Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 1

Solutions

Problem 1 8 marks

Write each of the following in polar form, that is, in $re^{j\theta}$.

(a) $1 + j\sqrt{3}$

$z_1 = a + jb = 1 + j\sqrt{3}$
$r = |z_1| = \sqrt{a^2 + b^2}$
$r = \sqrt{1^2 + (\sqrt{3})^2}$
$\implies r = 2$
$\theta = \tan^{-1}(b/a)$
$\theta = \tan^{-1}(\sqrt{3}/1)$
$\implies \theta = \pi/3$
$\therefore z_1 = e^{j\theta} = \boxed{2e^{j\pi/3}}$

(b) $\dfrac{e^{j\pi/3} - 1}{1 + j\sqrt{3}}$

$z_2 = \dfrac{e^{j\pi/3} - 1}{1 + j\sqrt{3}}$

Using the Euler's Identity
$e^{j\pi/3} = \cos(\pi/3) + j\sin(\pi/3) = -(1 - j\sqrt{3})/2$

$\therefore z_2 = -\dfrac{(1 - j\sqrt{3})}{2(1 + j\sqrt{3})}$

Which reduces to:
$z_2 = -\dfrac{(1 - j\sqrt{3})^2}{8}$
Solving further, we get

$z_2 = \dfrac{1 + j\sqrt{3}}{4}$
Putting value of $1 + j\sqrt{3}$ from part (a)
$z_2 = \dfrac{2e^{j\pi/3}}{4}$
$\therefore \boxed{z_2 = \dfrac{1}{2}e^{j\pi/3}}$

(c) j^7

$z_3 = j^7 = j^4 j^2 j$
$z_3 = -j = e^{j3\pi/2}$
$\therefore \boxed{z_3 = e^{j3\pi/2}}$

(d) j^j

$z_4 = j^j$
$z_4 = (e^{j\pi/2})^j = e^{j^2\pi/2}$
$z_4 = e^{-\pi/2} = 0.2079$
$\therefore \boxed{z_4 = 0.2079e^{j0}}$

Express each of the following complex numbers in Cartesian form $(x + jy)$.

(e) $e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j$

(f) $\sqrt{2}e^{j\pi/4} = \sqrt{2}(\cos\pi/4 + j\sin\pi/4) = 1 + j$

(g) $j^2 e^{j^{j^2}\pi/3} = -e^{-j\pi/3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}j$

Problem 2 12 marks

Evaluate the following summations or integrals and express your answers in Cartesian form $(x + jy)$.

(a)

$$\sum_{n=-5}^{5} e^{j\pi n/2}$$

$$x = \sum_{n=1}^{5}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) + e^{j\frac{\pi}{2}0}$$

As $e^{j\pi/2} = j \textit{ and } e^{-j\pi/2} = -j$, so,

$$x = \sum_{n=1}^{5}(j^n + (-j)^n) + 1$$

Expanding the summation
$x = (j - j) + (j^2 + j^2) + (j^3 - j^3) + (j^4 + j^4) + (j^5 - j^5) + 1$
$x = 2j^2 + 2j^4 + 1$
$x = 2(-1) + 2(1) + 1$
$\implies x = 1 = \boxed{1 + j0}$

(b)

$$\sum_{n=-2}^{\infty}\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)$$

Hint: Express cosine in the form of complex exponentials using the Euler identity, given by $e^{j\theta} = \cos\theta + j\sin\theta$.

$$x = \sum_{n-2}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2}\right)$$

Using the fact that: $e^{j\pi/2} = j$ and $e^{-j\pi/2} = -j$

$$x = \frac{1}{2}\sum_{n=-2}^{\infty}(\frac{1}{2})^n(j^n + (-j)^n)$$

$$x = \frac{1}{2}\left[\sum_{n=-2}^{\infty}(\frac{j}{2})^n + \sum_{n=-2}^{\infty}(\frac{-j}{2})^n\right]$$

Let m = n +2. Making this replacement in the above equation, we get,

$$x = \frac{1}{2}\left[\sum_{m=0}^{\infty}(\frac{j}{2})^{m-2} + \sum_{m=0}^{\infty}(\frac{-j}{2})^{m-2}\right]$$

$$x = \frac{1}{2}\left[(\frac{j}{2})^{-2} + \sum_{m=0}^{\infty}(\frac{j}{2})^m + (\frac{-j}{2})^{-2} + \sum_{m=0}^{\infty}(\frac{-j}{2})^m\right]$$

