Assignment 2 - Solutions

Q1 (a)

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau-2)\, d\tau$$

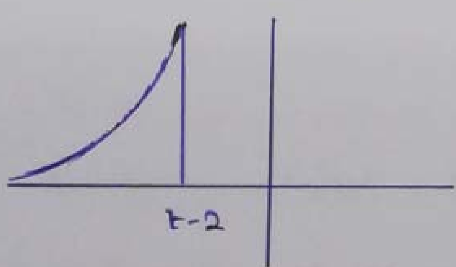
$m = \tau - 2$ new limits $t-2$ and ∞

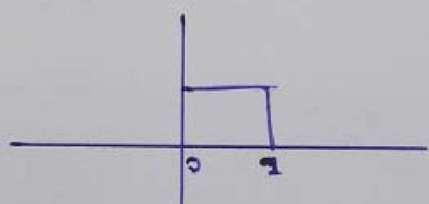
$$\int_{-\infty}^{t-2} e^{-(t-2-m)}\, x(m)\, dm$$

$$\boxed{h(t) = e^{-(t-2)}\, \underbrace{u(t-2)}_{\text{to accomodate the } t-2 \text{ limit instead of } +\infty}}$$

Q1 (b)

$h(t-\tau)$ $x(t) = u(t) - u(t-1)$

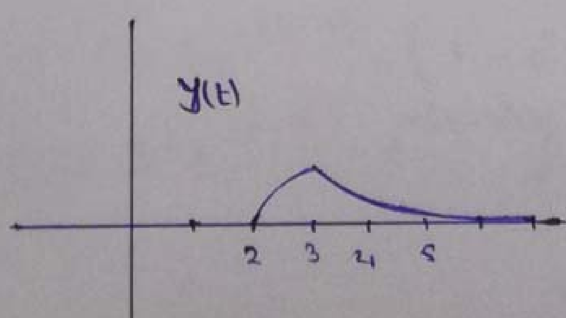




$t < 2$ $y(t) = 0$

$2 \le t \le 3$ $y(t) = \int_{2}^{t} e^{-(t-\tau)}\, d\tau = e^{0} - e^{-(t-2)} = 1 - e^{-(t-2)}$

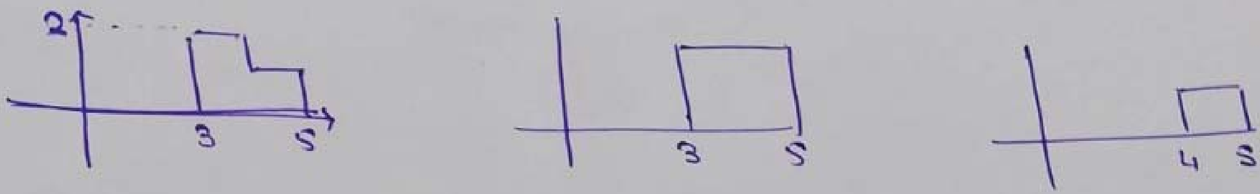
$t > 3$ $y(t) = \int_{2}^{3} e^{-(t-\tau)}\, d\tau = e^{-(t-3)} - e^{-(t-2)}$



Q1(c)

$$x(t) = 2u(t-3) - u(t-4) - u(t-5)$$

$$x(t) = x_1(t) - x_2(t)$$



$$y(t) = x(t) * h(t) = \underbrace{x_1(t) * h(t)}_{y_1(t)} - \underbrace{x_2(t) * h(t)}_{y_2(t)}$$

$y_1(t)$

$t < 3 \quad y_1(t) = 0$

$3 \le t \le 5 \quad y_1(t) = 2 - 2e^{-(t-3)}$

$5 < t \quad y_1(t) = 2\left(e^{-(t-5)} - e^{-(t-3)}\right)$

$y_2(t)$

$t < 4 \quad y_2(t) = 0$

$4 \le t \le 5 \quad y_2(t) = 1 - e^{-(t-4)}$

$5 < t \quad y_2(t) = e^{-(t-5)} - e^{-(t-4)}$

$y(t) = y_1(t) - y_2(t)$

$t < 3 \quad y(t) = 0$

$3 \le t < 4 \quad y(t) = 2 - 2e^{-(t-3)}$

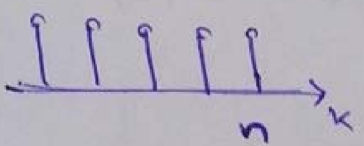
$4 \le t \le 5 \quad y(t) = 2 - 2e^{-(t-3)} - \left(1 - e^{-(t-4)}\right)$

