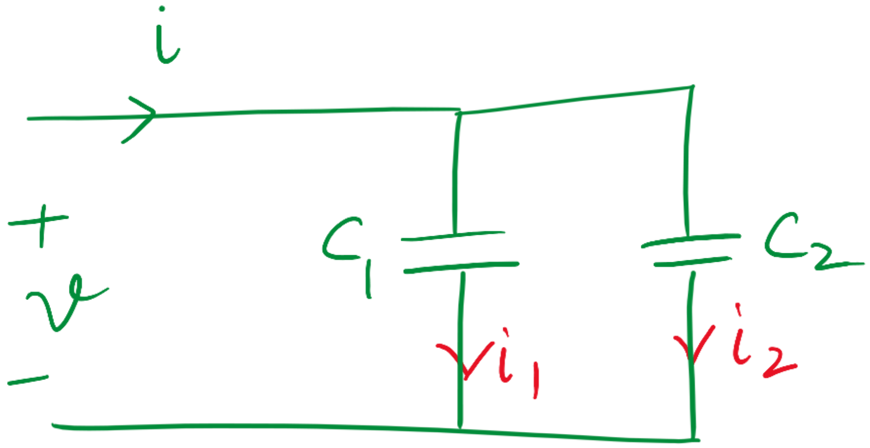
Capacitors in Series or Parallel



Same current $\longleftrightarrow$ same charge $\Rightarrow i \longleftrightarrow q$

$$v_1 = \frac{q}{C_1}, \quad v_2 = \frac{q}{C_2} \Rightarrow v = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

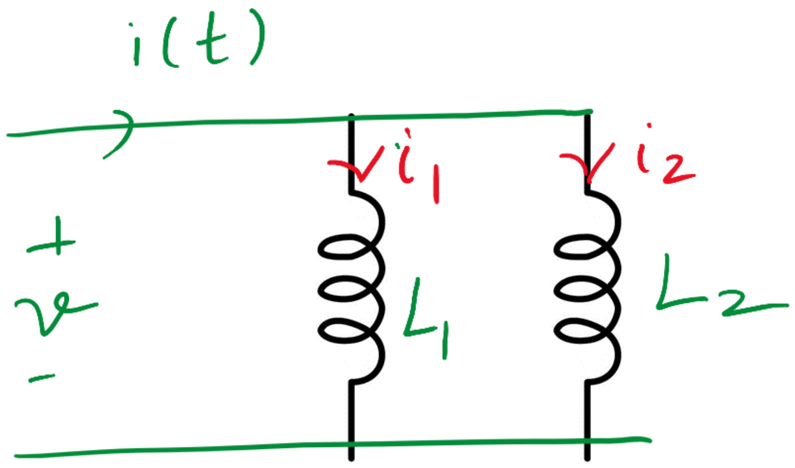
$$v = \frac{q}{C_{eq}} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$



Similarly

$$C_{eq} = C_1 + C_2$$

Inductors in Series or Parallel



* Since flux ∝ current in inductor, we can use flux to determine equivalent inductance (L_{eq}).

