Department of Electrical Engineering
School of Science and Engineering

EE310 Signals and Systems

ASSIGNMENT 5

Due Date: 5pm, Monday, Apr. 29, 2019 (Dropbox located outside 9-246A, EE Department)
Format: 8 problems, for a total of 125 marks
Instructions:

- You are not allowed to submit a group assignment. Each student must submit his/her own hand-written assignment.
- You are allowed to collaborate with your peers but copying your colleague's solution is strictly prohibited. Anybody found guilty would be subjected to disciplinary action in accordance with the university rules and regulations.

Problem 1 (15 marks)

Determine the Fourier transform of the following discrete-time signals. Also plot the magnitude of the Fourier transform for $-3\pi \leq 3\pi$.

(a) **[4 marks]** $\alpha^{-|n|}$

(b) **[3 marks]** $\cos(2\pi n/3) + 3$

(c) **[3 marks]** $\left(\frac{1}{2}\right)^n u[n]$

(d) **[5 marks]** $(n+1)\left(\frac{1}{4}\right)^n u[n]$

Problem 2 (15 marks)

Consider a causal LTI system described by the difference equation,

$$y[n] = x[n] - \frac{1}{2}y[n-1]$$

(a) **[4 marks]** Determine the frequency response $H(e^{j\omega})$ of this system.

(b) **[5 marks]** Determine the response $y[n]$ of the given system to the following input:

$$x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$$

(c) **[6 marks]** Determine the response $y[n]$ to the input $x[n]$ described by the following Fourier transform:

$$X(e^{j\omega}) = \frac{1}{(1 - \frac{1}{4}e^{-j\omega})(1 + \frac{1}{2}e^{-j\omega})}$$

Problem 3 (25 marks)

(a) **[10 marks]** The impulse response of a discrete-time LTI system is given by,

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Using the answers of Problem 1 or otherwise, find the response to each of the following inputs signals:
(i) $x[n] = (\frac{3}{4})^n u[n]$
(ii) $x[n] = (n+1)(\frac{1}{4})^n u[n]$
(iii) $x[n] = (-1)^n$

(b) **[10 marks]** Consider a discrete-time LTI system with impulse response given by,

$$h[n] = (\frac{1}{2})^n \cos(\frac{\pi n}{2}) u[n]$$

Find the response to each of the following input signals:
(i) $x[n] = (\frac{1}{2})^n u[n]$
(ii) $x[n] = \cos(\pi n/2)$

(c) **[5 marks]** Let $x[n]$ and $h[n]$ be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-3j\omega}$$

$$H(e^{j\omega}) = -e^{j\omega} + e^{j4\omega} + 2e^{-2j\omega}$$

Determine $y[n] = x[n] * h[n]$.

Problem 4 (10 marks)

Consider a discrete-time LTI system with input $x[n]$ and output $y[n]$, where the Fourier transforms of these signals are related by the following equation,

$$Y(e^{j\omega}) = 2X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) - \frac{dX(e^{j\omega})}{d\omega}$$

(a) **[2 marks]** Is the system linear? Clearly justify your answer.

(b) **[2 marks]** Is the system time invariant? Clearly justify your answer.

(c) **[6 marks]** Determine the impulse response the system.

Problem 5 (5 marks)

Consider a discrete-time LTI system for which the Fourier transform $Y(e^{j\omega})$ of the output $y[n]$ is related to the Fourier transform $X(e^{j\omega})$ of the input $x[n]$ through the relation,

$$Y(e^{j\omega}) = \int_{\omega-\frac{\pi}{4}}^{\omega+\frac{\pi}{4}} X(e^{j\omega}) d\omega$$

Express $y[n]$ in terms of $x[n]$.

Problem 6 (10 marks)

Suppose there are two discrete-time LTI systems connected in cascade(series). The frequency response of these LTI systems are given below,

$$H_1(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

and

$$H_2(j\omega) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}$$

(a) [**4 marks**] Find the difference equation describing the overall system.

(b) [**6 marks**] Determine the impulse response of the overall system.

