Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 4

Due Date: 5pm, Monday, Apr. 15, 2019 (Dropbox located outside 9-246A, EE Department)
Format: 9 problems, for a total of 100 marks
Instructions:

- You are not allowed to submit a group assignment. Each student must submit his/her own hand-written assignment.
- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. Anybody found guilty would be subjected to disciplinary action in accordance with the university rules and regulations.

Problem 1 (10 marks)

(Convolution property) The response of an LTI system to an input $x(t)$ is $y(t) = x(t) * h(t)$, where $h(t)$ is the system impulse response.

(a) [**4 marks**] If the Fourier transform of $x(t)$ is $X(j\omega)$ and $h(t)$ is $H(j\omega)$ then show that

$$Y(jw) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(t-\tau)e^{-jwt}d\tau dt.$$

(b) [**6 marks**] By appropriate change of variables, show that

$$Y(jw) = X(j\omega)H(j\omega).$$

Problem 2 (10 marks)

Using the analysis equation find the Fourier transform of the following signals. Also sketch the magnitude and phase as a function of frequency including both positive and negative frequencies.

(a) $x(t) = \delta(t-5)$.

(b) $x(t) = e^{-at}u(t)$. $\quad a$ real, $a > 0$

(c) $x(t) = e^{(-1+2j)t}u(t)$.

Problem 3 (10 marks)

Find the continuous time signal corresponding to the following Fourier transforms.

(a) [**2 marks**]

$$X(j\omega) = 1/(7 + j\omega).$$

(b) [**3 marks**]

$$Y(j\omega) = 2((\delta(\omega - 7) + \delta(\omega + 7))).$$

(c) [**5 marks**]

$$Z(j\omega) = X(j\omega)Y(j\omega),$$

where $X(j\omega)$ and $Y(j\omega)$ are given in parts (a) and (b) respectively.

Problem 4 (10 marks)

The frequency-shift property of Fourier transform is given

$$\mathcal{F}\left\{e^{j\omega_o t}x(t)\right\} = X(j(\omega - \omega_o)$$

(a) [**5 marks**] Prove the frequency shift property by applying frequency shift to the analysis equation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

(b) [**5 marks**] Using the properties of Fourier transform to show by induction that the Fourier transform of

$$x(t) = t^{n-1}/(n-1)!e^{-at}u(t) \qquad a > 0$$

is

$$X(j\omega) = 1/(a + j\omega)^n$$

Problem 5 (10 marks)

Using the duality property of Fourier transform, determine the inverse Fourier transform of $X(j\omega) = u(\omega)$.

Problem 6 (10 marks)

The following differential equation relates the output $y(t)$ of causal continuous LTI system to the input $x(t)$

$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

(a) [**4 marks**] Determine the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$ and sketch the magnitude of $H(j\omega)$.

(b) [**5 marks**] If $x(t) = e^{-t}u(t)$, determine $Y(j\omega)$ and $y(t)$.

Problem 7 (15 marks)

A casual and stable LTI system has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

and frequency domain description:

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

(a) [**6 marks**] Using the property:

$$\mathcal{F}\{\frac{d^k z(t)}{dt^k}\} = (j\omega)^k\, Z(j\omega), \quad k = 0, 1, 2, \ldots$$

where

$$\mathcal{F}\{z(t)\} = Z(j\omega),$$

find the **differential equation** relating the input $x(t)$ (and its derivatives) and output $y(t)$ (and its derivatives) of the system with frequency response $H(j\omega)$.

(b) [**4 marks**] Find the **partial fraction expansion** of $H(j\omega)$.

(c) [**5 marks**] Determine the **impulse response** $h(t)$ corresponding to frequency response $H(j\omega)$.

Problem 8 (10 marks)

(a) [**3 marks**] Consider two LTI systems with frequency response (Fourier transform of the impulse response) denoted by $H_1(j\omega)$ and $H_2(j\omega)$. If the two systems are inverse of one another, establish the relationship between $H_1(j\omega)$ and $H_2(j\omega)$.

(b) [**3 marks**] Consider an LTI system (Ideal Band-Pass Filter) with frequency response given by

$$H(j\omega) = \begin{cases} 1, & 2 \le |\omega| \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

Determine if we can find the inverse of this system. If yes, find the impulse response of the inverse system. If no, provide the justification.

(c) [**4 marks**] Determine the inverse Fourier transform of $H(j\omega)$ given in part (b). Determine whether $h(t)$ is real or not!

Problem 9 (15 marks)

(a) [**2 marks**] Using duality and the fact that the Fourier transform of $\delta(t+5)$ is $e^{j5\omega}$, determine the Fourier transform of e^{j5t}.

(b) [**5 marks**] Suppose that the signal $x(t) = e^{j5t}\cos(2\pi 1000t)$ is transmitted over a wireless channel. Determine and sketch the Fourier transform of $x(t)$. For your information, e^{j5t} is referred to as message signal and $\cos(2\pi 1000t)$ is referred to as the carrier signal. (You may use multiplication property.)

(c) [**5 marks**] Suppose that the receiver receives signal $x(t)$ of part (b) and multiplies it with the carrier to obtain a signal $y(t) = x(t)\cos(2\pi 1000t)$. Determine and sketch the Fourier transform of $y(t)$.

(d) [**3 marks**] The receiver then passes the signal $y(t)$ through an LTI system of impulse response $h(t)$ to recover the message signal e^{j5t}. Determine the impulse response $h(t)$.

— End of Assignment —