* Let's determine L_{eq} using

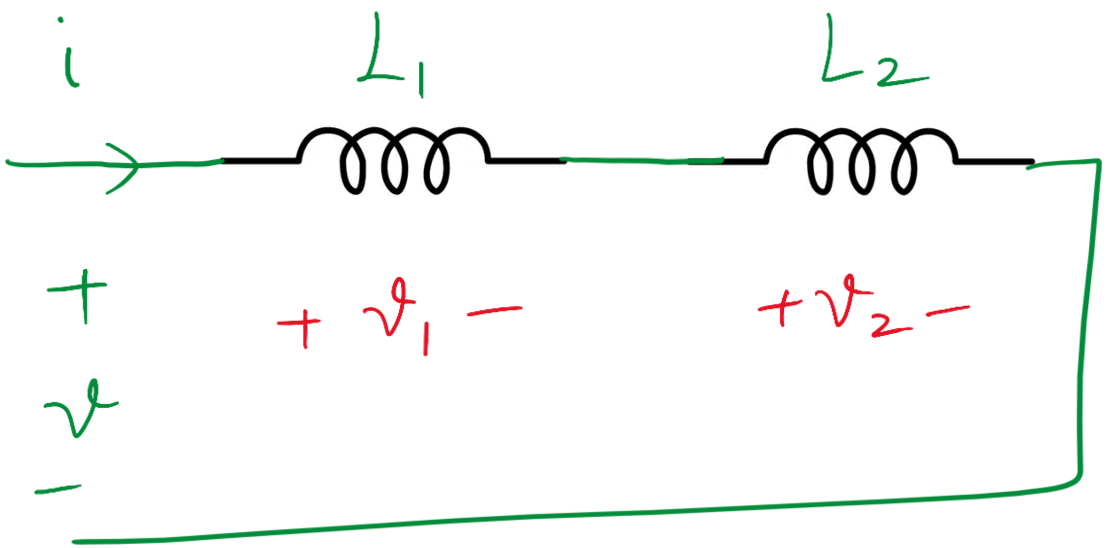
$$v = L_{eq} \frac{di}{dt}$$

Since $i = i_1 + i_2 \Rightarrow v = L_{eq}\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$

$\frac{di_1}{dt} = \frac{v}{L_1}$, $\frac{di_2}{dt} = \frac{v}{L_2} \Rightarrow v = L_{eq}\left(\frac{1}{L_1} + \frac{1}{L_2}\right) v$

$$\Rightarrow \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}} \quad \text{OR} \quad \boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

Inductors in Series or Parallel



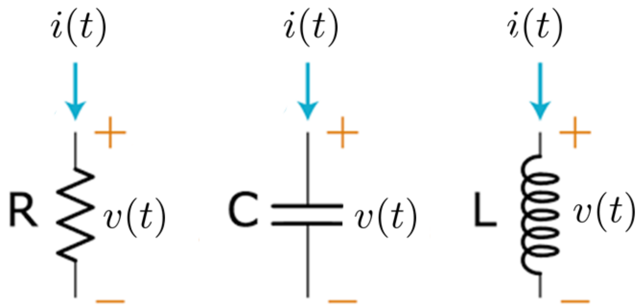
$$v = v_1 + v_2$$

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$\Rightarrow v = \underbrace{(L_1 + L_2)}_{L_{eq}} \frac{di}{dt}$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2}$$

Resistor, Capacitor, Inductor (R,C,L)



Summary:

R,C and L are passive elements

Resistor, R

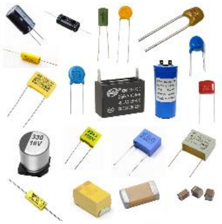
$v = iR$

$i = \frac{v}{R}$

$w(t) = \int_{\tau=-\infty}^{t} p(\tau)d\tau$

$p = vi = i^2R = \frac{v^2}{R}$

Capacitor, C



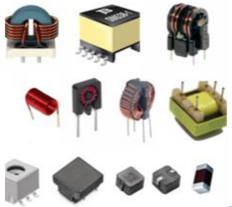
$v(t) = \frac{1}{C}\int_{\tau=-\infty}^{t} i(\tau)d\tau$

