LAHORE UNIVERSITY OF MANAGEMENT SCIENCES
Department of Electrical Engineering

EE 240 Circuits I
Quiz 5 Solution

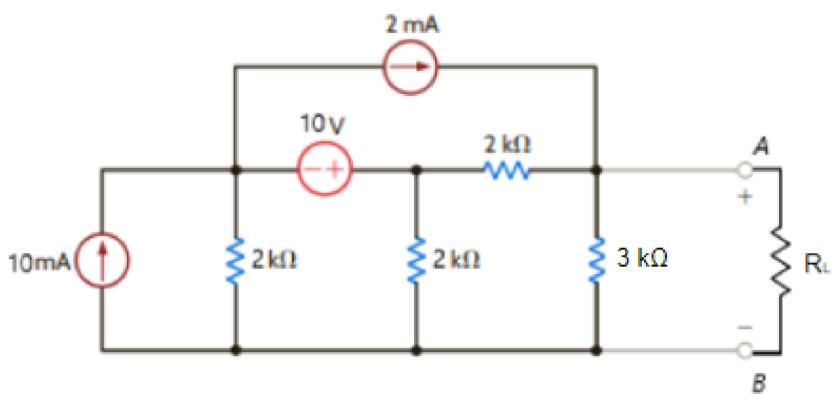
Name: ______________________

Campus ID: ______________________

Total Marks: 10
Time Duration: 15 minutes

Question 1 (10 marks)

Find the Norton equivalent circuit for the following circuit with respect to terminals A,B. Find Load Resistance R_L for maximum power transfer and the maximum power transferred to this load.



Solution:

Step 1: Finding R_{eq}

After replacing voltage sources with a short wire and current sources with open terminals we get the following circuit:

$$R_{\text{eq}} = ((2k \parallel 2k) + 2k) \parallel 3k = 1.5k\,\Omega$$

R_1 and R_2 are in parallel and can be replaced by a 1K resistor which would be in series with R_3. R_3 + 1K = 3K which is in parallel with R_4. R_4 and 3K can be resolved into one 1.5K across terminals A and B.

Step 2: Finding I_{sc}

Note: You can find I_{sc} either using the direct method of shorting the terminals and finding the current through them or by finding V_{oc} and then using source transformations to find I_{sc}.

Method 1

The first thing to note is that there is a short wire connected to Node 3. As a result, all the current will flow through this wire, bypassing the 3k resistor entirely. Therefore, we can replace the resistor with a short wire, which directly connects V_3 to ground, making $V_3 = 0V$.

Equations for Supernode (Node 1 and 2):

$10m = \frac{V_1}{2K} + \frac{V_2}{2K} + \frac{V_2}{2K} + 2m$ Eq 1

$V_2 = V_1 + 10$ Eq 2

Substituting Eq 2 in Eq 1 we find $V_1 = -\frac{4}{3}$ V and $V_2 = \frac{26}{3}$ V

$I_{\text{sc}} = 2\,\text{mA} + \frac{V_2 - V_3}{2k} = \frac{19}{3}\,\text{mA}$

Method 2

First we will find V_{oc} and then use source transformation to find I_{sc}. From the figure we observe that $V_{\text{oc}} = V_3$.

Equation for supernode (Node 1 and 2):

$10m = \frac{V_1}{2K} + \frac{V_2}{2K} + \frac{V_2 - V_3}{2k} + 2m$ Eq 1

$V_2 = V_1 + 10$ Eq 2

Equation for Node 3:

$2m + \frac{V_2 - V_3}{2k} = \frac{V_3}{3K}$ Eq 3

Eq 3 x 6K: $12 + 3V_2 - 3V_3 = 2V_3$ Eq 4

Plugging Eq 2 in Eq 4: $12 + 30 + 3V_1 - 3V_3 = 2V_3$ Eq 5

Now to find V_1 in terms of V_3, we substitute Eq 2 into Eq 1 to find the relationship:

$3V_1 = V_3 - 4$ Eq 6

Substituting Eq 6 in Eq 5 we find $V_3 = \frac{38}{4}V$

Using Source Transformation:

$I_{\text{sc}} = \frac{V_{\text{oc}}}{R_{\text{eq}}} = \frac{38}{4} \times \frac{1}{1.5 \times 10^3\,K} = \frac{19}{3}\,\text{mA}$

The Norton Equivalent Circuit:

Step 3: Find Load Resistance R_L for maximum power transfer and the maximum power transferred to this load.

By the maximum power transfer theorem, we know that when $R_L = R_{\text{eq}} = 1.5\,\text{k}\Omega$, maximum power is transferred to the load.The power can be calculated using:

$$P = I^2 R$$

Using the current divider rule, the current through the load is:

$$I_{\text{load}} = \frac{1.5\,\text{k}\Omega}{1.5\,\text{k}\Omega + 1.5\,\text{k}\Omega} \times \frac{19}{3}\,\text{mA} = \frac{19}{6}\,\text{mA}$$

Now, the power delivered to the load is:

$$P = \left(\frac{19}{6}\,\text{mA}\right)^2 \times 1.5\,\text{k}\Omega = 15.04\,\text{mW}$$