Which simplifies to,

$$x = \frac{1}{2}\left[-8 + \sum_{m=0}^{\infty}(\frac{j}{2})^m + \sum_{m=0}^{\infty}(\frac{-j}{2})^m\right]$$

Applying the identity $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ and simplifying gives,

$$x = \frac{1}{2}\left[-8 + \frac{2}{2-j} + \frac{2}{2+j}\right]$$

$$x = \frac{1}{2}\left[-8 + \frac{2(2+j)}{5} + \frac{2(2-j)}{2}\right]$$

$$x = \frac{1}{2}.\frac{2}{5}(-20 + 2 + j + 2 - j)$$

$\Longrightarrow$

$$\boxed{x = -16/5 = -16/5 + j0}$$

(c)

$$\int_{-\infty}^{\infty} e^{-7|t|}dt$$

$$x = 2\int_{0}^{\infty} e^{-7t}dt$$

$$x = 2\frac{e^{-7t}}{-7}\Big|_0^{\infty}$$

$$\Longrightarrow x = \frac{2}{7} = \boxed{\frac{2}{7} + j0}$$

(d)

$$\int_{0}^{\infty} e^{-t}e^{-j\pi t/2}\sin(5t)dt$$

This equation can also be written as,

$$x = \int_{0}^{\infty} e^{-(1+j\pi/2)t}\sin(5t)dt$$

Now, Applying the identity,

$$\int_0^\infty e^{-ax}.sin(bx)dx = \frac{b}{a^2+b^2}$$

We get,

$$x = \frac{5}{(1+j\pi/2)^2+5^2}$$

Which reduces to,

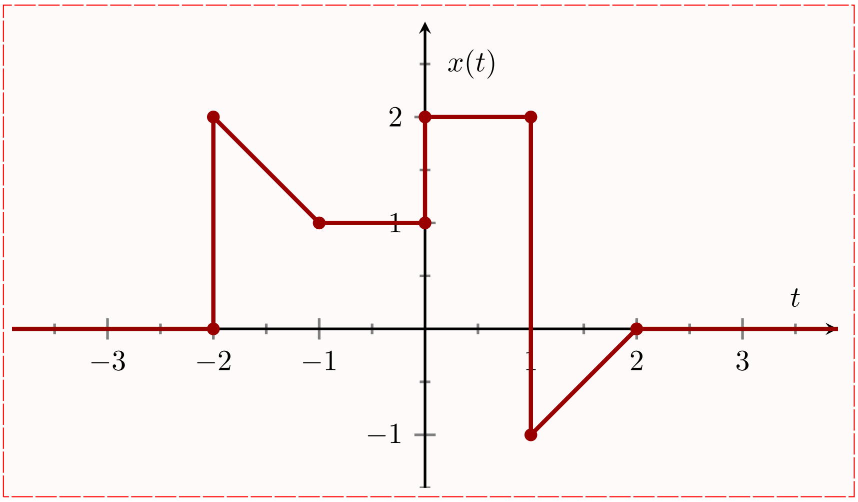
$$x = \frac{20}{104-\pi^2+j\pi}$$

Or,

$$\boxed{x = 0.2088 - j0.0279}$$

Problem 3 15 marks

For the CT signal $x(t)$ shown below,



sketch and carefully label:

(a) $\text{Ev}\{x(t)\}$

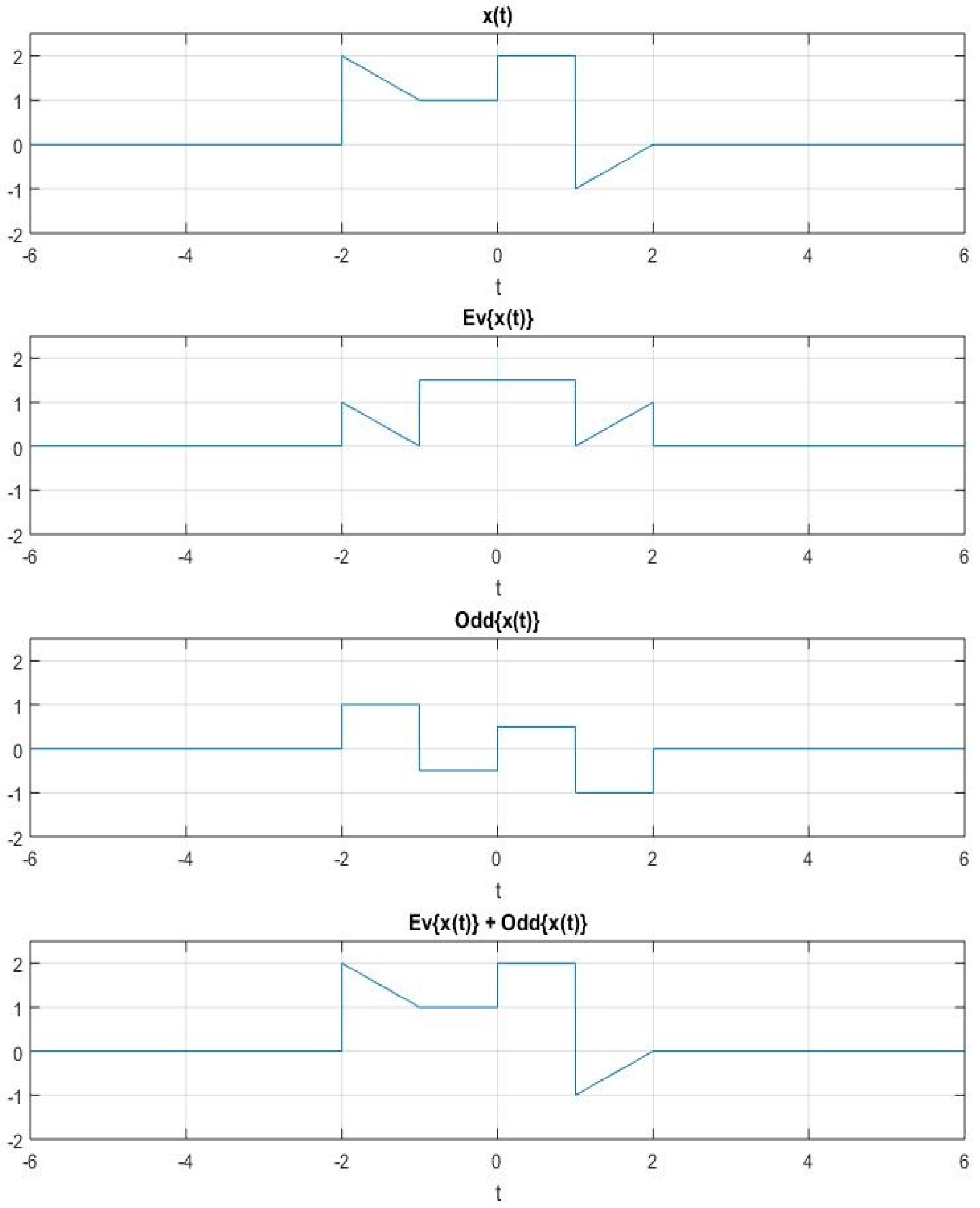
(b) $\text{Odd}\{x(t)\}$

(c) $x(t/2+3)$

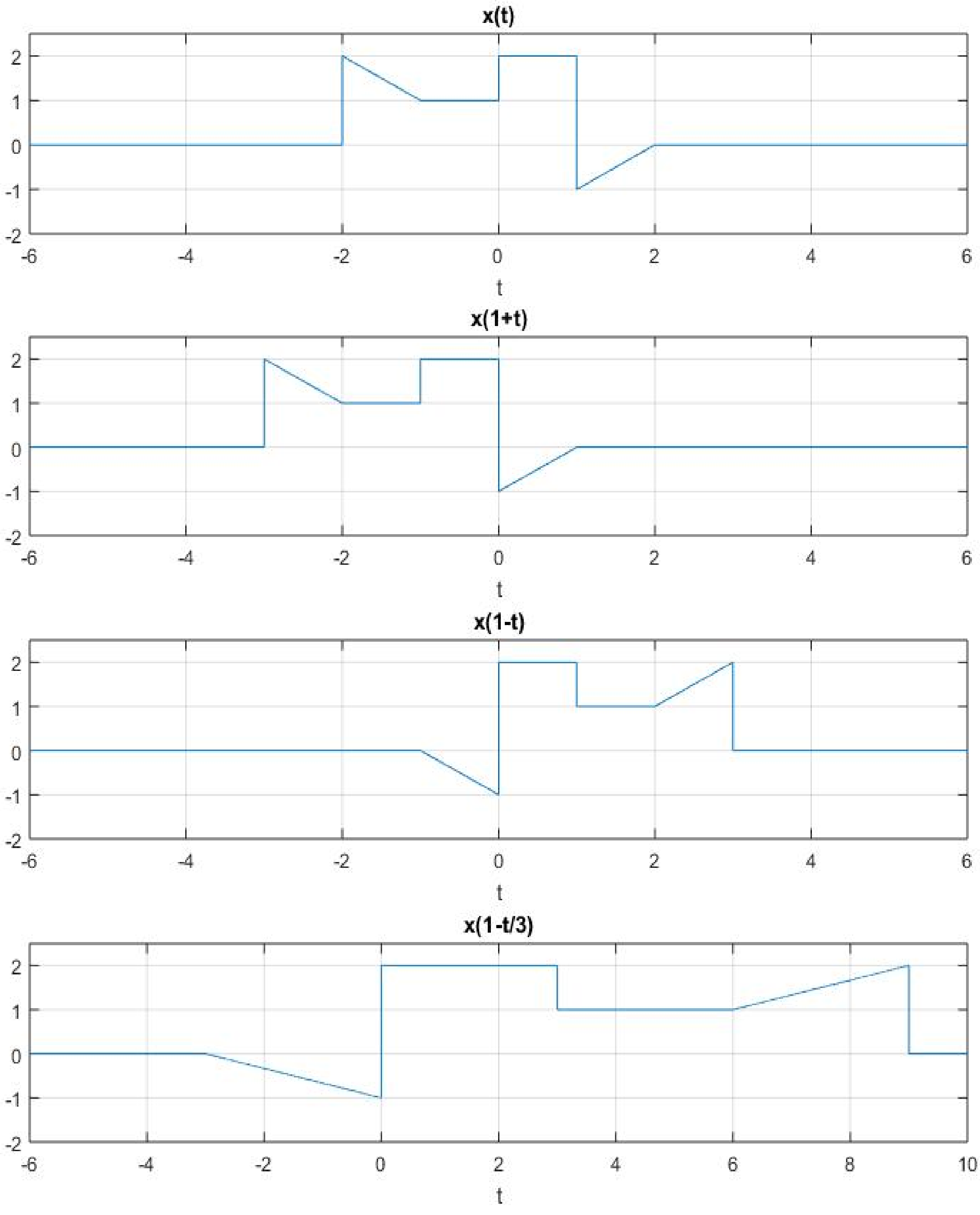
(d) $x(1-t/3)$

(e) $x(t-2)\big(\delta(t-1/2)+u(3-t)\big)$, where $\delta(t)$ is the delta function and $u(t)$ is the step function.