$i = C\frac{dv}{dt}$

$w = \frac{1}{2}Cv^2$

$q = Cv$

Inductor, L



$v = L\frac{di}{dt}$

$i(t) = \frac{1}{L}\int_{\tau=-\infty}^{t} v(\tau)d\tau$

$w = \frac{1}{2}Li^2$

$\Psi = Li$

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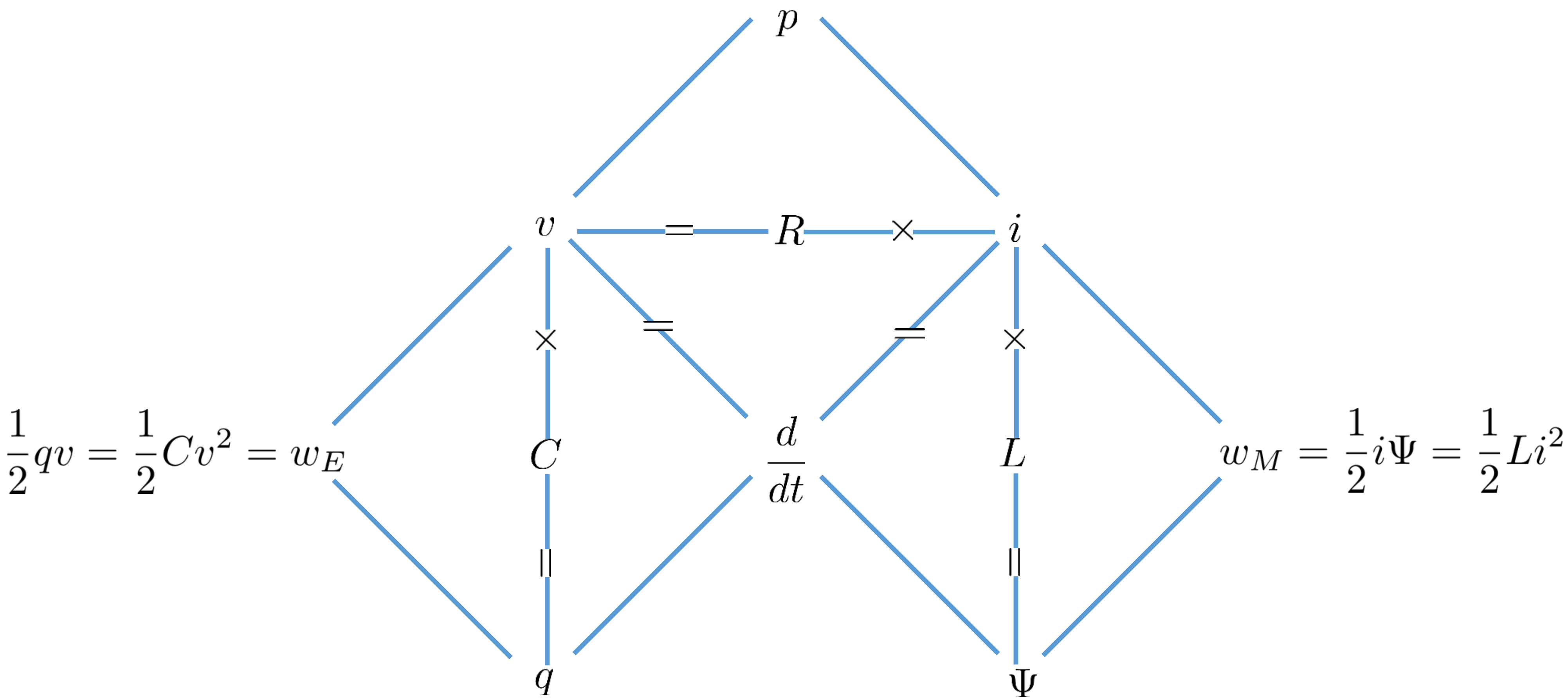
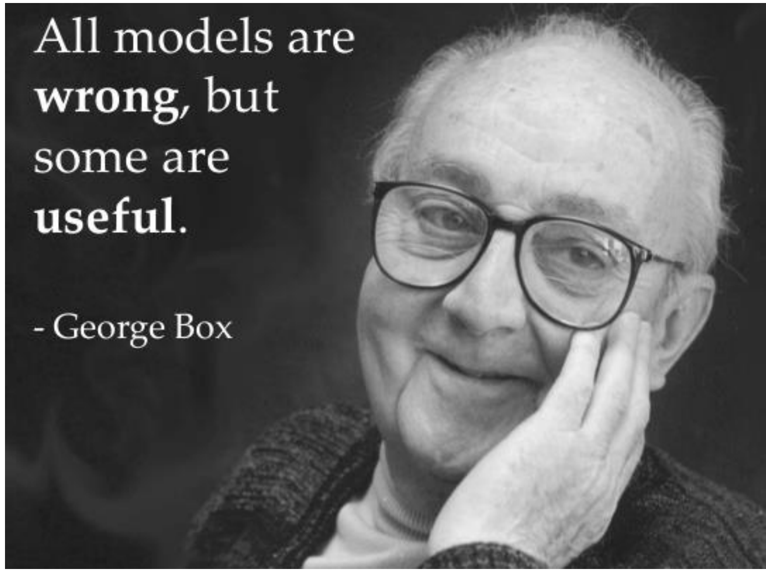
Resistor, Capacitor, Inductor (R,C,L)