$5 < t \quad y(t) = 2\left(e^{-(t-5)} - e^{-(t-3)}\right) - \left(e^{-(t-5)} - e^{-(t-4)}\right)$

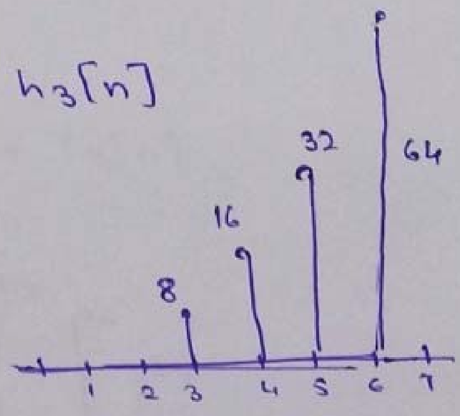
Q2 $y[n] = x[n] * (h_3[n] * (h_1[n] - h_2[n]))$

$$= \underbrace{(x[n] * h_3[n])}_{y_1[n]} * (h_1[n] - h_2[n])$$

$y_1[n] = x[n] * h_3[n]$



$u[m-k]$



$h_3[k]$

$y_1[3] = 8$

$y_1[4] = 8 + 16$

$y_1[5] = 24 + 32$

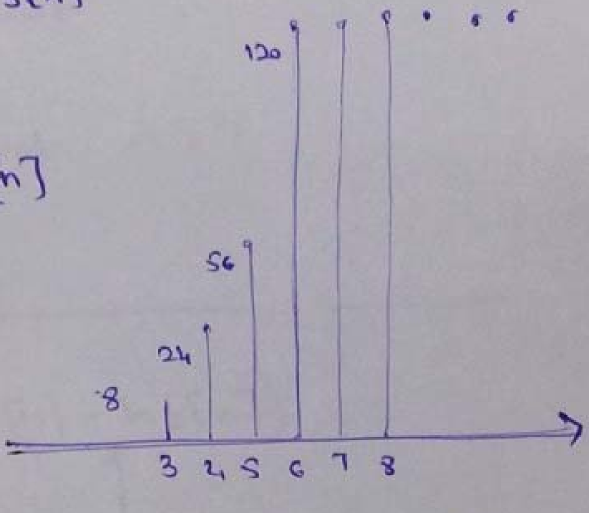
$y_1[6] = 56 + 64$

$y_1[7] = 120$

$y_1[8] = 120$

⋮

$y_1[n]$



$y_1[n] = 0$ for $n < 3$

$y_1[n] = 120$ for $n > 5$

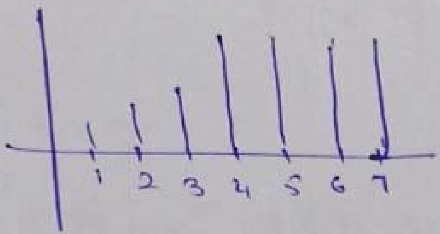
$$y[n] = \underbrace{(y_1[n] * h_1[n])}_{y_2[n]} - \underbrace{(y_1[n] * h_2[n])}_{y_3[n]}$$

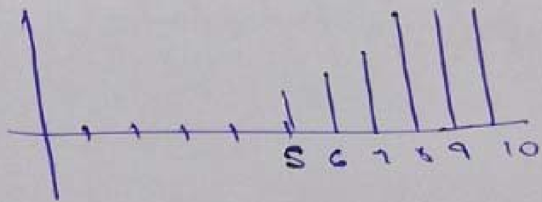
$y_2[n] - y_3[n]$

Convolutions with delta functions result in copying and shifting same signal.