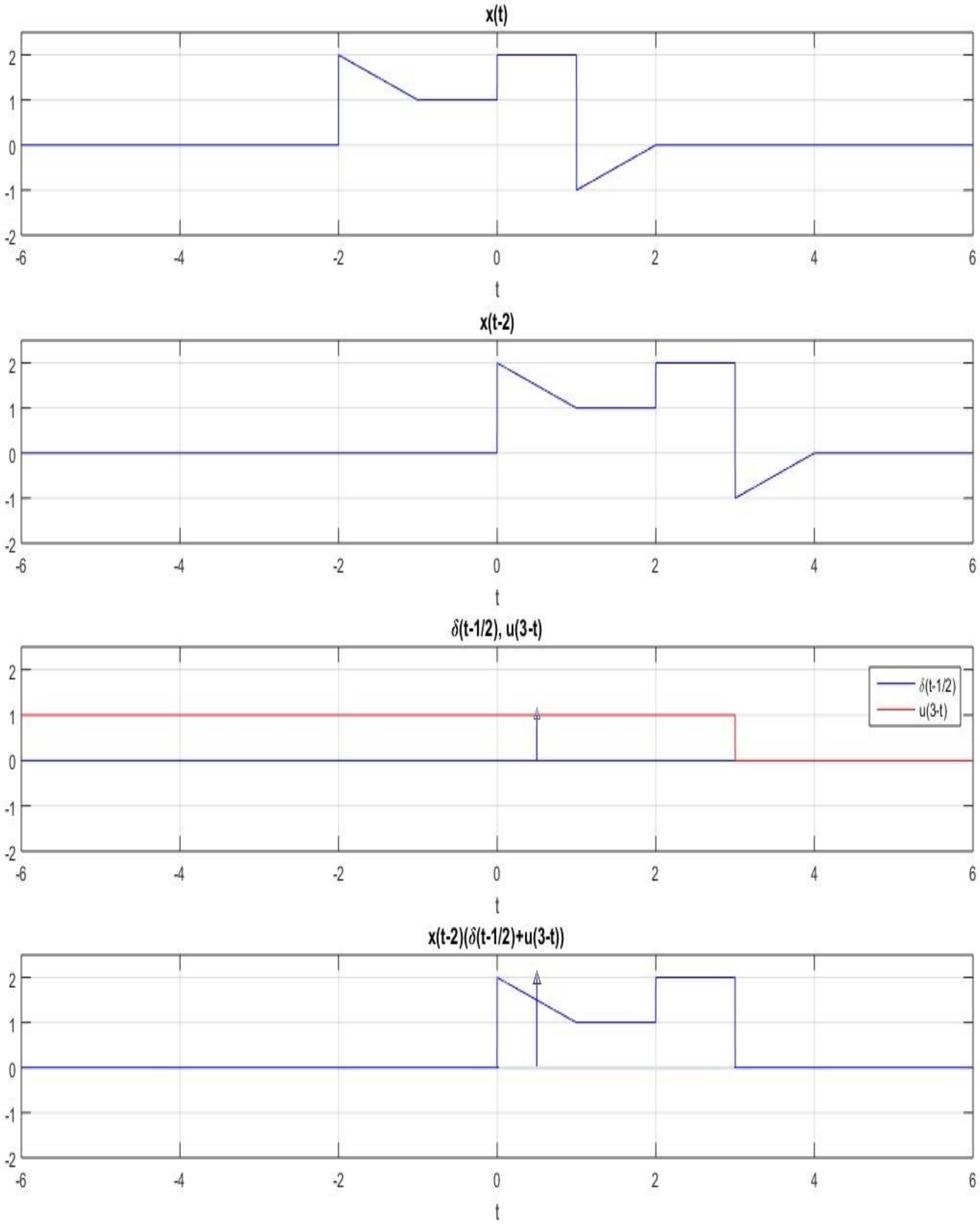
Ev$\{x(t)\}$ and Odd$\{x(t)\}$



$x(1 - t/3)$



$x(t-2)(\delta(t-1/2)+u(3-t))$



Problem 4 15 marks

Determine whether or not each of the following signals is periodic. If the signal is periodic determine its fundamental period. Please note that the period of the product of two periodic functions is not the same as we have for the sum of two periodic functions.

(a)

$$x[n] = \cos(4n + \frac{\pi}{4})$$

Comparing with $cos(\omega_0 n + \phi)$
$\omega_0 = 4$
For $x[n]$ to be periodic,
$N = \frac{2\pi m}{\omega_0}$
$\implies N = \frac{\pi}{2}m$
Since there is no such integer m which will make the N an integer, therefore $x[n]$ is not periodic.

(b)

$$x[n] = (-1)^n \cos(\frac{2\pi n}{7})$$

Using the following identities,
$cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$ and $e^{j\pi} = -1$
$x[n] = e^{j\pi n}.\frac{e^{j2\pi n/7}+e^{-j2\pi n/7}}{2}$
Which reduces to,
$x[n] = \frac{e^{j9\pi n/7}+e^{j5\pi n/7}}{2}$
As, period of $e^{j\omega_0 n} = N = \frac{2\pi}{\omega_0}m$
So, period of $e^{j9\pi n/7} = N_1 = \frac{2\pi * 7}{9\pi}m = 14$
and period of $e^{j5\pi n/7} = N_2 = \frac{2\pi * 7}{5\pi}m = 14$
Which gives an LCM of 14.
$\therefore$ The given signal is periodic with fundamental period of 14.

(c)

$$x(t) = \sum_{n=-\infty}^{\infty} e^{(2t-n)}$$

The given signal is sum of real exponentials and real exponential signals are aperiodic. Also, the sum of aperiodic signals is also aperiodic.
$\therefore$ The given signal is not periodic.

(d)

$$x(t) = \sin^2(4t) \equiv \big(\sin(4t)\big)^2$$

$x(t) = \sin^2(4t) \equiv (\sin(4t))^2$
$x(t) = \sin^2(4t)$
$x(t) = \frac{1-\cos(8t)}{2}$
Fundamental period of $\cos(8t)$ is $\pi/4$.
$\therefore x(t)$ is periodic with fundamental period $\pi/4$

(e)

$$x[n] = \cos\big(\frac{\pi}{2}n\big)\cos\big(\frac{\pi}{4}n\big)$$

Applying the identity
$cos(\alpha).cos(\beta) = \frac{1}{2}[cos(\alpha+\beta) + cos(\alpha-\beta)]$
We get
$x[n] = \frac{1}{2}[cos(3\pi n/4) + cos(\pi n/4)]$

Period of $cos(3\pi n/4) = 2\pi/\omega = \frac{8}{3}m = 8$
Period of $cos(\pi n/4) = 2\pi/\omega = 4m = 4$
LCM of 4 and 8 = 8
$\therefore$ Fundamental Period = 8.