Encapsulated:



Practical Models – Resistor

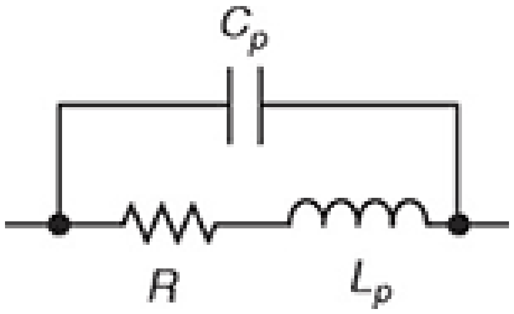


Practical Resistor:

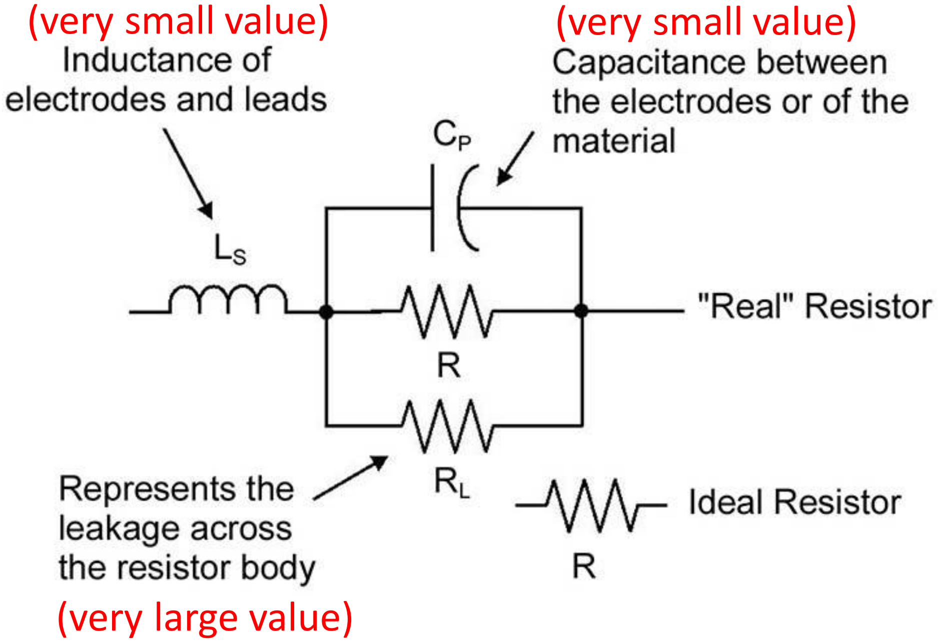
- Stray capacitance – or Parasitic (unwanted) Capacitance
- Parasitic Inductance
- Frequency dependency (prominent effect of stray capacitance and inductance at higher frequencies)
- Non-linear relationship between current and voltage
- Change in resistor due to variations in the temperature and voltage levels.

Circuit Model 1:

L_p - Parasitic Inductance
C_p - Parasitic Capacitance



Circuit Model 2:



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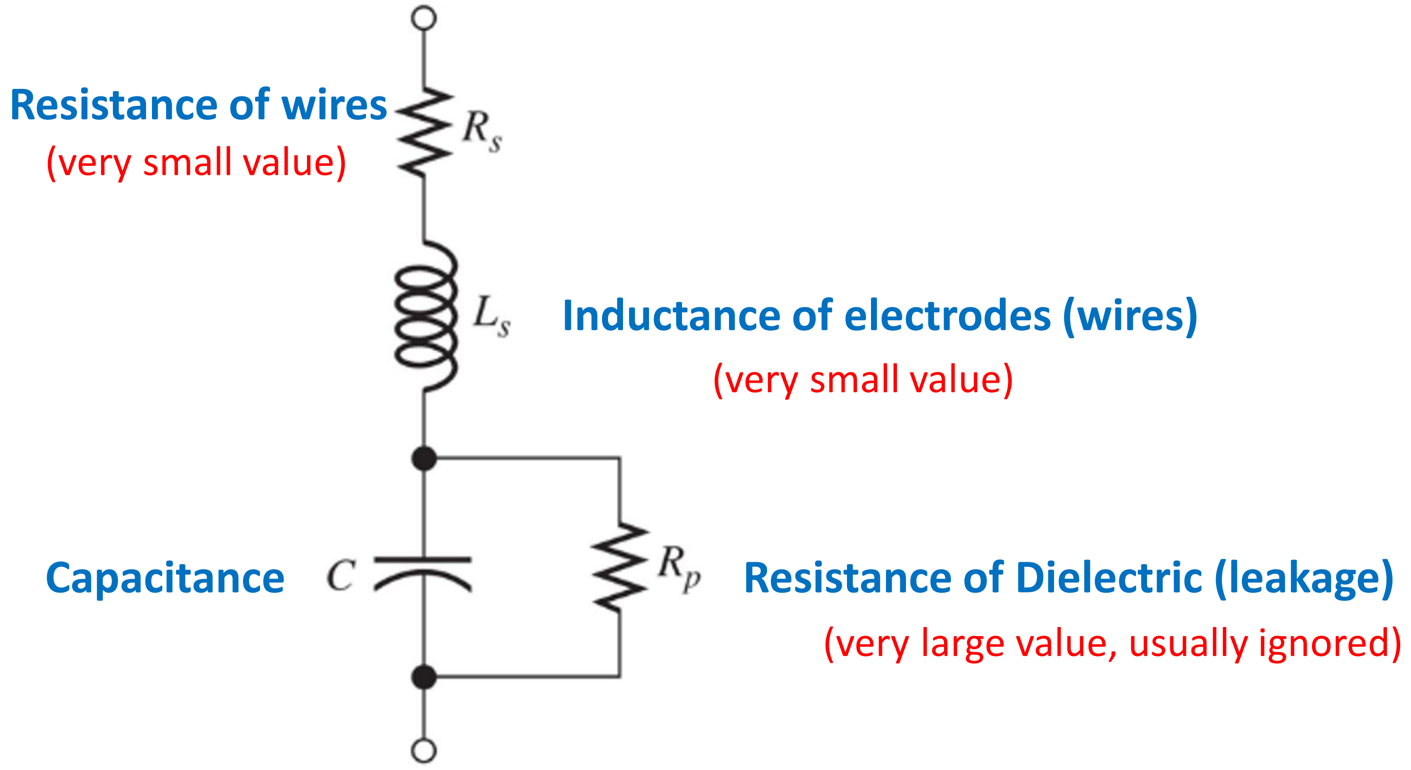
Practical Models – Capacitor

Circuit Model:



L_s - also referred to as equivalent series inductance (ESL)

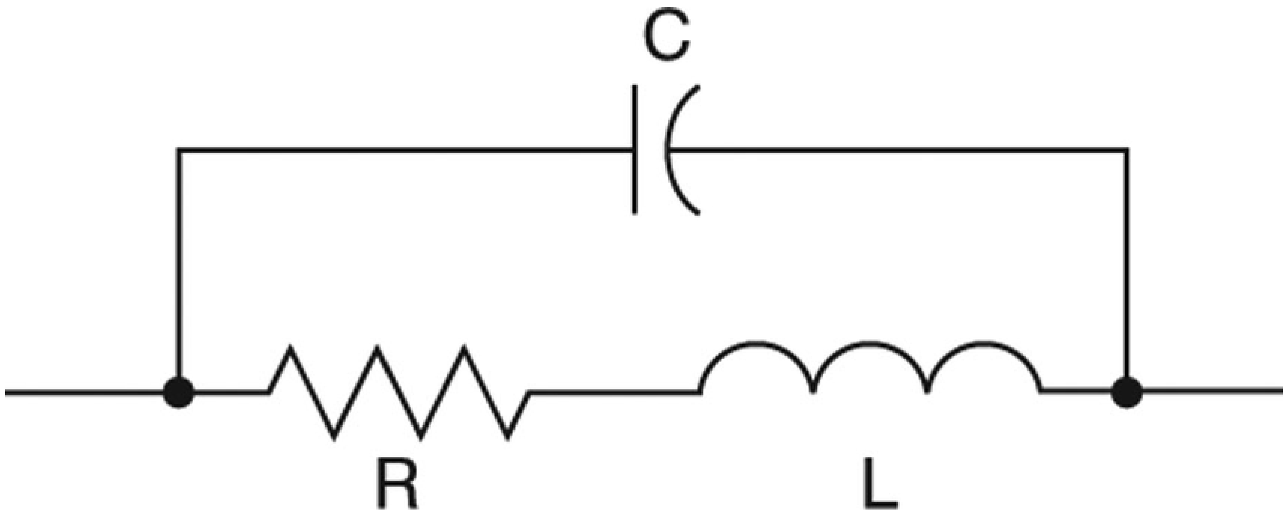
R_s - also referred to as equivalent series resistance (ESR)