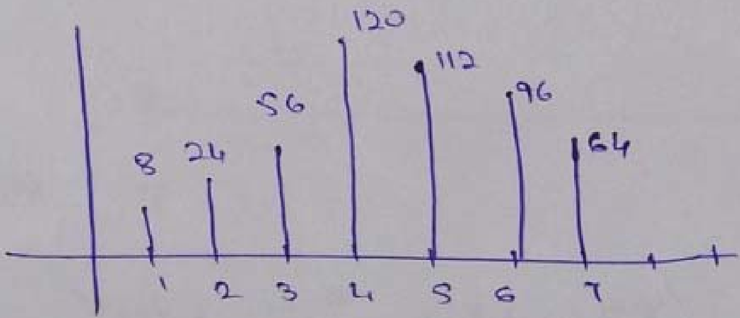
P.T.O →

$y_2[n]$ $-$ $y_3[n]$





$$y[n] = y_2[n] - y_3[n]$$

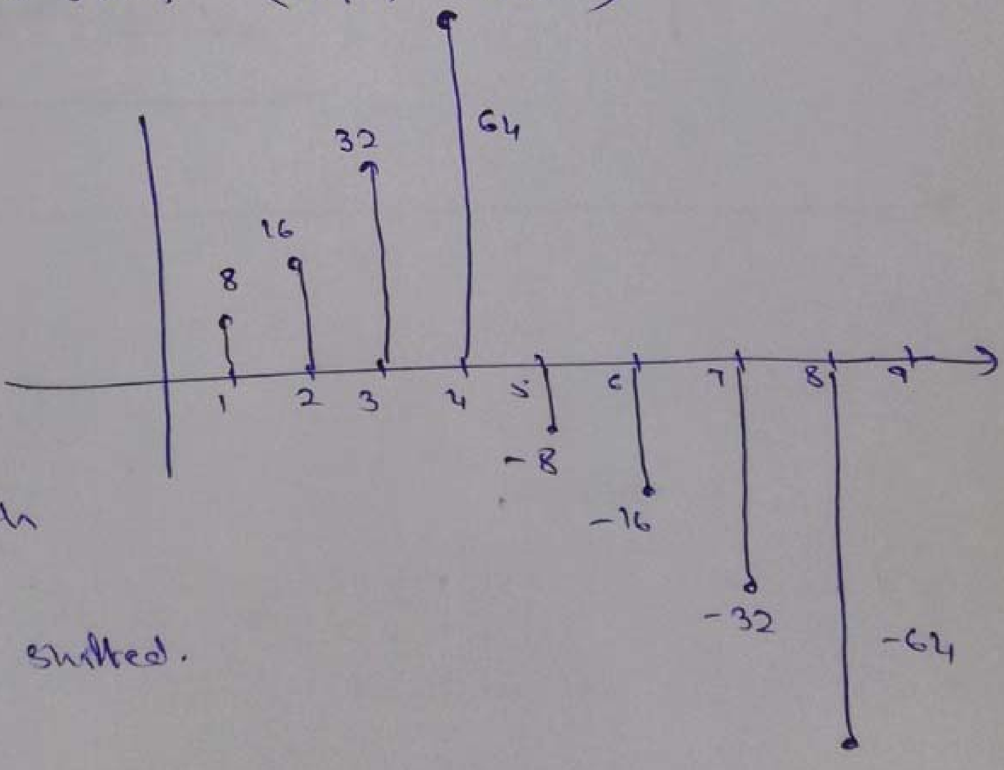


$y[n] = 0 \quad n < 1$ and $n > 7$

~~y[n]~~

(b) $h[n] = h_3[n] * (h_1[n] - h_2[n])$

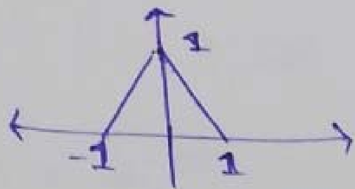
$h[n] =$



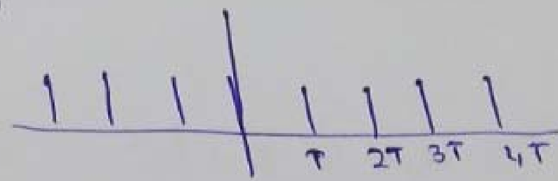
Convolution with Impulses copied and shifted.

