Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 1

Due Date: 5pm, Friday. February 08, 2019 (Dropbox located outside 9-246A, EE Department)
Format: 8 problems, for a total of 110 marks
Instructions:

- You are not allowed to submit a group assignment. Each student must submit his/her own hand-written assignment.
- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. Anybody found guilty would be subjected to disciplinary action in accordance with the university rules and regulations.

Problem 1 8 marks

Write each of the following in polar form, that is, in $re^{j\theta}$.

(a) $1 + j\sqrt{3}$

(b) $\dfrac{e^{j\pi/3} - 1}{1 + j\sqrt{3}}$

(c) j^7

(d) j^j

Express each of the following complex numbers in Cartesian form $(x + jy)$.

(e) $e^{j\pi/2}$

(f) $\sqrt{2}e^{j\pi/4}$

(g) $j^2 e^{j j^2 \pi/3}$

Problem 2 12 marks

Evaluate the following summations or integrals and express your answers in Cartesian form $(x + jy)$.

(a)

$$\sum_{n=-5}^{5} e^{j\pi n/2}$$

(b)

$$\sum_{n=-2}^{\infty} \left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)$$

Hint: Express cosine in the form of complex exponentials using the Euler identity, given by $e^{j\theta} = \cos\theta + j\sin\theta$.

(c)

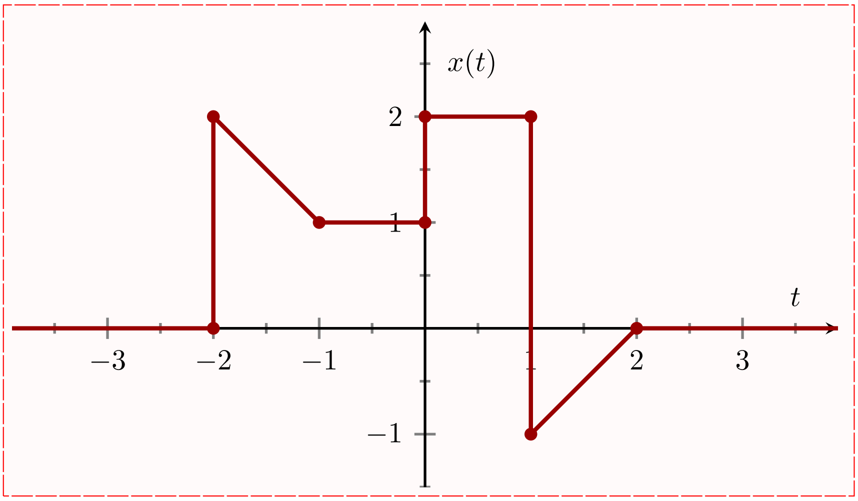
$$\int_{-\infty}^{\infty} e^{-7|t|}dt$$

(d)

$$\int_{0}^{\infty} e^{-t}e^{-j\pi t/2}\sin(5t)dt$$

Problem 3 15 marks

For the CT signal $x(t)$ shown below,



sketch and carefully label:

(a) $\text{Ev}\{x(t)\}$

(b) $\text{Odd}\{x(t)\}$

(c) $x(t/2 + 3)$

(d) $x(1 - t/3)$

(e) $x(t-2)\big(\delta(t-1/2) + u(3-t)\big)$, where $\delta(t)$ is the delta function and $u(t)$ is the step function.

Problem 4 15 marks

Determine whether or not each of the following signals is periodic. If the signal is periodic determine its fundamental period. Please note that the period of the product of two periodic functions is not the same as we have for the sum of two periodic functions.

(a)

$$x[n] = \cos(4n + \frac{\pi}{4})$$

(b)

$$x[n] = (-1)^n \cos(\frac{2\pi n}{7})$$

(c)

$$x(t) = \sum_{n=-\infty}^{\infty} e^{(2t-n)}$$

(d)

$$x(t) = \sin^2(4t) \equiv \big(\sin(4t)\big)^2$$

(e)

$$x[n] = \cos\big(\frac{\pi}{2}n\big)\cos\big(\frac{\pi}{4}n\big)$$

(f)

$$x[n] = 2\cos\big(\frac{\pi}{4}n\big) + \sin\big(\frac{\pi}{8}n\big) - 2\cos\big(\frac{\pi}{2}n + \frac{\pi}{6}\big)$$

Problem 5 25 marks

For each of the following systems described by the input-output relationship:

SYSTEM 1: $y[n] = \cos\big(2\pi\, x[n+1]\big) + x[n]$

SYSTEM 2: $y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$

SYSTEM 3: $y(t) = \int_{-10}^{t} x(\tau)\, d\tau$

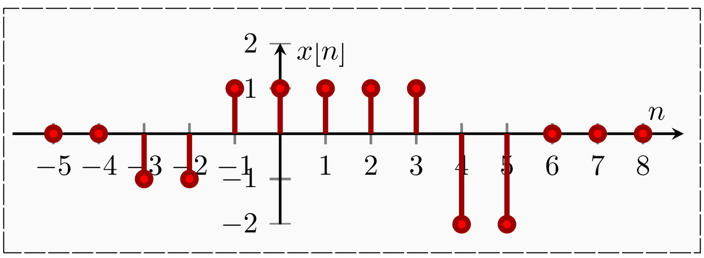
SYSTEM 4: $y[n] = \Big(-\frac{1}{5}\Big)^{2n} \big(x[n] + 1.5\big)$

SYSTEM 5: $y(t) = x(t)\, \cos(\omega_c t)$

where $x(t)$ or $x[n]$ is the system input and $y(t)$ or $y[n]$ is the system output, determine whether the system is linear, time-invariant, causal, stable and memoryless. Briefly justify your answer for each case.

Problem 6 15 marks

Consider the discrete-time signal $x[n]$ given below



(a) Sketch the signal $x[-n+3]$

(b) Sketch the signal $x[3n-1]$

(c) Express the signal as a summation of DT impulse functions

(d) Express the signal in terms of DT unit step functions.

MATLAB Based Problems

Problem 7 10 marks

Visualization of Continuous Time Complex Exponential

Consider a continuous time complex exponential $x(t) = Ae^{\alpha t}$ where 'A' and 'α' can take real as well as complex values. The equations below express 'A' in polar form and 'α' in rectangular form that is:

$$A = |A|e^{j\theta},$$
$$\alpha = r + j\omega_o$$

Then

$$Ae^{\alpha t} = |A|e^{j\theta}e^{(r+j\omega_o)t} = |A|e^{rt}e^{j(\omega_o t+\theta)}$$

Using Euler's relation, this equation can be expanded further into

$$Ae^{\alpha t} = |A|e^{rt}cos(\omega_o t + \theta) + j|A|e^{rt}sin(\omega_o t + \theta)$$

ω_o determines the rate of oscillation of the signal. For r=0, the real and imaginary parts of a complex exponential are sinusoidal. An important property of this signal is that it is periodic.The sign of 'r' determines if the sinusoid will be a decaying sinusouid ('damped' sinusoid) or a growing sinusoid.

The code given below generates a plot of continuous time complex exponential along with its real and imaginary parts. Run it on MATLAB in order to see the result and try analyzing what effect would varying ω_o have on the rate of oscillation and how would the sinusoid change for r>0 and r<0.

```
fs=500;
t= -10:1/fs:30;
a=j;
A=1;
y=A*exp(a*t);
h=plot3(t,imag(y),real(y),'b');
hold on;
h=plot3(t,ones(size(t)),real(y),'r');
h=plot3(t,imag(y),-ones(size(t)),'g');
hold off;
grid on;
```

```
xlabel('Time(s)');
ylabel('Imaginary Part');
zlabel('Real Part');
title('Blue: Complex,Red: Real,Green: Imaginary');
View(27.5,22);
```

How does the plot change when $a = \pm 0.05 + j2$? Include the plots in your submission.

Problem 8 10 marks

Periodicity of Discrete Time and Continuous Time Signals

(a) **[5 marks]** Plot the continuous time signal $x(t) = \cos(\frac{3\pi t}{5})$ for $t = 0 : 0.0005 : 10$. On the same figure, plot $x_1(t) = \cos(\frac{7\pi t}{5})$ in different color. What do you observe from the given plots?

(b) **[5 marks]** Now on a separate figure, use the 'stem' command to plot $x[n] = \cos(\frac{3\pi n}{5})$ for $n = 1 : 1 : 10$. On the same figure plot $x_1[n] = \cos(\frac{7\pi n}{5})$ and $x_2[n] = \cos(\frac{-7\pi n}{5})$ in different, observe the 3 plots and include your observations in your submission.

— End of Assignment —