Practical Models – Inductor

Circuit Model:

(small value)

Inter-winding Capacitive Effect



Resistance of coil

(small value)

Inductance

Coupled Inductors and Mutual Inductance

The two inductors are said to be **coupled** if the flux produced due to the current in one inductor is linked to the other inductor. In other words, the inductors (two or more) are said to be coupled if they are magnetically linked together by a common magnetic flux.

This linking or coupling is quantified by the 'Mutual inductance'.

Let's understand this in more detail. Consider iron core with two coils as shown below

Current I produces magnetic flux density B

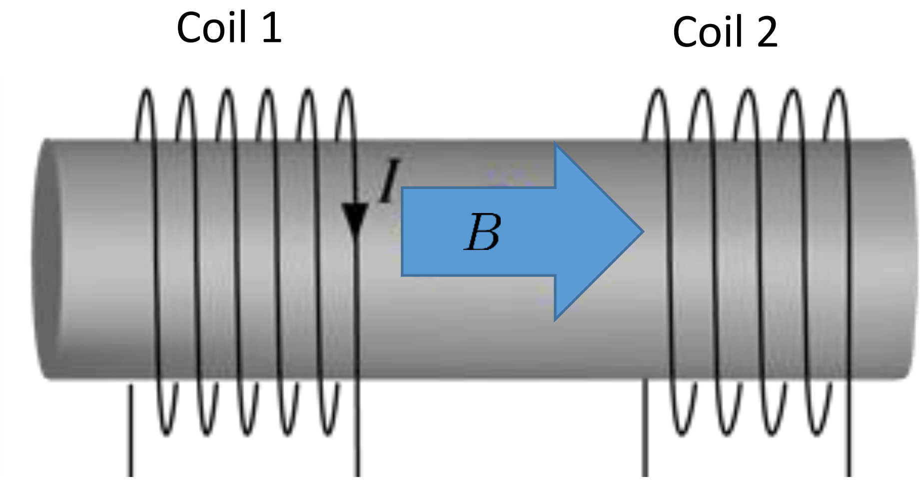
This magnetic field desnity is linked to coil 2 as well.

What we know already that flux in coil 1 is given by

$$\phi_1 = L_1 I$$

Flux in coil 2 is given by

$$\phi_2 = M_{21} I$$



M_{21} Mutual inductance, relates the flux in Coil 2 due to the current in Coil 1

Coupled Inductors

Position of the coils on a common core or by increasing the number of turns of either of the coils increases the flux linkage and consequently increases the mutual inductance.

For example; Transformer

Reciprocity of Mutual Inductance:

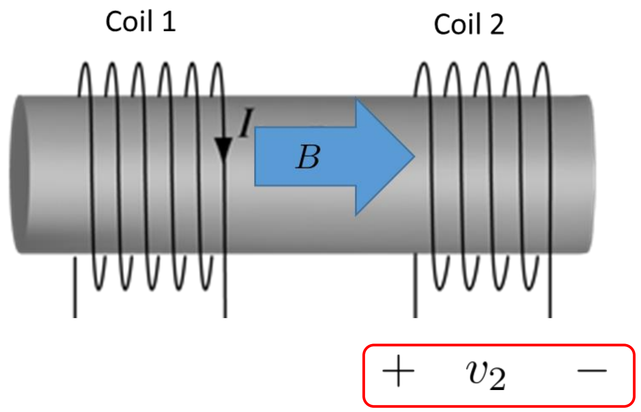
It follows from the ***Reciprocity Theorem*** *(proof is beyond the scope here) that Mutual inductance is reciprocate from one side to other equally, that is,*

$$M_{12} = M_{21} = M$$

Induced voltage due to mutual induction:

Due to flux $\phi_2 = M_{12} I$ in coil 2 due to current in coil 1, the voltage v_2 is induced in coil 2 (Faraday's Law), that is

$$v_2 = \frac{d\phi_2}{dt} = M \frac{dI}{dt}$$



Q: How do we determine the polarity of the induced voltage?

