Problem 1 (20 marks)
In this problem, consider a continuous-time signal \( x(t) \) in Fig. 1.

(a) [12 marks] Determine the Fourier series coefficients of the signal \( x(t) \) using the analysis equation.
(b) [8 marks] The Fourier series coefficients of periodic Impulse train defined as

\[ y(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), \]

are given by

\[ b_k = \frac{1}{T}. \]

Using the Fourier series coefficients \( b_k \) of the impulse train \( y(t) \), the properties of Fourier series and the result of Example 3.5 (textbook), determine the Fourier series coefficients of the signal \( x(t) \).

Problem 2 (15 marks)

(a) [6 marks] (Convolution property of Fourier series) Periodic convolution of two signals \( x(t) \) and \( y(t) \), both periodic with period \( T \), is defined as

\[ z(t) = \int_{T} x(\tau)y(t-\tau)d\tau, \]

where the integral is taken over one period \( T \) only. Show that the Fourier coefficients of \( z(t) \) are given by

\[ c_k = Ta_kb_k. \]

(b) [9 marks] Using the result of Example 3.5 (textbook), the convolution property and other properties of Fourier series, determine the Fourier series coefficients of the signal \( z(t) \) shown in Fig. 2.

![Figure 2: Signal z(t).](image)

Problem 3 (20 marks)

(a) [7 marks] (Multiplication property of Fourier series)

Let \( x(t) \) and \( y(t) \) both be continuous-time periodic signals having period \( T \). Show that the Fourier series coefficients of the signal \( z(t) = x(t)y(t) \) are given by

\[ c_k = \sum_{\ell=-\infty}^{\infty} a_\ell b_{k-\ell}. \]

(b) [5 marks] (Parseval’s Relation) Using the result of multiplication property, or otherwise, derive the Parseval’s relation, which relates the average energy of the signal to its Fourier series coefficients and is given by

\[ \frac{1}{T} \int_{T} |x(t)|^2dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \]

for a continuous-time periodic signal \( x(t) \) having period \( T \).
(c) [8 marks] Using the multiplication property of Fourier series, determine the Fourier series coefficients of the signal $h(t)$ with period 4 and description over one period $-1 \leq t < 3$ given by

$$h(t) = \begin{cases} e^{-2t} \cos(20\pi t) & -1 \leq t \leq 1, \\ 0 & 1 < t < 3. \end{cases} \quad (1)$$

**Problem 4** (15 marks)

In each of the following, the Fourier Series coefficients of a continuous time signals with period 4 is specified. Determine the signal $x(t)$ in each case:

(a) $a_k = \begin{cases} 0, & k = 0 \\ (j)^k \frac{\sin k\pi/4}{k\pi}, & \text{otherwise} \end{cases}$

(b) $a_k = (-1)^k \frac{\sin k\pi/8}{2k\pi}, \quad a_0 = \frac{1}{16}$

(c) $a_k = \begin{cases} 1, & k \text{ even} \\ 2, & k \text{ odd} \end{cases}$

**Problem 5** (15 marks)

For each of the following discrete-time periodic signals, determine the Fourier series coefficients

(a) [7 marks] $x[n] = \sin \frac{2\pi n}{4} \cos \frac{\pi n}{2}$

(b) [8 marks] $x[n] = 1 - \sin \frac{\pi n}{4}$ for $0 \leq n \leq 11$ and periodic with period 12.

**Problem 6** (15 marks)

Let $x[n]$ be a periodic signal with period $N$ and the Fourier series representation

$$x[n] = \sum_{k=-<N>} a_k e^{j(2\pi/N)n}. \quad (2)$$

For each of the following signals, determine the Fourier series coefficients in terms of $a_k$.

(a) [2 marks] $Ax[n] + Bx[-n]$

(b) [2 marks] $x[n]e^{jL(2\pi/N)}$

(c) [1 mark] $x[n - n_0]$

(d) [2 marks] $x[n] + x[n - N/2]$ (assume that $N$ is even )

(e) [2 marks] $(x[-n])^*$ (* denotes conjugation)

(f) [3 marks] $(-1)^n x[n]$ (assume that $N$ is even)

(g) [3 marks]

$$y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$