(f)

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$

Period of $cos(\frac{\pi}{4}n) = T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi*4}{\pi} = 8$
Period of $sin(\frac{\pi}{8}n) = T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi*8}{\pi} = 16$
Period of $cos(\frac{\pi n}{2} + \frac{\pi}{6}) = T_3 = \frac{2\pi}{\omega_3} = \frac{2\pi*2}{\pi} = 4$
LCM of T_1, T_2 and $T_3 = 16$
$\therefore x[n]$ is periodic with fundamental period of 16.

Problem 5 25 marks

For each of the following systems described by the input-output relationship:

$$\text{System 1:} \qquad y[n] = \cos\big(2\pi\, x[n+1]\big) + x[n]$$

$$\text{System 2:} \qquad y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$$

$$\text{System 3:} \qquad y(t) = \int_{-10}^{t} x(\tau)\, d\tau$$

$$\text{System 4:} \qquad y[n] = \left(-\frac{1}{5}\right)^{2n} \big(x[n] + 1.5\big)$$

$$\text{System 5:} \qquad y(t) = x(t)\, \cos(\omega_c t)$$

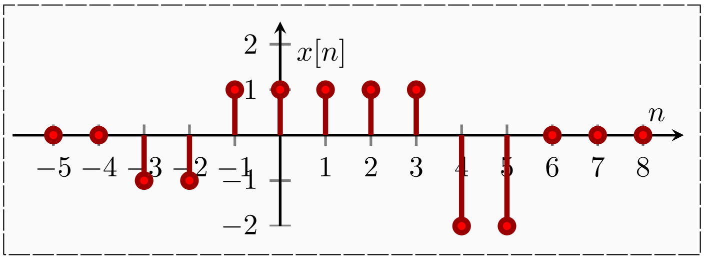
where $x(t)$ or $x[n]$ is the system input and $y(t)$ or $y[n]$ is the system output, determine whether the system is linear, time-invariant, causal, stable and memoryless. Briefly justify your answer for each case.

System	Linear	Time-Invariant	Causal	Stable	Memoryless
System 1	X	✓	X	✓	X
System 2	✓	X	✓	✓	✓
System 3	✓	X	X	X	X
System 4	X	X	✓	X	✓
System 5	✓	X	✓	✓	✓

You need to show each property of the system by following the first definition approach.

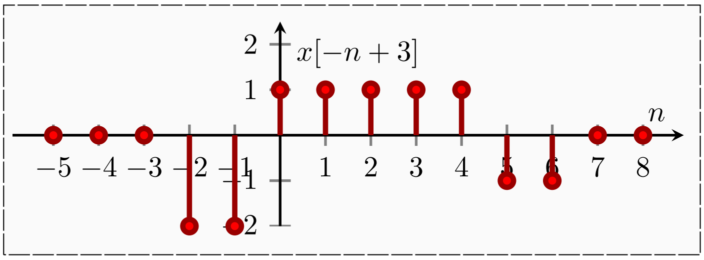
Problem 6 15 marks

Consider the discrete-time signal $x[n]$ given below

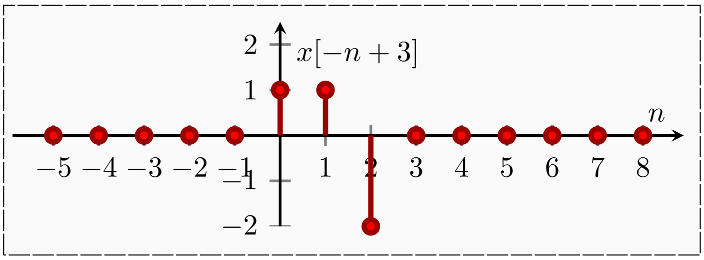


(a) Sketch the signal $x[-n+3]$

Shift by 3 to the left (time advance) followed by time reversal (flip around y-axis).



(b) Sketch the signal $x[3n-1]$ Shift by 1 to the right (time delay) followed by time compression (downsampling) by a factor of 3.



(c) Express the signal as a summation of DT impulse functions

$x[n] = -\delta[n+3] - \delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - 2\delta[n-5]$

(d) Express the signal in terms of DT unit step functions.

$$x[n] = -u[n+3] + 2u[n+1] - 3u[n-4] + 2u[n-6]$$