Q 3(a)

h(t)



m(t)

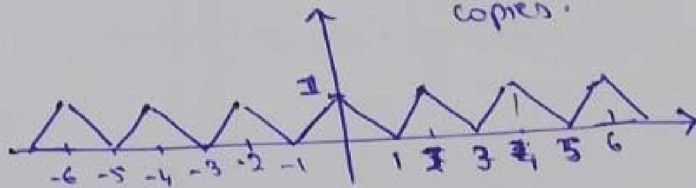


$T = 2$

$$y(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) * h(t)$$

$$= \sum_{k=-\infty}^{\infty} h(t - k\overset{\curvearrowright 2}{T})$$

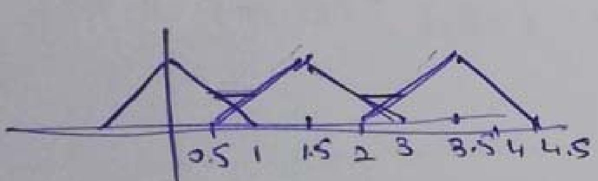
Infinitely many copies.



Q3(b) $T = 1.5$

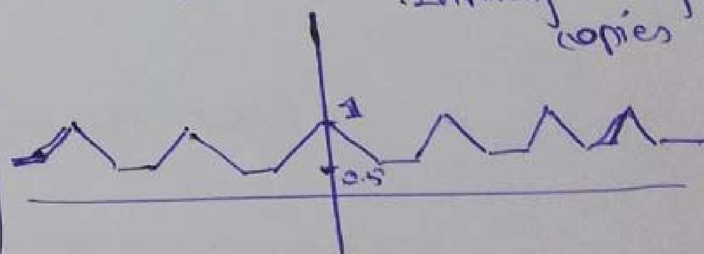
$$Y(t) = \sum_{k=-\infty}^{\infty} h(t - 1.5k)$$

Rough sketch on what's going on



y(t)

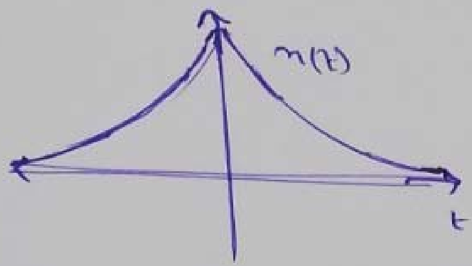
Infinitely many copies

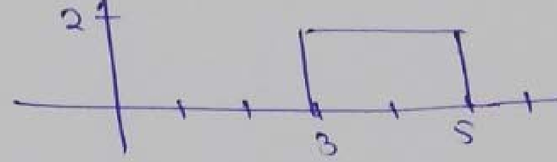


Q4 $y(t) = x(t) * h(t)$

(a)

