Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 2

Due Date: 5pm, Monday. Feb 25, 2019 (Dropbox located outside 9-246A, EE Department)
Format: 5 problems, for a total of 70 marks
Instructions:

- You are not allowed to submit a group assignment. Each student must submit his/her own hand-written assignment.
- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. Anybody found guilty would be subjected to disciplinary action in accordance with the university rules and regulations.

Problem 1 (15 marks)

(a) [**4 marks**] A Continuous Time LTI system has input signal $x(t)$ and output signal $y(t)$ related through

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau$$

What is the impulse response for this system? That is, given $y(t) = x(t) * h(t)$, find $h(t)$.

(b) [**8 marks**] For an LTI system given in part (a), determine the output $y(t)$ for an input $x(t)$ given by

$$x(t) = u(t) - u(t-1).$$

Show all working and also plot the output.

(c) [**3 marks**] Using the result of part(b), determine the output $y(t)$ for an input $x(t)$ given by

$$x(t) = 2\,u(t-3) - u(t-4) - u(t-5).$$

Problem 2 (15 marks)

Let the three LTI systems are connected in a configuration shown in Fig. 1, where the impulse response of each system is given by:

$$h_1[n] = \delta[n+2], \quad h_2[n] = \delta[n-2], \quad \text{and} \quad h_3[n] = (2)^n \big(u[n-3] - u[n-7]\big).$$

Since convolution is a linear operation, the output $y[n]$ and the input $x[n]$ of the overall system in Fig. 1 are related by

$$y[n] = x[n] * \big(h_3[n] * (h_1[n] - h_2[n])\big).$$

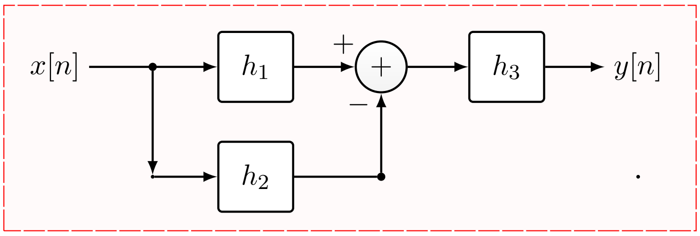

Figure 1: Cascade connection of systems.

(a) **[10 marks]** For a unit step function as an input signal, that is, $x[n] = u[n]$, determine the output $y[n]$.

(b) **[5 marks]** Using the result of part (a) or otherwise, determine the impulse response $h[n]$ of the overall system.

Problem 3 (10 marks)

Let $h(t)$ be the triangular pulse shown in Fig. 2 and $x(t)$ be the impulse train depicted in Fig. 3. That is,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT).$$

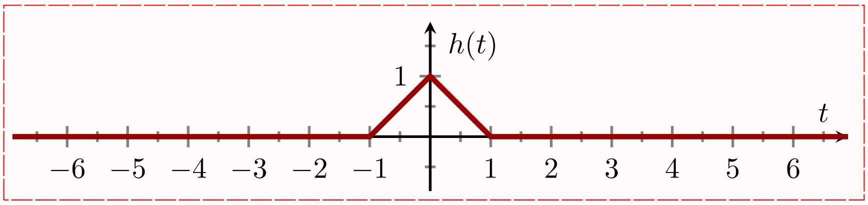

Figure 2: Triangular pulse $h(t)$.

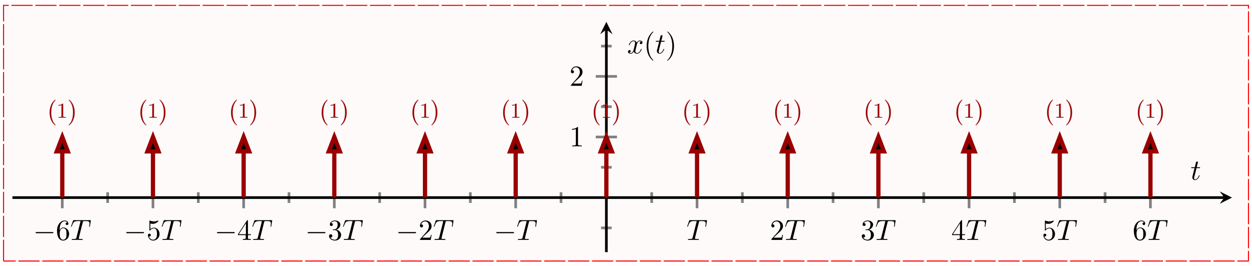

Figure 3: Impulse train $x(t)$ with spacing T.

Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T:

(a) **[5 marks]** $T = 2$

(b) **[5 marks]** $T = 1.5$

Problem 4 (15 marks)

Consider the following convolution

$$y(t) = x(t) * h(t)$$

where

$$x(t) = e^{-|t|} \quad \text{and} \quad h(t) = 2\,u(t-3) - 2\,u(t-5).$$

(a) [**8 marks**] Draw $x(t)$ and $h(t)$ and determine the convolution

$$y(t) = x(t) * h(t) \triangleq \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

Also draw $y(t)$.

(b) [**7 marks**] Now, determine the convolution by first evaluating the convolution

$$z(t) = x(t) * h'(t),$$

where $h'(t)$ denotes the first derivative of $h(t)$ with respect to time. Using $z(t)$, determine $y(t) = \int_{-\infty}^{t} z(\tau)d\tau$. Provide brief comments on determining the convolution this way?

Problem 5 (15 marks)

(a) [**3 marks**] Suppose we observe that the impulse response of an LTI system satisfies $1 \leq h(t) \leq 3$. Can we decide the system's stability (or instability)?

(b) [**3 marks**] Give an example of a causal LTI system whose inverse in not causal.

(c) [**3 marks**] A discrete-time FIR system (a system whose impulse response is of finite duration) is always

1. Stable.
2. Unstable.
3. Causal.
4. With Memory.

Select all that apply with short justifications.

(d) [**3 marks**] For a CT LTI system, the step response is

1. Running Integral of its impulse response.
2. Derivative of its impulse response.
3. Not related to its impulse response.
4. Moving average of its impulse response.

Give a short explanation.

(e) [**3 marks**] We claim that a DT LTI system is causal if and only if its **step response** is zero for all $n < 0$. Agree or refute with a short argument.

— End of Assignment —