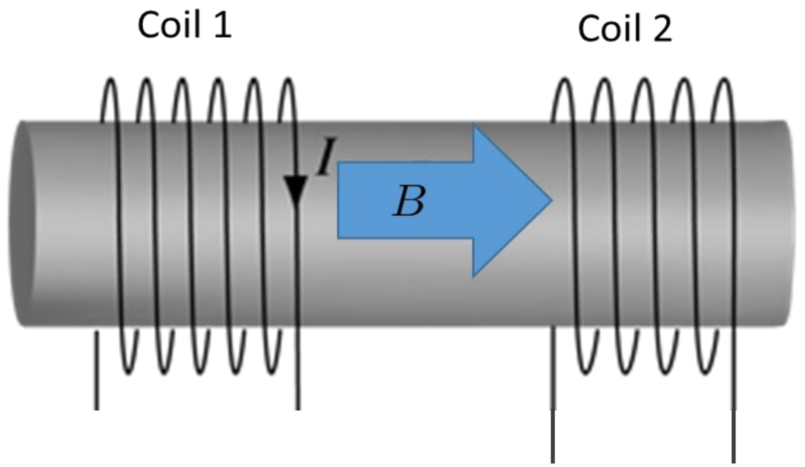
Coupled Inductors

Determine polarity of the voltage:

Using Lenz's Law.

Idea: The current produced in Coil 2 due to the induced voltage across coil 2 creates a magnetic field that should oppose the magnetic field due to the current in coil 1 (the current that is causing induced voltage to develop).

Let's understand this further.



Coupled Inductors

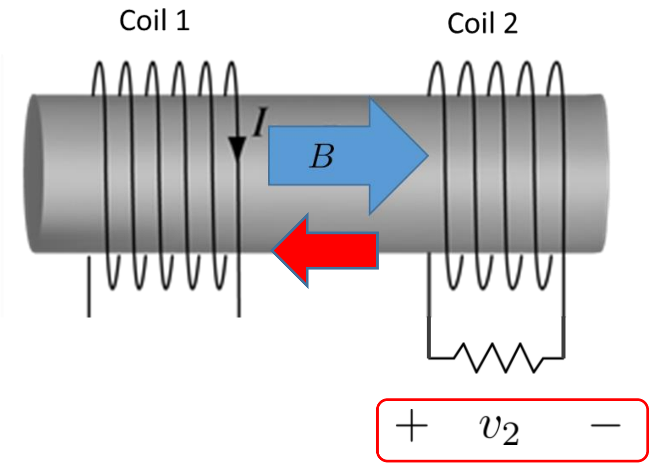
Determine polarity of the voltage:

Using Lenz's Law.

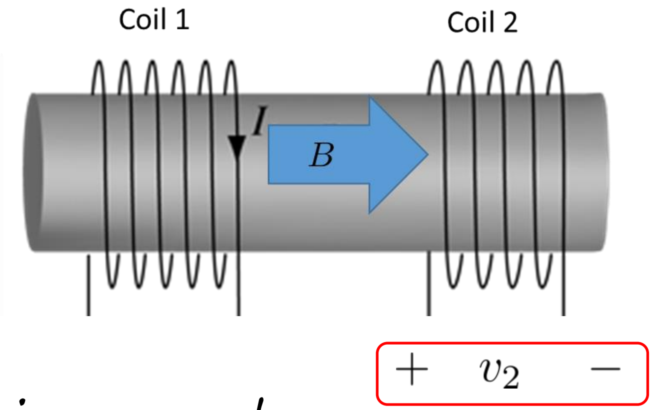
Idea: The current produced in Coil 2 due to the induced voltage across coil 2 creates a magnetic field that should oppose the magnetic field due to the current in coil 1 (the current that is causing induced voltage to develop).

Let's understand this further.