Problems related to Continuous-time Fourier Transform

Problem 7 (20 marks)

Let $x(t)$ be a continuous time signal whose Fourier transform $X(j\omega)$ is given by

$$X(j\omega) = \delta(\omega) + \frac{1}{2}\,\delta(\omega - \frac{\pi}{2}) + 6\,\delta(\omega - 2) - 2\,\delta(\omega - \frac{\pi}{3}) - 3\,\delta(\omega - 3),$$

and let $h(t)$ be another continuous time signal given by

$$h(t) = u(t+\alpha) - u(t-\alpha), \quad \text{for Real } \alpha.$$

(a) [**4 marks**] Prove that $x(t)$ is aperiodic?

(b) [**4 marks**] We know that the periodicity in one (time/frequency) domain implies discreteness in other (freqeuncy/time) domain. Here, the frequency domain description of the signal contains impulses (discrete) but the signal in the time-domain is aperiodic. Under what conditions, the impulses in the frequency domain correspond to the periodic time-domain signal?

(c) [**4 marks**] Determine and sketch $H(j\omega)$ with proper labeling.

(d) [**4 marks**] Determine the *smallest* value of α which makes $h(t) \star x(t)$ periodic with period $T = 12$.

(e) [**4 marks**] Determine the *smallest* value of α which makes $h(t) \star x(t)$ periodic with period $T = 2\pi$.

Problem 8 (25 marks)

Consider a continuous-time signal $x(t)$, with Fourier transform $X(j\omega)$, shown in Figure 1

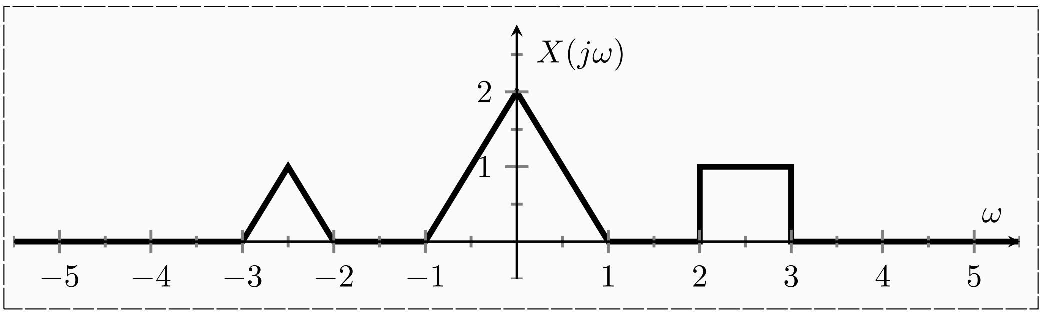

Figure 1: The Fourier transform $X(j\omega)$ of some signal $x(t)$ (Question 4, part (a) and (b)).

(a) [**2 marks**] What is the maximum frequency component, ω_M in radians/sec, of the signal $x(t)$ such that $X(j\omega) = 0$, $\forall |\omega| > \omega_M$?

(b) [**3 marks**] Let $x_p(t)$ be the sampled version of $x(t)$ given by:

$$x_p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT).$$

where T is the sampling interval. Also let $\omega_s = \frac{2\pi}{T}$ is the sampling frequency (in radians/sec). Find the expression of $X_p(j\omega)$ in terms of $X(j\omega)$.

(c) [**5 marks**] Plot, with proper labeling, $X_p(j\omega)$ for each $T = \pi/6$, $T = \pi/3$ and $T = \pi/2$ over the frequency range $|\omega| \le 15$.

(d) **[5 marks]** Determine T and ω_s such that the signal $x(t)$ can be recovered from the sampled signal $x_p(t)$ by passing the sampled signal $x_p(t)$ through the ideal low-pass filter. We refer to such maximum value of T as Nqyuist Sampling Time and the corresponding minimum value of the frequency ω_s as Nyquist sampling frequency or Nyqyuist rate.

(e) **[4 marks]** The signal $x_p(t)$ sampled at Nyquist rate is passed through an ideal low-pass filter (LTI system) of cut-off frequency w_c. Determine the impulse response of the low-pass filter such that the output of the filter is $x(t)$ (original signal).

(f) **[6 marks]** Here we assume that we can design a filter using the combination of low-pass filters and band-bass filter in series or parallel to recover the signal $x(t)$ from $x_p(t)$. Determine the minimum value of the sampling frequency ω_s and design the filter (LTI system as combination of band-pass and low-pass filters) such the $x(t)$ can be recovered from $x_p(t)$.

— End of Assignment —