Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 3

Due Date: 5pm, Friday, Mar 15, 2019 (Dropbox located outside 9-246A, EE Department)
Format: 6 problems, for a total of 100 marks
Instructions:

- You are not allowed to submit a group assignment. Each student must submit his/her own hand-written assignment.
- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. Anybody found guilty would be subjected to disciplinary action in accordance with the university rules and regulations.

Notation:

- In this assignment, the Fourier series coefficients of continuous-time periodic signals $x(t)$, $y(t)$ and $z(t)$ are denoted by a_k, b_k and c_k respectively, that is,

$$x(t) \overset{FS}{\longleftrightarrow} a_k,$$

$$y(t) \overset{FS}{\longleftrightarrow} b_k,$$

$$z(t) \overset{FS}{\longleftrightarrow} c_k,$$

where the shorthand notation $\overset{FS}{\longleftrightarrow}$ is adopted to relate the signal with its Fourier series coefficients.

Problem 1 (20 marks)

In this problem, consider a continuous-time signal $x(t)$ in Fig. 1.

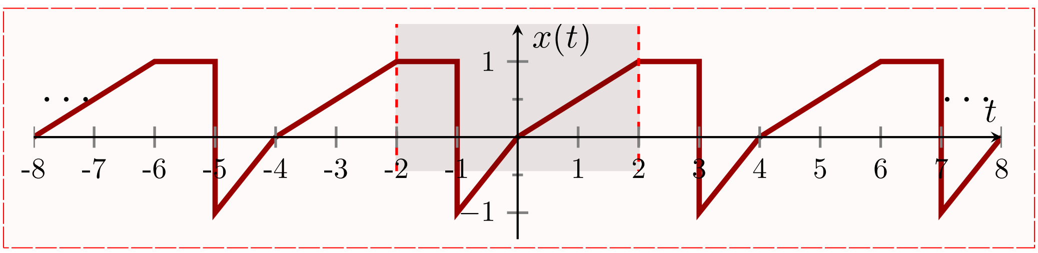

Figure 1: Signal $x(t)$.

(a) **[12 marks]** Determine the Fourier series coefficients of the signal $x(t)$ using the analysis equation.

(b) [**8 marks**] The Fourier series coefficients of periodic Impulse train defined as

$$y(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT),$$

are given by

$$b_k = \frac{1}{T}.$$

Using the Fourier series coefficients b_k of the impulse train $y(t)$, the properties of Fourier series and the result of Example 3.5 (textbook), determine the Fourier series coefficients of the signal $x(t)$.

Problem 2 (15 marks)

(a) [**6 marks**] (Convolution property of Fourier series) Periodic convolution of two signals $x(t)$ and $y(t)$, both periodic with period T, is defined as

$$z(t) = \int_T x(\tau)y(t - \tau)d\tau,$$

where the integral is taken over one period T only. Show that the Fourier coefficients of $z(t)$ are given by

$$c_k = Ta_k b_k.$$

(b) [**9 marks**] Using the result of Example 3.5 (textbook), the convolution property and other properties of Fourier series, determine the Fourier series coefficients of the signal $z(t)$ shown in Fig. 2.

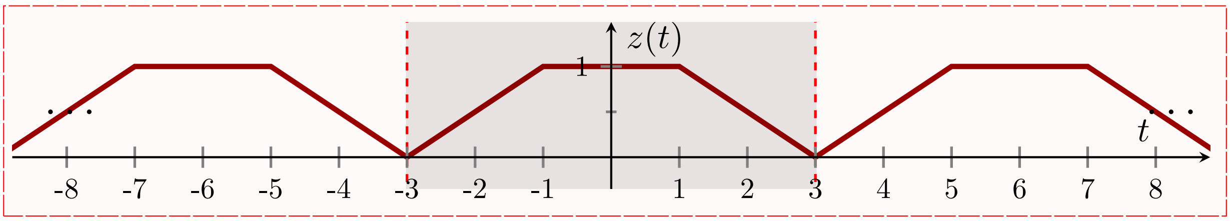

Figure 2: Signal $z(t)$.

Problem 3 (20 marks)

(a) [**7 marks**] (Multiplication property of Fourier series)
Let $x(t)$ and $y(t)$ both be continuous-time periodic signals having period T. Show that the Fourier series coefficients of the signal $z(t) = x(t)y(t)$ are given by

$$c_k = \sum_{\ell=-\infty}^{\infty} a_\ell b_{k-\ell}\,.$$

(b) [**5 marks**] (Parseval's Relation) Using the result of multiplication property, or otherwise, derive the Parseval's relation, which relates the average energy of the signal to its Fourier series coefficients and is given by

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

for a continuous-time periodic signal $x(t)$ having period T.

(c) **[8 marks]** Using the multiplication property of Fourier series, determine the Fourier series coefficients of the signal $h(t)$ with period 4 and description over one period $-1 \le t < 3$ given by

$$h(t) = \begin{cases} e^{-2t}\cos(20\pi t) & -1 \le t \le 1, \\ 0 & 1 < \text{t} < 3. \end{cases} \tag{1}$$

Problem 4 (15 marks)

In each of the following, the Fourier Series coefficients of a continuous time signals with period 4 is specified. Determine the signal $x(t)$ in each case:

(a) $a_k = \begin{cases} 0, & k = 0 \\ (j)^k \dfrac{\sin k\pi/4}{k\pi}, & \text{otherwise} \end{cases}$

(b) $a_k = (-1)^k \dfrac{\sin k\pi/8}{2k\pi}, \quad a_0 = \dfrac{1}{16}$

(c) $a_k = \begin{cases} 1, & k \text{ even} \\ 2, & k \text{ odd} \end{cases}$

Problem 5 (15 marks)

For each of the following discrete-time periodic signals, determine the Fourier series coefficients

(a) **[7 marks]** $x[n] = \sin \frac{2\pi n}{3} \cos \frac{\pi n}{2}$

(b) **[8 marks]** $x[n] = 1 - \sin \frac{\pi n}{4}$ for $0 \le n \le 11$ and periodic with period 12.

Problem 6 (15 marks)

Let $x[n]$ be a periodic signal with period N and the Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}. \tag{2}$$

For each of the following signals, determine the Fourier series coefficients in terms of a_k.

(a) **[2 marks]** $Ax[n] + Bx[-n]$

(b) **[2 marks]** $x[n]e^{jL(2\pi/N)}$

(c) **[1 mark]** $x[n - n_0]$

(d) **[2 marks]** $x[n] + x[n - N/2]$ (assume that N is even)

(e) **[2 marks]** $\big(x[-n]\big)^*$ ($*$ denotes conjugation)

(f) **[3 marks]** $(-1)^n\, x[n]$ (assume that N is even)

(g) **[3 marks]**

$$y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

— End of Assignment —