- To determine the polarity, connect a resistor across ends of coil 2.
- Once the resistor is connected, current will flow out of the coil from the positive terminal and enter into the coil from the negative terminal.
- Applying right hand-rule, this will produce magnetic field in the direction indicated by the red arrow, that is, opposing the flux indicated in blue (that is due to the current in coil 1).



Coupled Inductors



Determine polarity of the voltage:

So the polarity indicated is correct.

If coil 2 winding direction is reversed, the polarity of the induced voltage is reversed.

Therefore, the polarity of the voltage depends on the construction of mutual inductors.

Once the inductors are packaged, the user does not know the direction of the winding.

To facilitate users and indicate the polarity of the voltage, engineers use the dot convention.

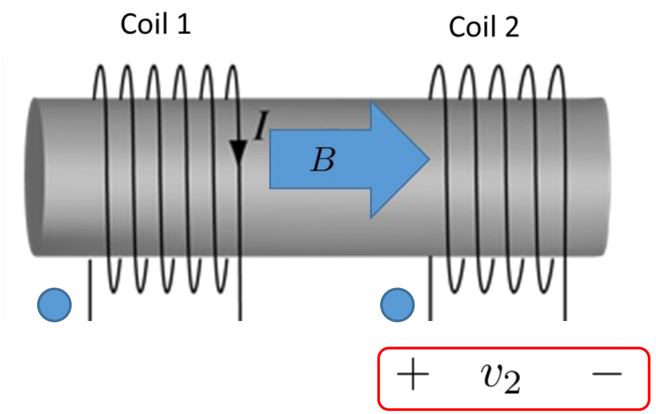
Dot Convention:

- How to use the dots marked on the coupled inductors?
- How to mark the dots given the construction (core, windings directions)?

Coupled Inductors

Dot Convention:

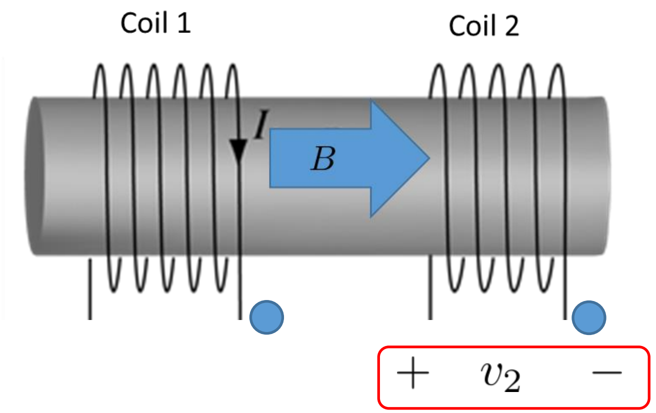
- *How to use the dots marked on the coupled inductors?*



If the current enters at the dotted terminal of one inductor, it induces a voltage at another (coupled) inductor with positive polarity at the dotted terminal.

OR

If the current leaves at the dotted terminal of one inductor, it induces a voltage at another (coupled) inductor with negative polarity at the dotted terminal.

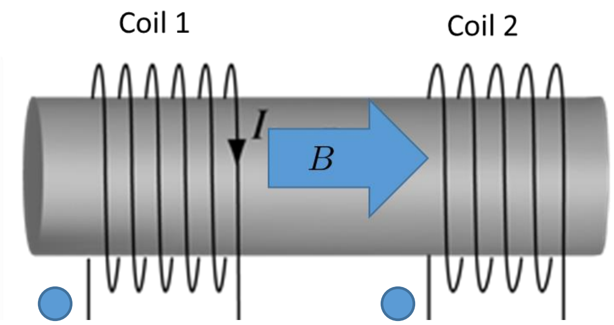


● *Dot adopted to indicate the coupling*

Coupled Inductors

Dot Convention:

- ***How to mark the dots given the construction (core, windings directions)?***
  - *Place the dot arbitrarily on the one winding.*
  - *Determine the direction of the magnetic field (B_1) for the current entering the dotted terminal.*
  - *Place the dot on the second winding on the terminal such that when current enters (or leaves) the dotted terminal, it produces a magnetic field in the direction that enhances (or opposes) B_1.*



Note: *If there are more than two coupled inductors, a separate mark is used for each pair of windings.*