$x(t) = e^{-|t|}$ $\qquad$ $h(t) = 2[u(t-3) - u(t-5)]$

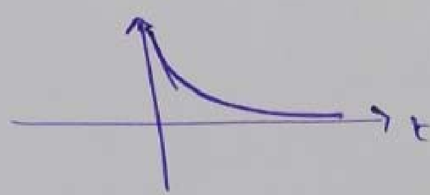


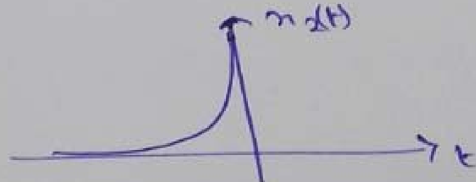


$x(t) = x_1(t) + x_2(t)$

$x_1(t) = e^{-t} u(t)$

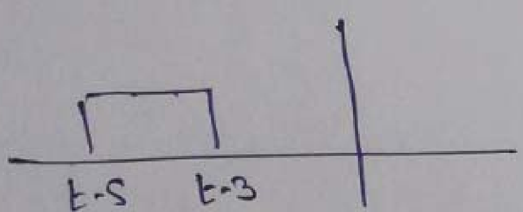
$x_2(t) = e^{t} u(-t)$

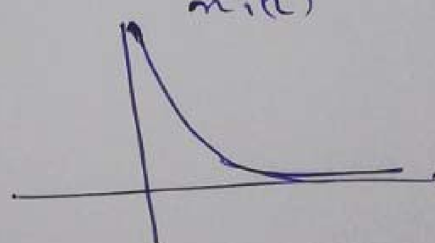




$y(t) = (x_1(t) * h(t)) + (x_2(t) * h(t))$

$h(t-\tau)$ $\qquad$ $x_1(\tau)$





$y_1(t) = x_1(t) * h(t)$

PTO $\longrightarrow$

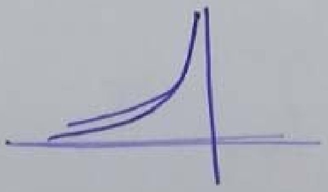
For $t < 3$ $\quad y_1(t) = 0$

For $3 \le t \le 5$ $\quad y_1(t) = 2\left(e^0 - e^{-(t-3)}\right)$

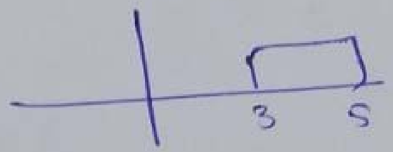
For $5 < t$ $\quad y_1(t) = 2\left(e^{-(t-5)} - e^{-(t-3)}\right)$

$y_2(t) = m_2(t) * h(t)$

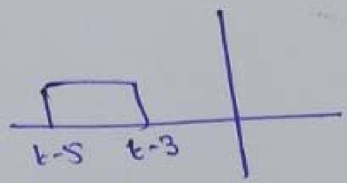
$m_2(t)$



$h(t)$



$h(t-\tau)$



~~for $t < 5$~~

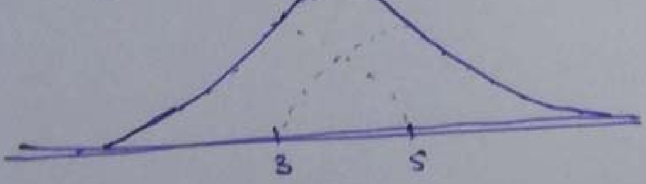
for $t > 5$ $\quad y_2(t) = 0$

for $3 \le t \le 5$ $\quad y_2(t) = 2\left(1 - e^{+(t-5)}\right)$

for $t < 3$ $\quad y_2(t) = 2\left(e^{(t-3)} - e^{(t-5)}\right)$

$y(t) = y_1(t) + y_2(t)$

$y(t)$



Q4 (b)

$$z(t) = x(t) * h'(t)$$

$$h(t) = 2u(t-3) - 2u(t-5)$$

$$h'(t) = 2\delta(t-3) - 2\delta(t-5)$$

$$z(t) = 2\left(x(t) * \delta(t-3) - x(t) * \delta(t-5)\right)$$

$$z(t) = 2e^{-|t-3|} - 2e^{-|t-5|}$$

$$y(t) = \int_{-\infty}^{t} z(\tau)\,d\tau$$

$$y(t) = 2\int_{-\infty}^{t} e^{-|\tau-3|} - e^{-|\tau-5|}\,d\tau$$

$$= 2\int_{-\infty}^{t} e^{-|\tau-3|}\,d\tau - 2\int_{-\infty}^{t} e^{-|\tau-5|}\,d\tau$$

Compute for final form

Q5

(a) we can not decide on the stability of the system based on only the given information.

we need to know about the duration of the signal.

(b) $x(t) = \delta(t-1)$

$x(t) * x^{-1}(t) = \delta(t)$

$x^{-1}(t) = \delta(t+1)$ non causal

(c) a finite duration signal is summable and hence stable.

we cannot however comment on the causality of the system.

(d) 1. Running Integral of its impulse Response. (By definition)

(e) Because the step Response is the Running integral of the impulse Response and since it is nessecary for causality that the impulse Response be $\boxed{\text{zero}}$ for all $n < 0$. The step Response i.e the Runing integral of those Impulse Responses must also be zero.